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KINEMATIC TRANSFORMATIONS OF SEVERAL MECHANISMS

by David W. Lewis

Prepared by
UNIVERSITY OF VIRGINIA
Charlottesville, Va.

for

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CODE:
DISPL.
VEL.
ACC.

INTRODUCTION

INTRODUCTION

This work presents a selection of kinematic information, on several different mechanisms, in graphical form. Hopefully, this will provide the means for "getting a feeling" for these several mechanisms and aid in the selection of a mechanism to obtain a specific input-output motion transformation.

The considerable range in form of the motion transformations presented here indicate an almost unlimited scope of possible transformations. It is expected that this information will afford the design engineer a means for solving many kinematic problems by selecting the appropriate mechanism directly from this compilation. In addition, minor changes to the parameters (link lengths or gear diameters), which determine the motion transformation of a particular mechanism, may produce more exact solutions to specific kinematic problems.

Kinematic solutions of precision better than can be obtained from the graphs are possible using a variety of numerical approaches. These are particularly suitable when used with a high speed digital computer. Several different means have been employed successfully including random selection of parameters (with limited ranges specified), systematic searches based on a specified grid size, and techniques that are local linearizations of an error function.

The graphs were produced "off line" by an electro-mechanical plotter from an ALGOL program run on a Burrough's B5000 computer. The considerable programming efforts were by Mr. Chang-Shi Lu whose work is hereby warmly acknowledged. This work would not have been possible without the financial support of a National Aeronautics and Space Administration Institutional Grant to the University of Virginia.

Considerable detail is presented including the manipulations of units. This should make the work of more value to those who do not count themselves experts in the field of kinematics. Hopefully, the superfluous detail will be skimmed over by those more familiar with the area.

The seven mechanisms considered here were selected arbitrarily. They do represent a rather broad scope of complexity ranging from a modified four-bar mechanism and a form of the slider-crank mechanism through a multiple-input mechanism. As might be anticipated, the forms of the motion transformations obtainable from these seven mechanisms range from the rather simple nearly sine-shaped form to extremely intricate forms. The spectrum of transformations offers almost untold possible applications. With the more complex forms, a great deal more study seems necessary before a real appreciation and widespread application is to be expected.

The pagination provides a cross linking between the graphical presentation and the underlying theory. For example, Plate 3-6 is the 6^{th} graph presented for Mechanism #3. The theoretical information for this particular mechanism can be found in the section of this work under the pages 3-1, 3-2, et cetera.

The several mechanisms may be considered singly so that there is no reason to begin reading at Mechanism #1 if it appears that Mechanism #4 is one suitable for your needs.

One general observation applies to all of the mechanisms. As a means of minimizing the number of identifiers on the figures letters are used for both identification of a link and the length of a link. For example, the input link of Mechanism #1 is referred to as link A (whose angular position with respect to the ground is given by θ in Figure 0-1) and the length of the link is also denoted by the letter A. Also, the sign convention for all of the mechanisms is the same with the direction of positive velocity and positive acceleration being the same as that for the indicated displacement (positive).

An example in calculating the velocity and acceleration of a mechanism may be instructive. Suppose that one wishes to know the greatest linear velocity of the slider of Mechanism #1 for the proportions given on Plate 1-1 ($A = 1.00$, $B = 4.50$, $C = 4.50$, et cetera) with link A turning at a constant speed of 100 rpm. Assume that the link lengths are given in the units inches. The greatest velocity (largest magnitude, disregarding sign) may be calculated

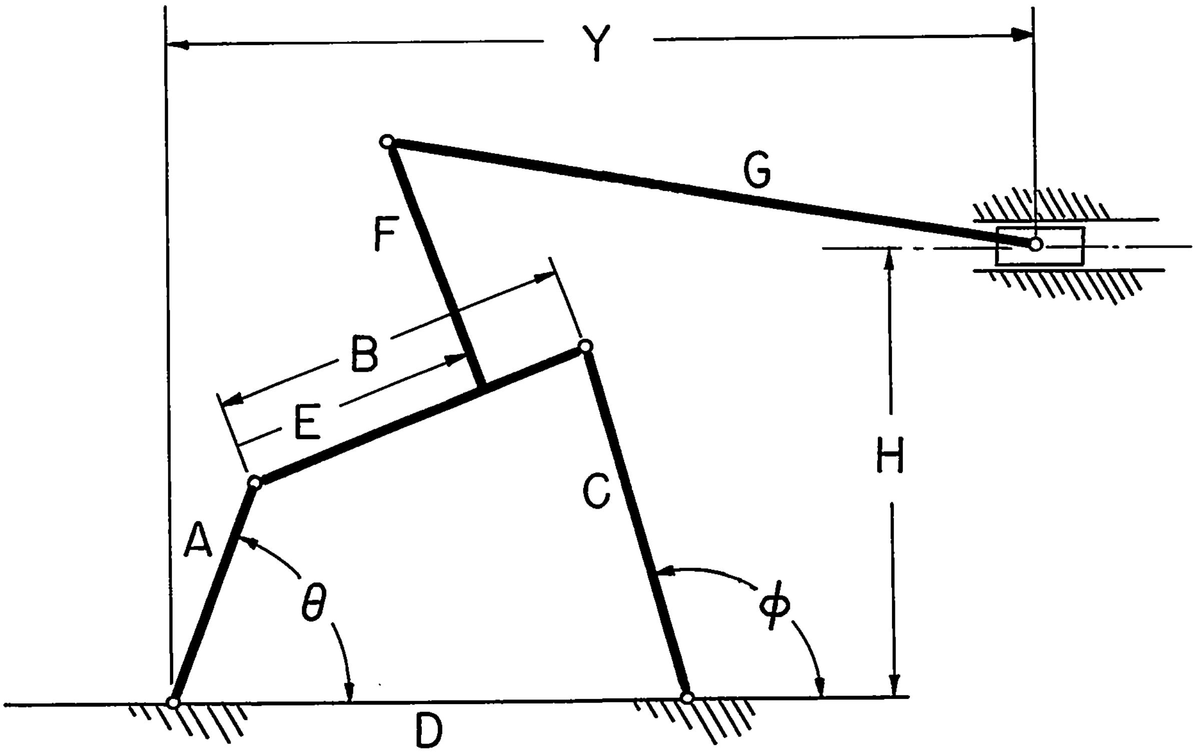


Figure 0-1

by first noting that the title shows VEL. MIN = -0.57. This is the largest (in magnitude) value that $dY/d\theta$ takes on. The largest positive value for the velocity is also shown on the title (VEL. MAX = 0.34) but in this case the magnitude of VEL. MIN is larger than that of the VEL. MAX. Using VEL. MIN in Eq. 1-2

$$\frac{dY}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{dY}{d\theta}$$

$$= \frac{\pi}{30} \times 100 \times (-0.57)$$

$$= -5.97 \text{ inches/second .}$$

The minus sign indicates that the velocity is toward the left (when looking at Figure 0-1). This maximum velocity occurs, scaling off of Plate 1-1, at $\theta - 165 = 56$. The complete graph has been shifted by the amount of 165 degrees (indicated by the abscissa title) causing the maximum displacement to appear on the graph at the left ordinate intersect. So the maximum (in magnitude) velocity occurs at

$$\theta = 56 + 165$$

$$= 221 \text{ degrees.}$$

For this same mechanism and with the same dimensions (in inches) of Plate 1-1, imagine that one wishes to determine the maximum positive acceleration of the slider (that is, the maximum acceleration of the slider toward the right as pictured in Figure 0-1). From the title of the graph, ACC. MAX = 0. 65. The position of this maximum positive value of acceleration is at approximately 112 degrees on the graph (corresponding to $\theta = 165 + 112 = 277$ degrees). This 0. 65 is the maximum positive value of $d^2Y/d\theta^2$. The maximum positive linear acceleration may be computed with Equation 1-4 as:

$$\frac{d^2Y}{dt^2} = \left[\frac{d^2Y}{d\theta^2}\right] \left[\frac{\pi}{30} \times \text{rpm}\right]^2$$

$$= \left[0.65\right] \left[\frac{\pi}{30} \times 100\right]^2$$

$$= 71.3 \frac{\text{inches}}{\text{second}^2} .$$

Positive acceleration indicates an acceleration of the slider toward the right as pictured in Figure 0-1.

MECHANISM #1

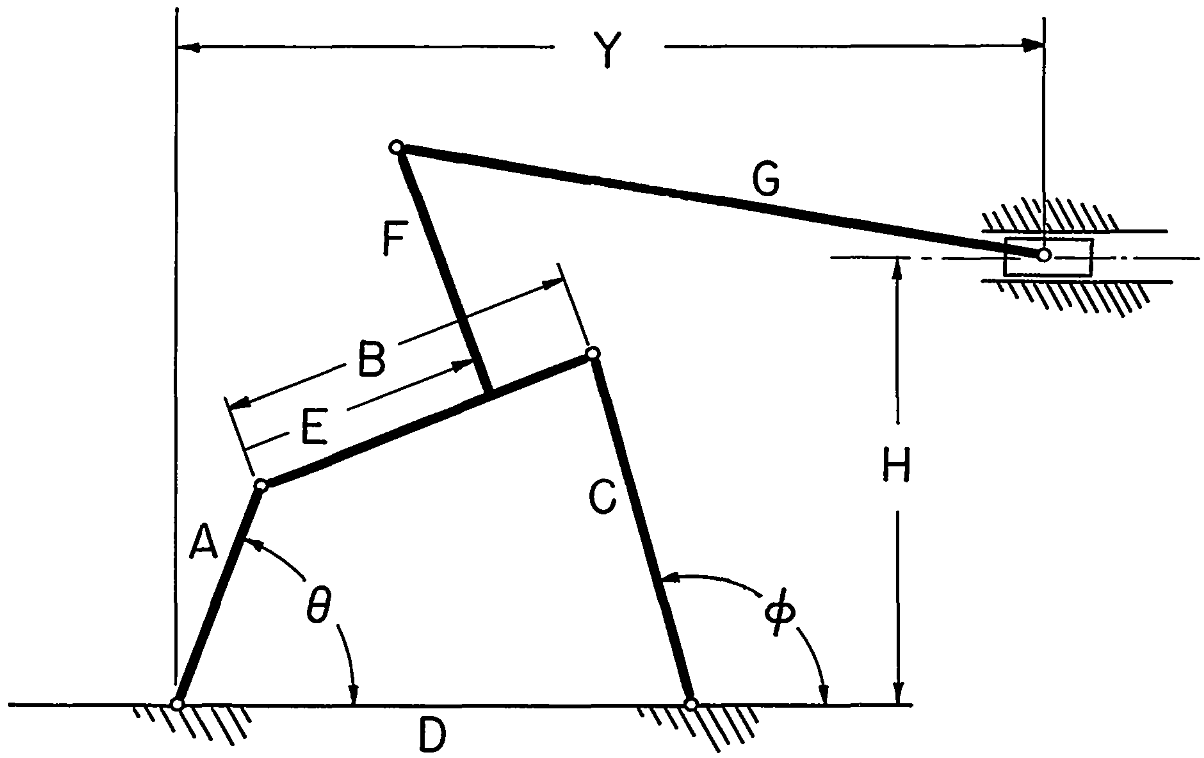


Figure 1-1

Figure 1-1 defines Mechanism #1. It is a four-bar mechanism with a slider linked to a coupler point. The input for the mechanism is the angular position, theta, of link A; the output is the linear position of the slider, Y. Upon specifying values for A through H, the output, Y, may be determined for any value of the input, θ. The dimensions E and F define the "coupler point" for the four-bar portion of this mechanism.

Each of the graphs for this mechanism shows Y versus θ as a solid line, the derivative of Y with respect to θ versus θ, as a dashed line, and the second derivative of Y with respect to θ versus θ as a series of small triangles. Each curve begins with the maximum displacement Y. This has been accomplished by shifting the θ axis. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. The variable θ is presented in the units degrees. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement.

Scales have not been presented for the derivatives but each graph heading includes the maximum and minimum of both the velocity and acceleration. The units for displacement, velocity, and acceleration will correspond to that chosen for the quantities A, B, C et cetera. For example, if the link lengths are specified in inches then the velocity, $dY/d\theta$, will be in the unit inches per radian. A more conventional engineering unit for velocity may be obtained as:

$$\frac{dY}{dt} = \frac{dY}{d\theta} \times \frac{d\theta}{dt} \qquad (1\text{-}1)$$

in which

$$\frac{d\theta}{dt} = \text{rpm} \times 2\pi \times \frac{1}{60} \left(\frac{\text{rad}}{\text{sec}} = \frac{\text{rev}}{\text{min}} \times \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{\text{sec}} \right).$$

With this modification Eq. 1-1 may be rewritten:

$$\frac{dY}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{dY}{d\theta}, \quad \frac{\text{inches}}{\text{second}}. \qquad (1\text{-}2)$$

Expressed in words, Eq. 1-2 indicates that the linear velocity of the slider (inches/ second) is obtained by multiplying the angular speed of link A (revolutions per minute) by $\pi/30$ and then multiplying this product by $dY/d\theta$ (inches/ radian). Values for this latter term may be obtained from a graph (the dashed line), or the extreme values may be obtained from the heading of a graph (VEL. MAX or VEL. MIN), or from the equations which follow.

The acceleration of the slider may be written:

$$\frac{d^2Y}{dt^2} = \frac{d}{dt}\left[\frac{dY}{dt}\right]$$

$$= \frac{d}{dt}\left[\frac{dY}{d\theta} \times \frac{d\theta}{dt}\right]$$

$$= \frac{d}{d\theta}\left[\frac{dY}{d\theta}\right]\left[\frac{d\theta}{dt}\right]^2 + \left[\frac{dY}{d\theta}\right]\left[\frac{d^2\theta}{dt^2}\right]$$

$$= \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{d\theta}{dt}\right]^2 + \left[\frac{dY}{d\theta}\right]\left[\frac{d^2\theta}{dt^2}\right]. \qquad (1\text{-}3)$$

If the angular speed of link A remains constant then the angular acceleration, $d^2\theta/dt^2$, is zero. The expression for the linear acceleration of the slider, with link A turning with constant speed, simplifies to:

$$\frac{d^2Y}{dt^2} = \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{d\theta}{dt}\right]^2$$

$$= \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{\pi}{30} \times \text{rpm}\right]^2, \quad \frac{\text{inches}}{\text{second}^2}. \qquad (1\text{-}4)$$

Values for $d^2Y/d\theta^2$ may be obtained from a graph (the series of small triangles), or the extreme values may be obtained from the heading of a graph (ACC. MAX or ACC. MIN), or from the equations which follow.

Referring to Figure 1-2 for this mechanism the equations relating the output, Y, to the input, θ, may be derived. The four-bar portion of this mechanism is defined by the links A, B, C, and D. Link D is considered as the fixed or ground link. Projecting this part of the mechanism onto first a horizontal line and then a vertical line will yield:

$$B\cos\gamma = D - A\cos\theta + C\cos\phi \qquad (1\text{-}5)$$

$$B\sin\gamma = C\sin\phi - A\sin\theta \qquad (1\text{-}6)$$

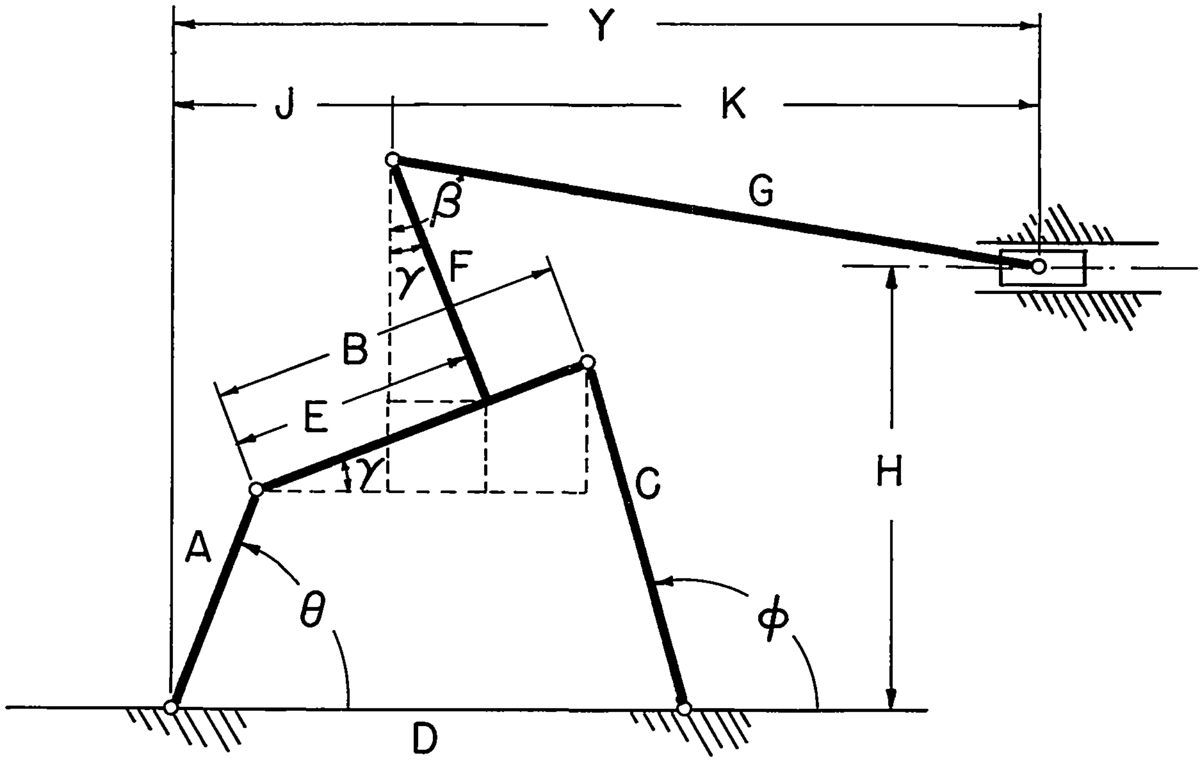


Figure 1-2

in which ϕ, the Greek letter phi, defines the angular position of link C. Squaring each of these equations and then adding the resulting equations together will produce:

$$A^2 - B^2 + C^2 - 2AC \cos(\phi - \theta) + D^2$$

$$+ 2D (C\cos\phi - A\cos\theta) = 0. \qquad (1\text{-}7)$$

Using the Newton-Raphson's method, ϕ may be solved from this equation for given values of θ, A, B, C, and D. Note that ϕ may be solved for in explicit form instead of by a numerical technique from Eq. 1-7.

Regardless of the means, once ϕ is determined the values for sin γ and cos γ may be obtained from Eqs. 1-5 and 1-6. From Figure 1-2:

$$K = G \sin \beta$$

$$J = A\cos \theta + E\cos \gamma - F\sin \gamma. \tag{1-8}$$

And, also from the figure:

$$G \cos \beta + H = A \sin \theta + E \sin \gamma + F \cos \gamma. \tag{1-9}$$

Eq. 1-9 contains the single unknown β which may be determined therefrom. This in turn permits J and K to be calculated and Y determined as:

$$Y = J + K$$

$$= A \cos \theta + E \cos \gamma - F \sin \gamma + G \sin \beta. \tag{1-10}$$

The several graphs for this mechanism follow from Eq. 1-10, Y versus θ being plotted as a solid line.

Linear Velocity

By means of Eq. 1-2, the linear velocity of the slider may be determined upon knowing the value of $dY/d\theta$. This latter term may be written by differentiating Eq. 1-10 with respect to θ as:

$$\frac{dY}{d\theta} = - A \sin \theta - E \sin \gamma \frac{d\gamma}{d\theta} - F \cos \gamma \frac{d\gamma}{d\theta} + G \cos \beta \frac{d\beta}{d\theta}. \tag{1-11}$$

An expression for $d\gamma/d\theta$ may be obtained from the second of Eq. 1-6 by differentiation with respect to θ:

$$B \cos \gamma \frac{d\gamma}{d\theta} = C \cos \phi \frac{d\phi}{d\theta} - A \cos \theta$$

or

$$\frac{d\gamma}{d\theta} = \frac{C \cos \phi \dfrac{d\phi}{d\theta} - A \cos \theta}{B \cos \gamma} . \tag{1-12}$$

This equation involves $d\phi/d\theta$ which may be obtained by differentiating Eq. 1-7 with respect to θ producing:

$$\frac{d\phi}{d\theta} = \frac{A\,[C \sin (\phi - \theta) - D \sin \theta]}{C\,[A \sin (\phi - \theta) - D \sin \phi]} . \tag{1-13}$$

To employ Eq. 1-11 requires an expression for $d\beta/d\theta$ which may be arrived at by differentiating Eq. 1-9 with respect to θ:

$$\frac{d\beta}{d\theta} = \frac{A \cos \theta + (E \cos \gamma - F \sin \gamma) \dfrac{d\gamma}{d\theta}}{- G \sin \beta} . \tag{1-14}$$

With Eqs. 1-12, 1-13, and 1-14 the expression for $dY/d\theta$, may be calculated. From Eq. 1-11 and in turn Eq. 1-2, the value for the linear velocity of the slider is obtained. The dashed line of each graph presents $dY/d\theta$ which may be converted into the more common unit of velocity by Eq. 1-2.

Linear Acceleration

In order to determine the linear acceleration of the slider depicted in Figure 1-1, Eq. 1-4 indicates that $d^2Y/d\theta^2$ must be known. This term may be evaluated by differentiating Eq. 1-11 with respect to θ. Doing so yields:

$$\frac{d^2Y}{d\theta^2} = - A \cos \theta - (E \sin \gamma + F \cos \gamma) \frac{d^2\gamma}{d\theta^2} + (F \sin \gamma - E \cos \gamma) \left[\frac{d\gamma}{d\theta}\right]^2$$
$$+ G \cos \beta \frac{d^2\beta}{d\theta^2} - G \sin \beta \left[\frac{d\beta}{d\theta}\right]^2 . \tag{1-15}$$

To evaluate Eq. 1-15 requires knowledge of $d^2\gamma/d\theta^2$ and $d^2\beta/d\theta^2$. The former term may be derived from Eq. 1-12 by differentiation with respect to θ:

$$\frac{d^2\gamma}{d\theta^2} = \left\{ B \sin\gamma \left[\frac{d\gamma}{d\theta}\right]^2 + C \cos\phi \, \frac{d^2\phi}{d\theta^2} + A \sin\theta - C \sin\phi \left[\frac{d\phi}{d\theta}\right]^2 \right\} \Big/ (B \cos\gamma). \tag{1-16}$$

By differentiating Eq. 1-14 with respect to θ, the following is produced:

$$\frac{d^2\beta}{d\theta^2} = \left\{ G \cos\beta \left[\frac{d\beta}{d\theta}\right]^2 - A \sin\theta + (E \cos\gamma - F \sin\gamma) \frac{d^2\gamma}{d\theta^2} - (E \sin\gamma + F \cos\gamma) \left[\frac{d\gamma}{d\theta}\right]^2 \right\} \Big/ (- G \sin\beta) . \tag{1-17}$$

These last two equations in addition to previously derived equations permit $d^2Y/d\theta^2$ to be evaluated in Eq. 1-15. Curves of $d^2Y/d\theta^2$ are presented as a series of small triangles on the graphs for this mechanism. Numerical values for the linear acceleration of the slider may be determined via Eq. 1-4. Of course, Eq. 1-4 is based on the assumption that the angular velocity of the input, link A, is constant and if this is not so, then Eq. 1-3 instead of Eq. 1-4 must be used.

A= 1.00, B= 4.50, C= 4.50, D= 7.00,
E= 4.00, F= 6.00, G= 7.00, H= 3.00,
VEL.MAX= 0.34, VEL.MIN= -0.57, ACC.MAX= 0.65, ACC.MIN= -0.82,
PEAK TO PEAK DISPLACEMENT = 0.78

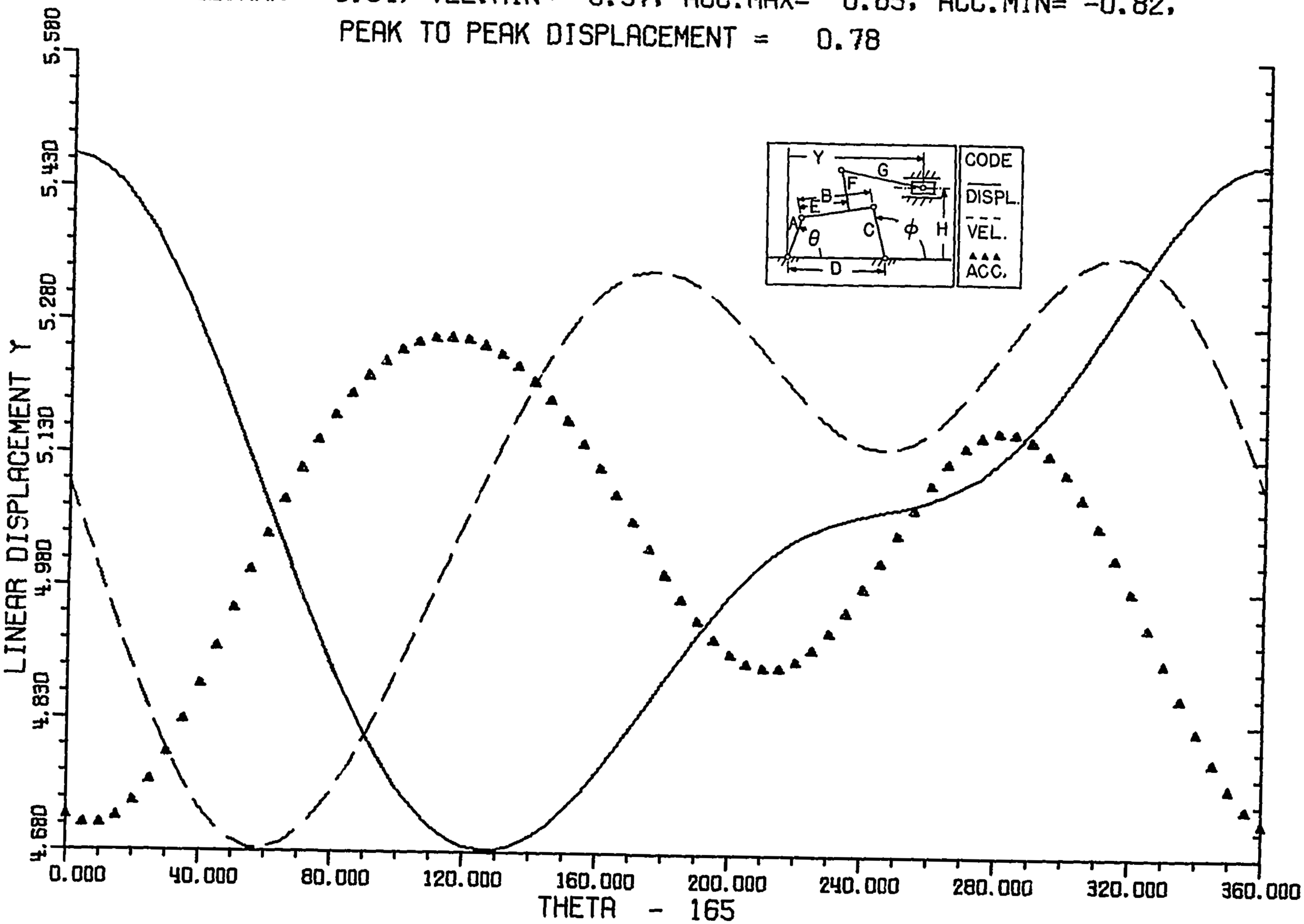
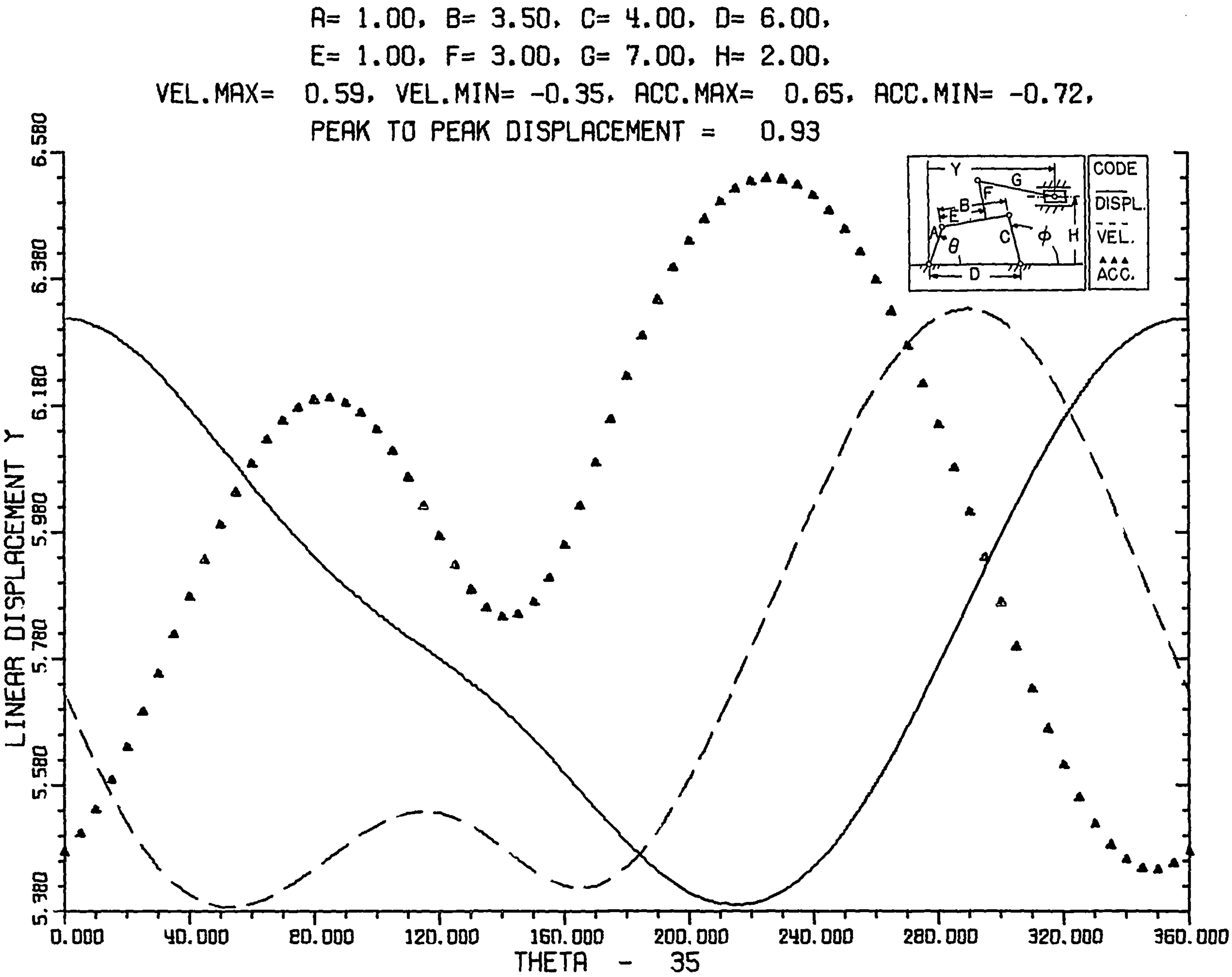


PLATE 1-1

PLATE I-2



A= 1.00, B= 3.50, C= 4.00, D= 6.00,
E= 1.00, F= 3.00, G= 7.00, H= 2.00,
VEL.MAX= 0.59, VEL.MIN= -0.35, ACC.MAX= 0.65, ACC.MIN= -0.72,
PEAK TO PEAK DISPLACEMENT = 0.93
LINEAR DISPLACEMENT Y
6.580
6.380
6.180
5.980
5.780
5.580
5.380
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 35
Y
G
B
F
E
A
C
θ
φ
H
D
CODE
DISPL.
VEL.
ACC.

A= 1.00, B= 1.50, C=10.00, D=10.00,
E= 1.00, F= 4.00, G= 7.00, H= 2.00,
VEL.MAX= 0.78, VEL.MIN= -0.81, ACC.MAX= 1.19, ACC.MIN= -1.55,
PEAK TO PEAK DISPLACEMENT = 1.09

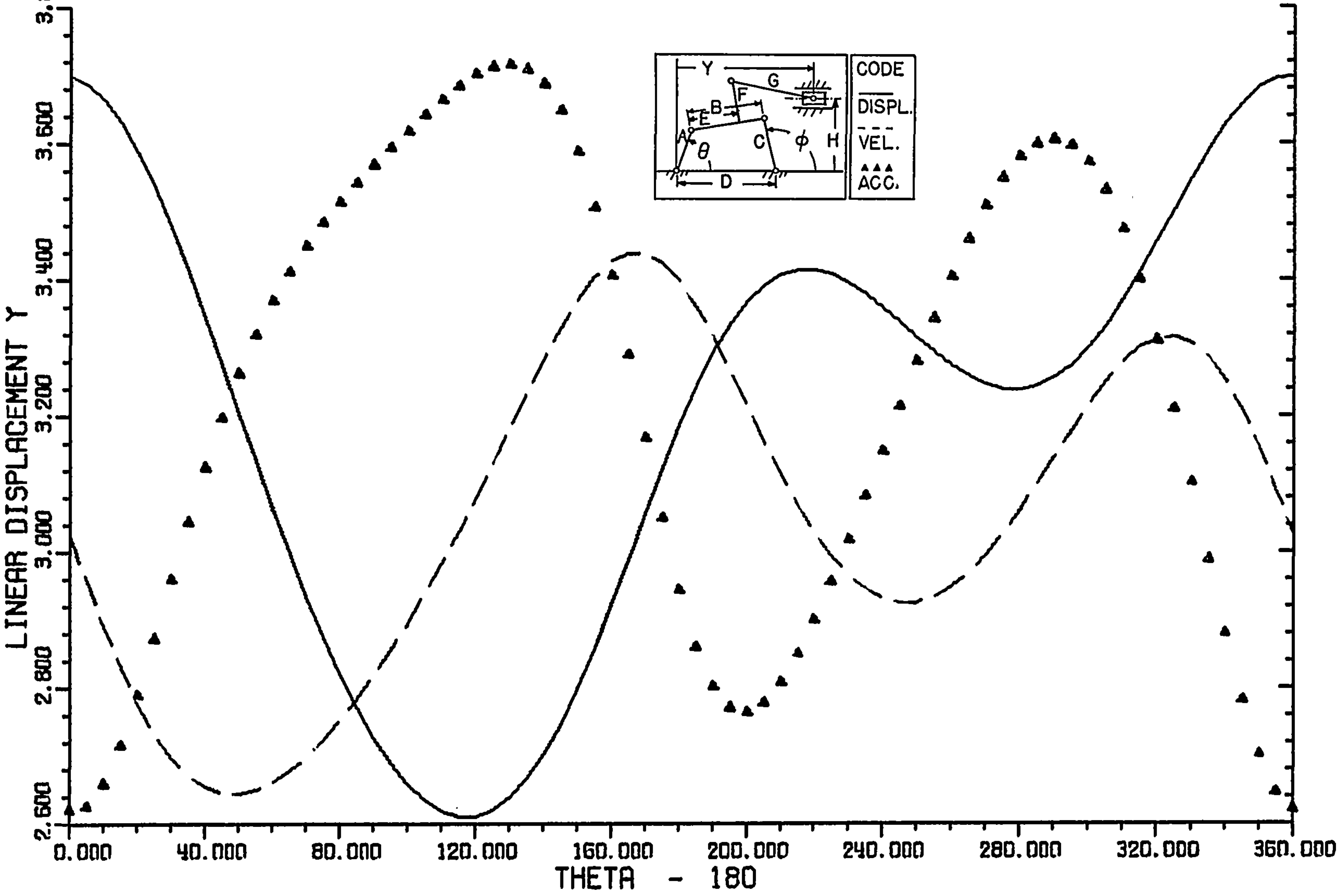


PLATE I-3

A= 1.00, B= 5.00, C= 5.00, D= 8.00,
E= 2.00, F= 3.00, G= 9.00, H= 3.00,
VEL.MAX= 0.64, VEL.MIN= -0.47, ACC.MAX= 0.59, ACC.MIN= -0.72,
PEAK TO PEAK DISPLACEMENT = 1.11

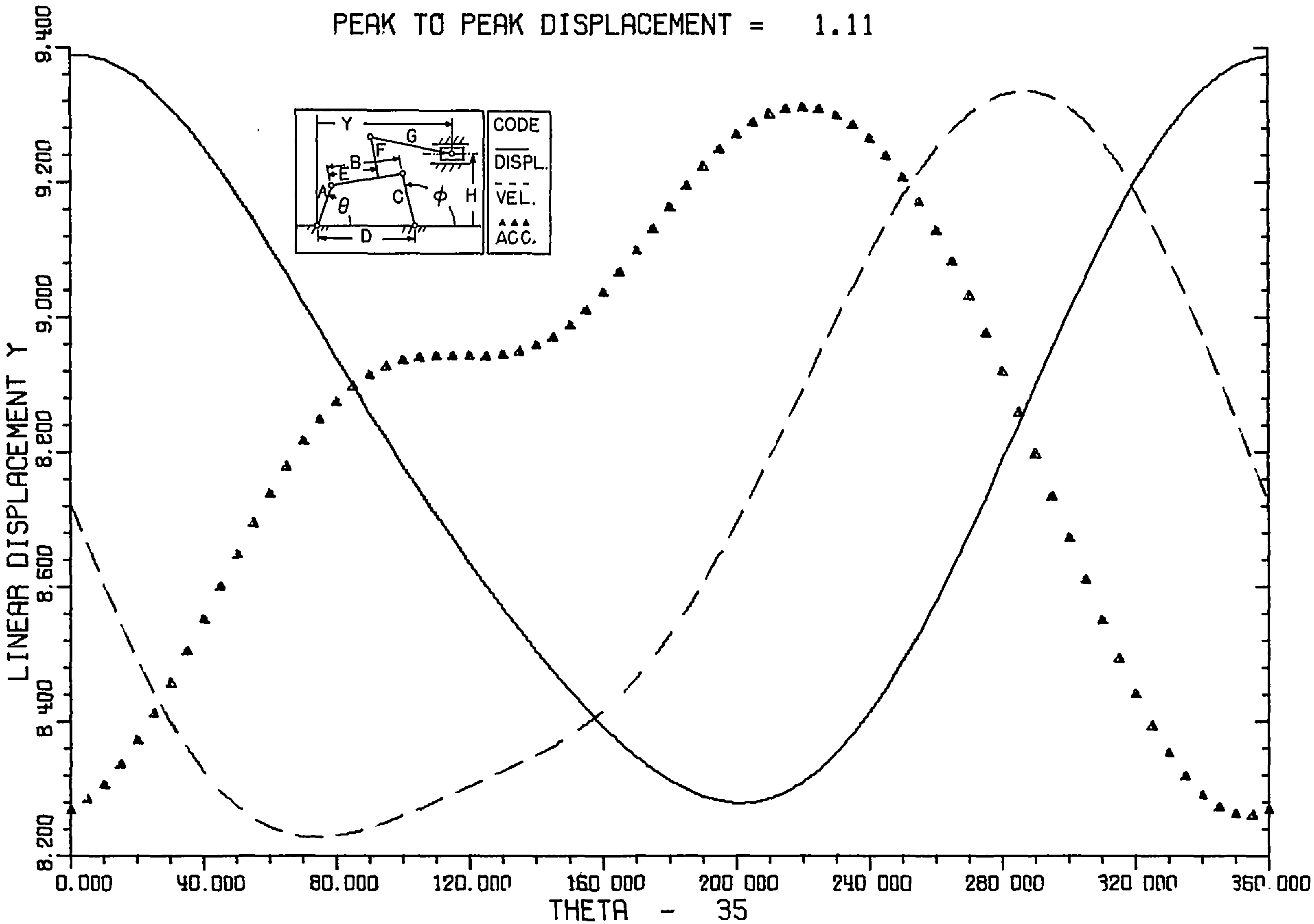
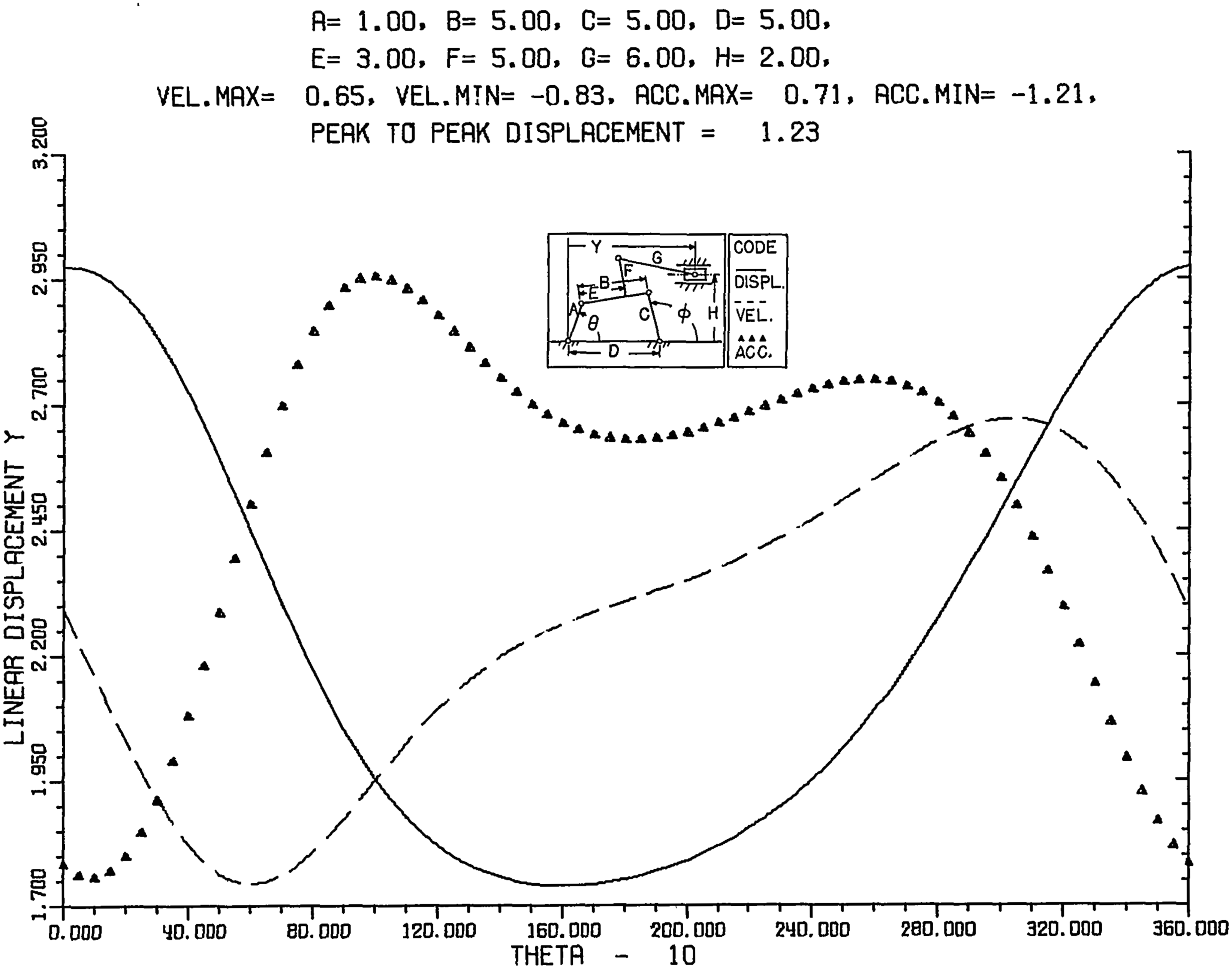


PLATE I-4



A= 1.00, B= 5.00, C= 5.00, D= 5.00,
E= 3.00, F= 5.00, G= 6.00, H= 2.00,
VEL.MAX= 0.65, VEL.MIN= -0.83, ACC.MAX= 0.71, ACC.MIN= -1.21,
PEAK TO PEAK DISPLACEMENT = 1.23
LINEAR DISPLACEMENT Y
3.200
2.950
2.700
2.450
2.200
1.950
1.700
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 10
Y
G
F
B
E
A
θ
C
φ
H
D
CODE
DISPL.
VEL.
ACC.

PLATE I-5

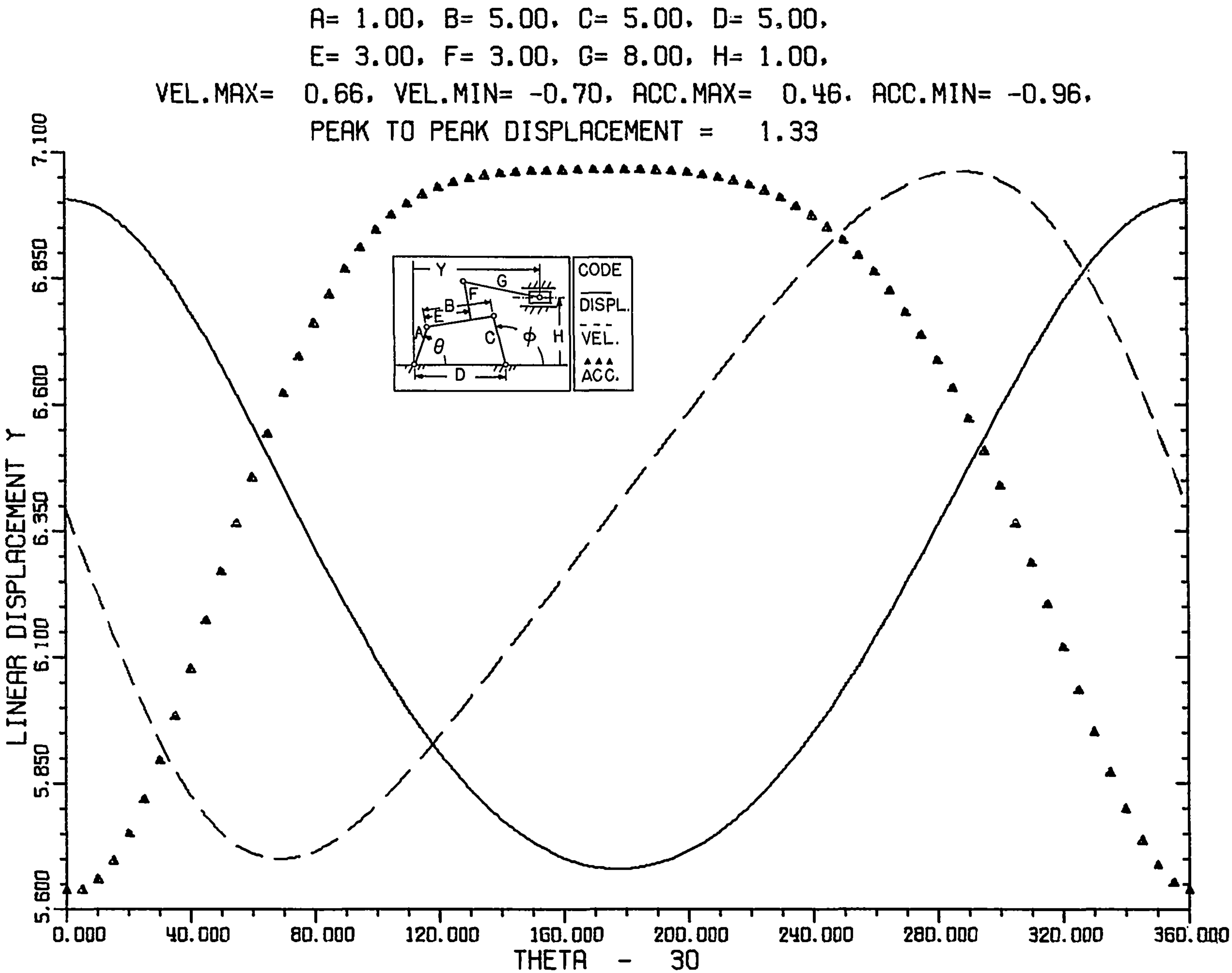

A= 1.00, B= 5.00, C= 5.00, D= 5.00,
E= 3.00, F= 3.00, G= 8.00, H= 1.00,
VEL.MAX= 0.66, VEL.MIN= -0.70, ACC.MAX= 0.46, ACC.MIN= -0.96,
PEAK TO PEAK DISPLACEMENT = 1.33
LINEAR DISPLACEMENT Y
THETA - 30
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
5.600
5.850
6.100
6.350
6.600
6.850
7.100
CODE
DISPL.
VEL.
ACC.
Y
G
B
F
E
A
C
H
D

PLATE I-6

A= 1.00, B= 3.00, C= 3.00, D= 4.00,
E= 1.00, F= 6.00, G=10.00, H= 2.00,
VEL.MAX= 0.84, VEL.MIN= -0.58, ACC.MAX= 0.84, ACC.MIN= -0.94,
PEAK TO PEAK DISPLACEMENT = 1.36

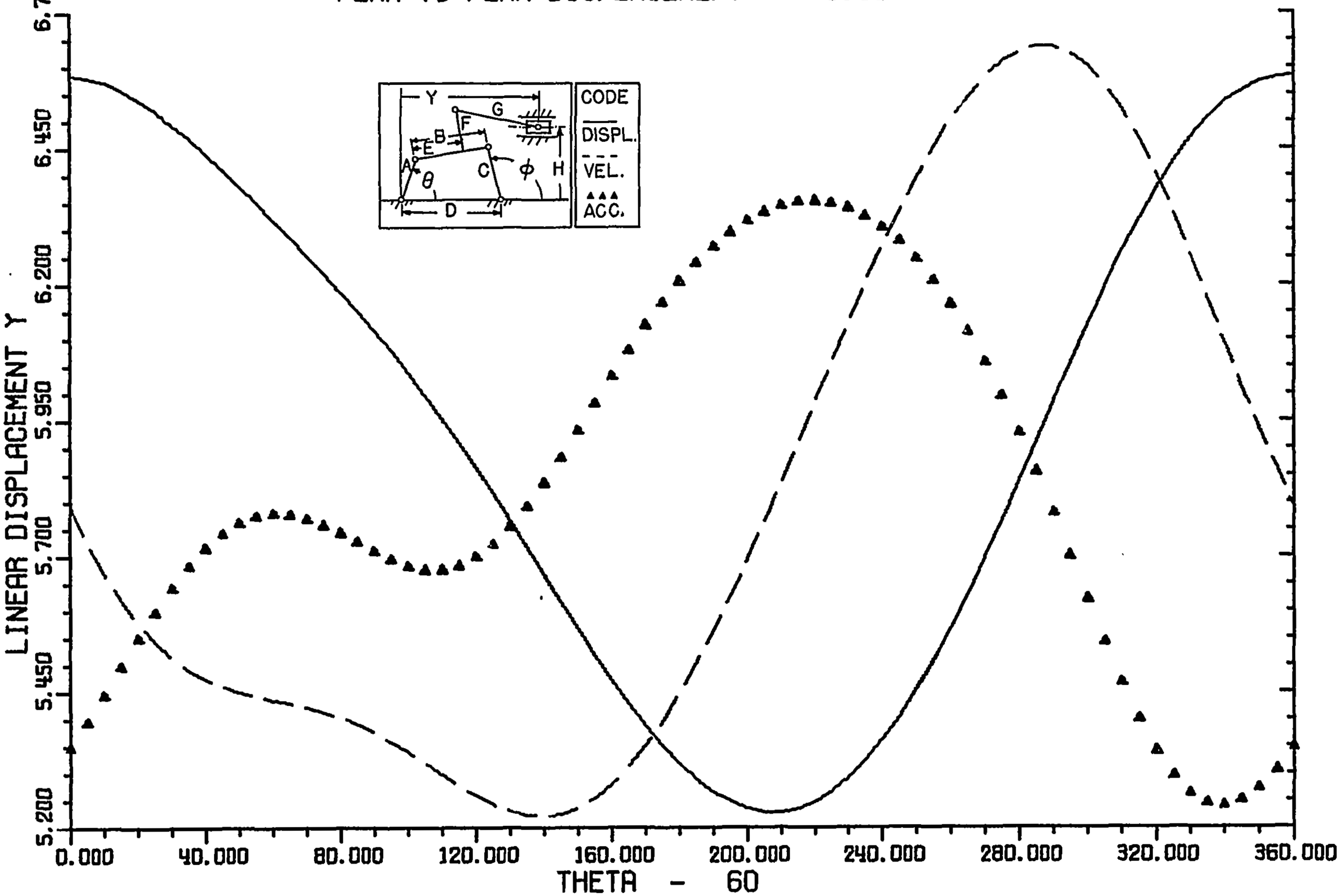


PLATE I-7

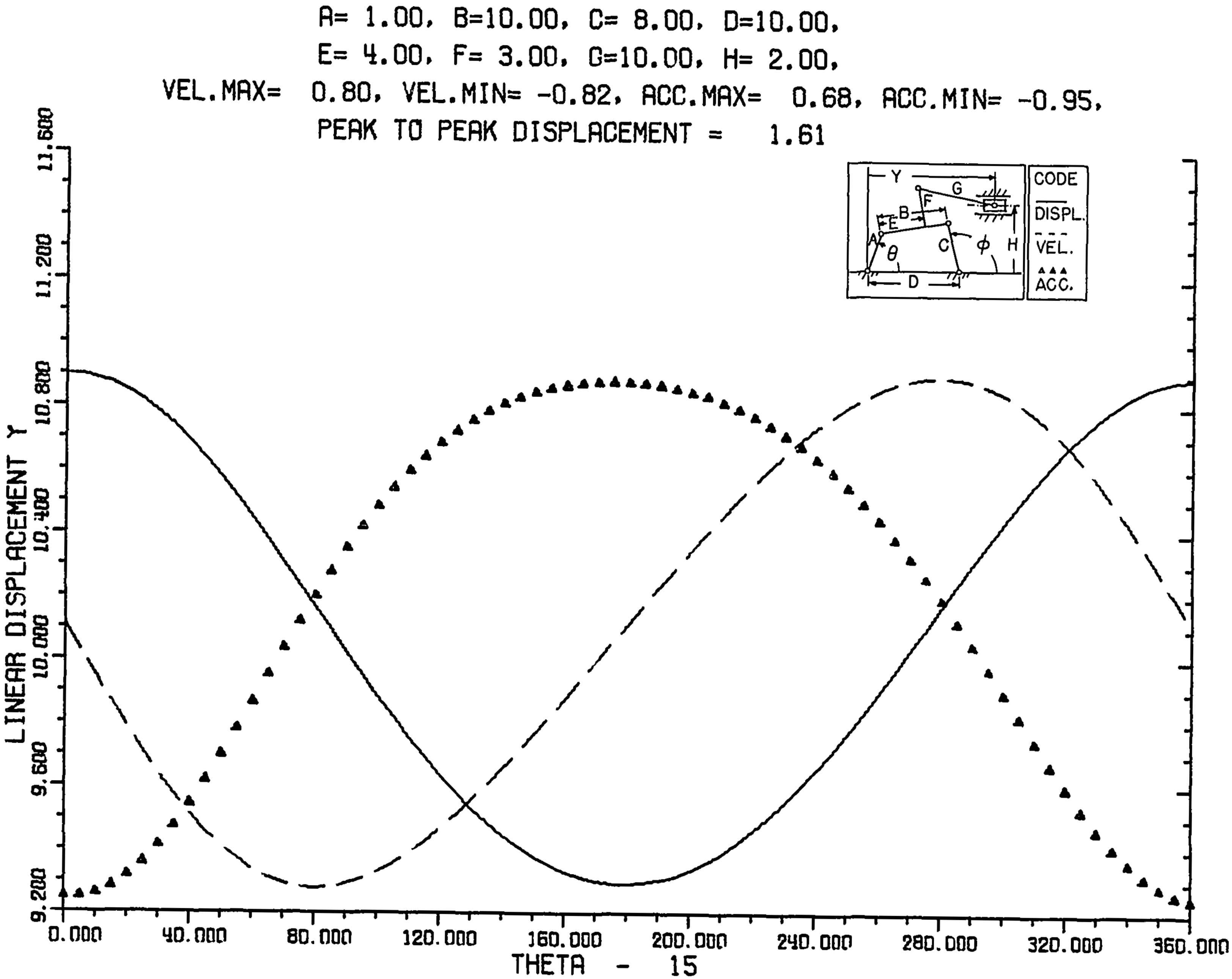

A= 1.00, B=10.00, C= 8.00, D=10.00,
E= 4.00, F= 3.00, G=10.00, H= 2.00,
VEL.MAX= 0.80, VEL.MIN= -0.82, ACC.MAX= 0.68, ACC.MIN= -0.95,
PEAK TO PEAK DISPLACEMENT = 1.61
LINEAR DISPLACEMENT Y
9.200
9.600
10.000
10.400
10.800
11.200
11.600
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 15
Y
G
F
B
E
A
θ
C
φ
H
D
CODE
DISPL.
VEL.
ACC.

PLATE I-8

A= 1.00, B= 1.50, C= 4.00, D= 4.00,
E= 2.00, F= 4.00, G= 9.00, H= 2.00,
VEL.MAX= 0.60, VEL.MIN= -1.20, ACC.MAX= 1.19, ACC.MIN= -1.63,
PEAK TO PEAK DISPLACEMENT = 1.83

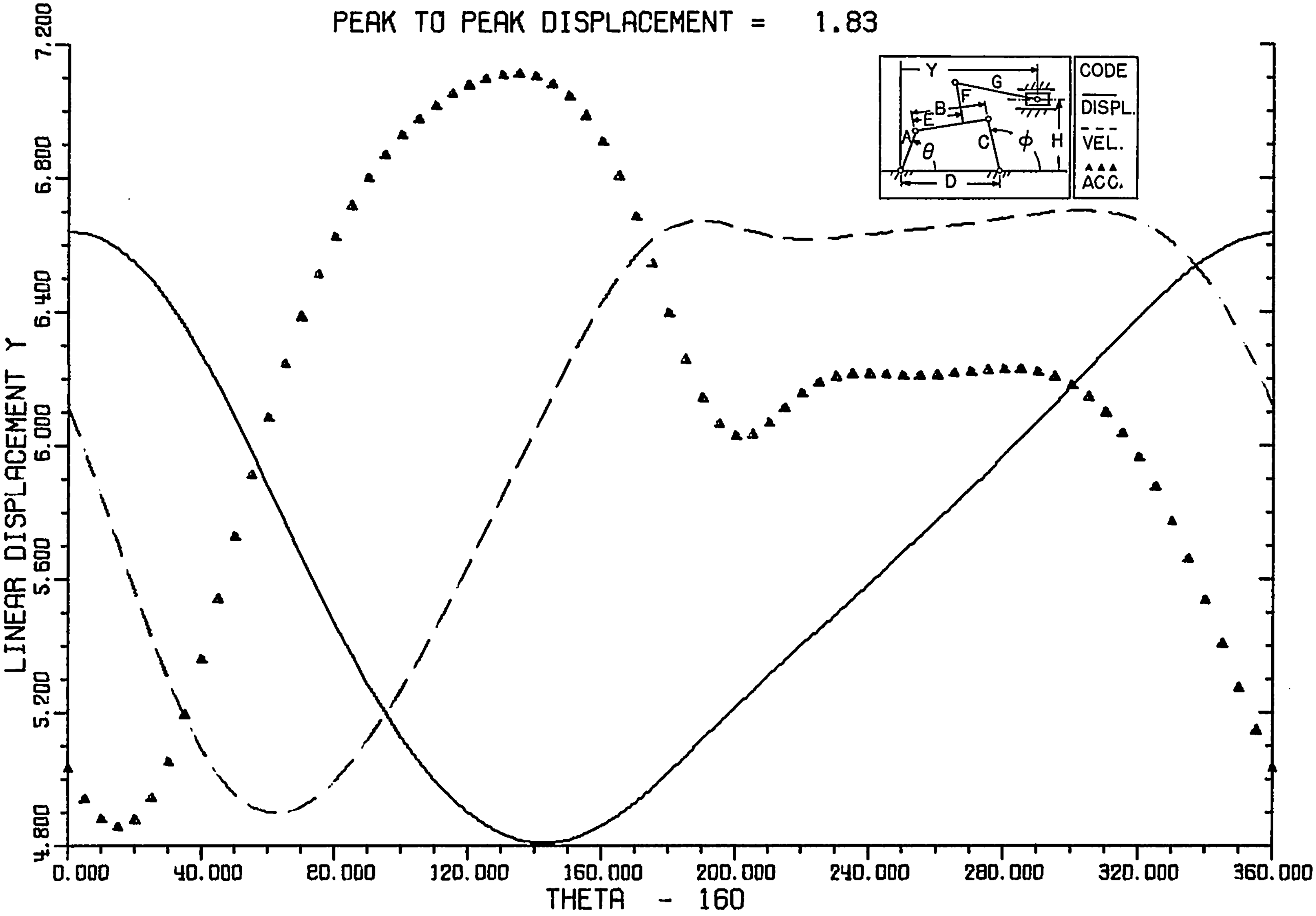


PLATE I-9

A= 1.00, B= 5.00, C= 5.00, D= 3.00,
E= 1.00, F= 6.00, G=10.00, H= 1.00,
VEL.MAX= 0.92, VEL.MIN= -1.01, ACC.MAX= 0.64, ACC.MIN= -1.50,
PEAK TO PEAK DISPLACEMENT = 1.87

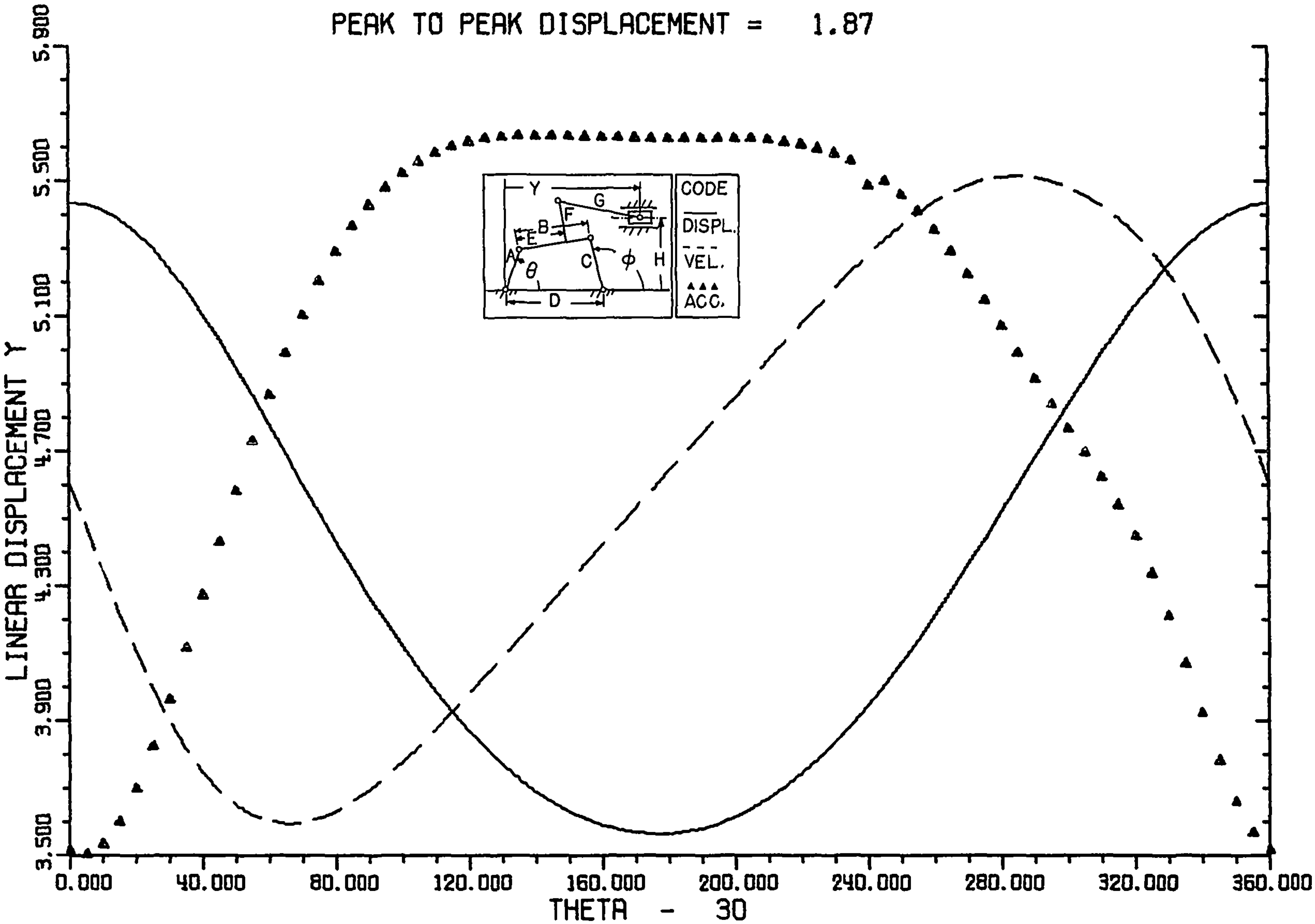


PLATE I-10

A= 1.00, B= 4.00, C= 4.00, D= 4.00,
E= 1.00, F= 4.00, G= 8.00, H= 3.00,
VEL.MAX= 1.10, VEL.MIN= -0.86, ACC.MAX= 0.85, ACC.MIN= -1.39,
PEAK TO PEAK DISPLACEMENT = 1.92

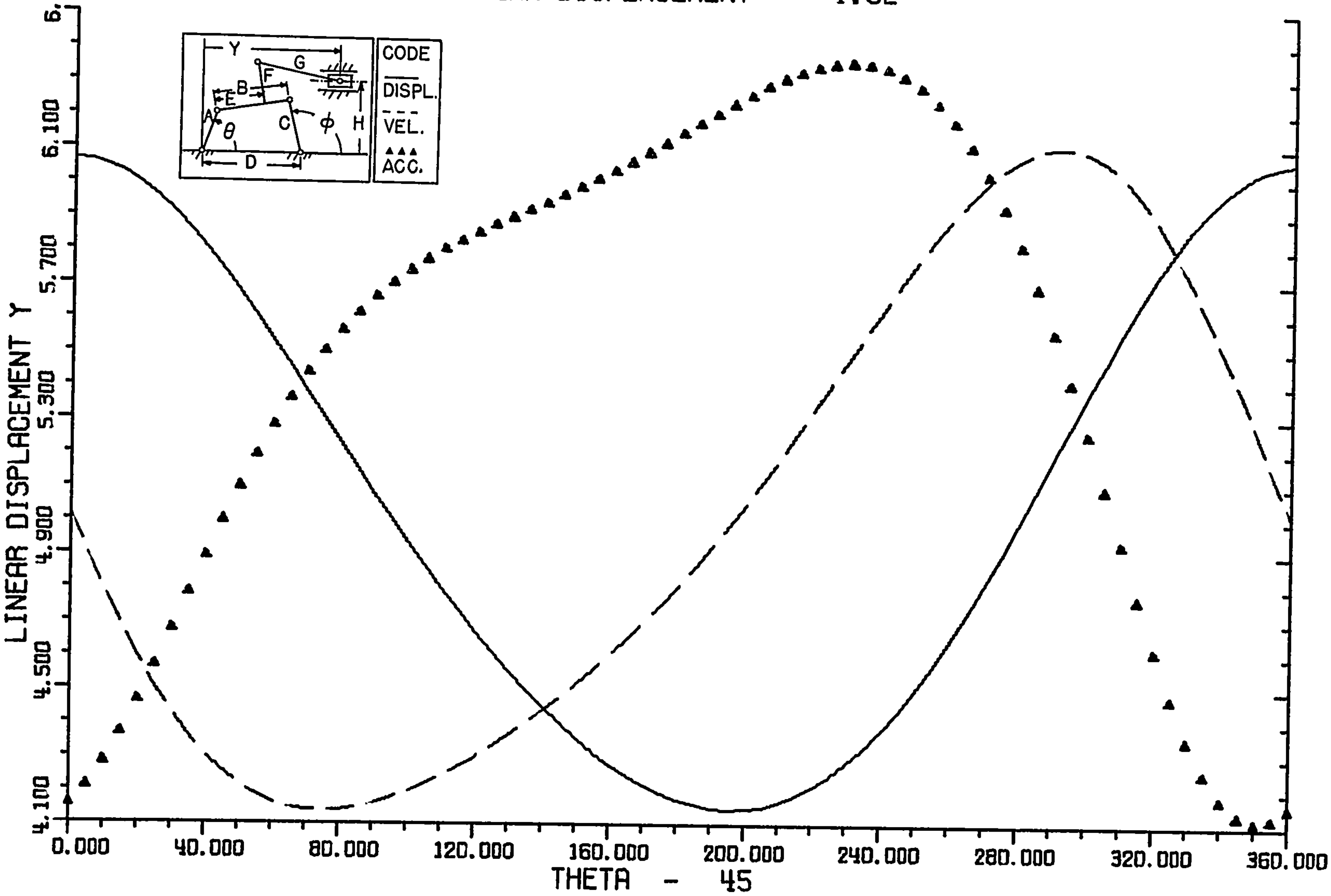


PLATE 1-11

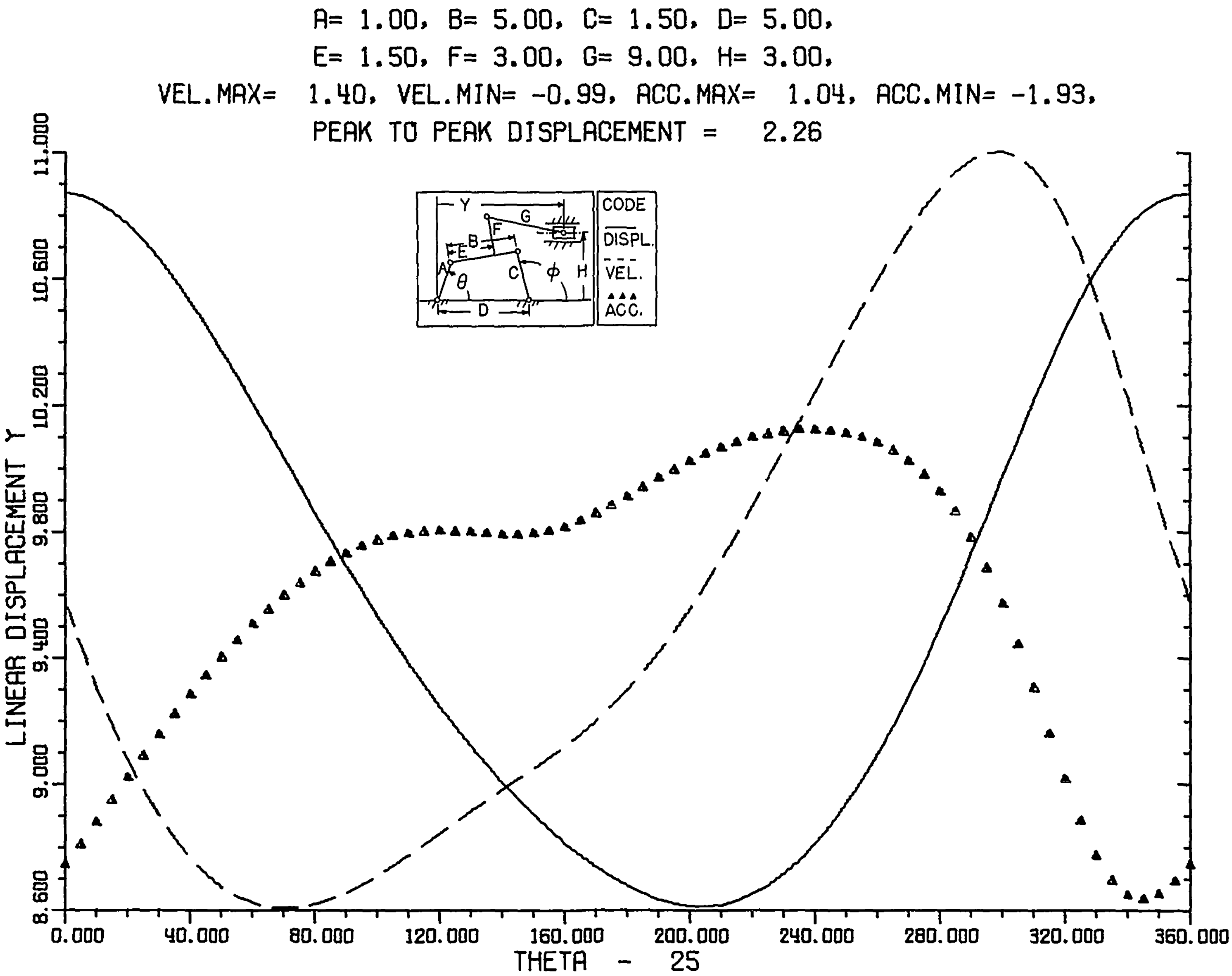
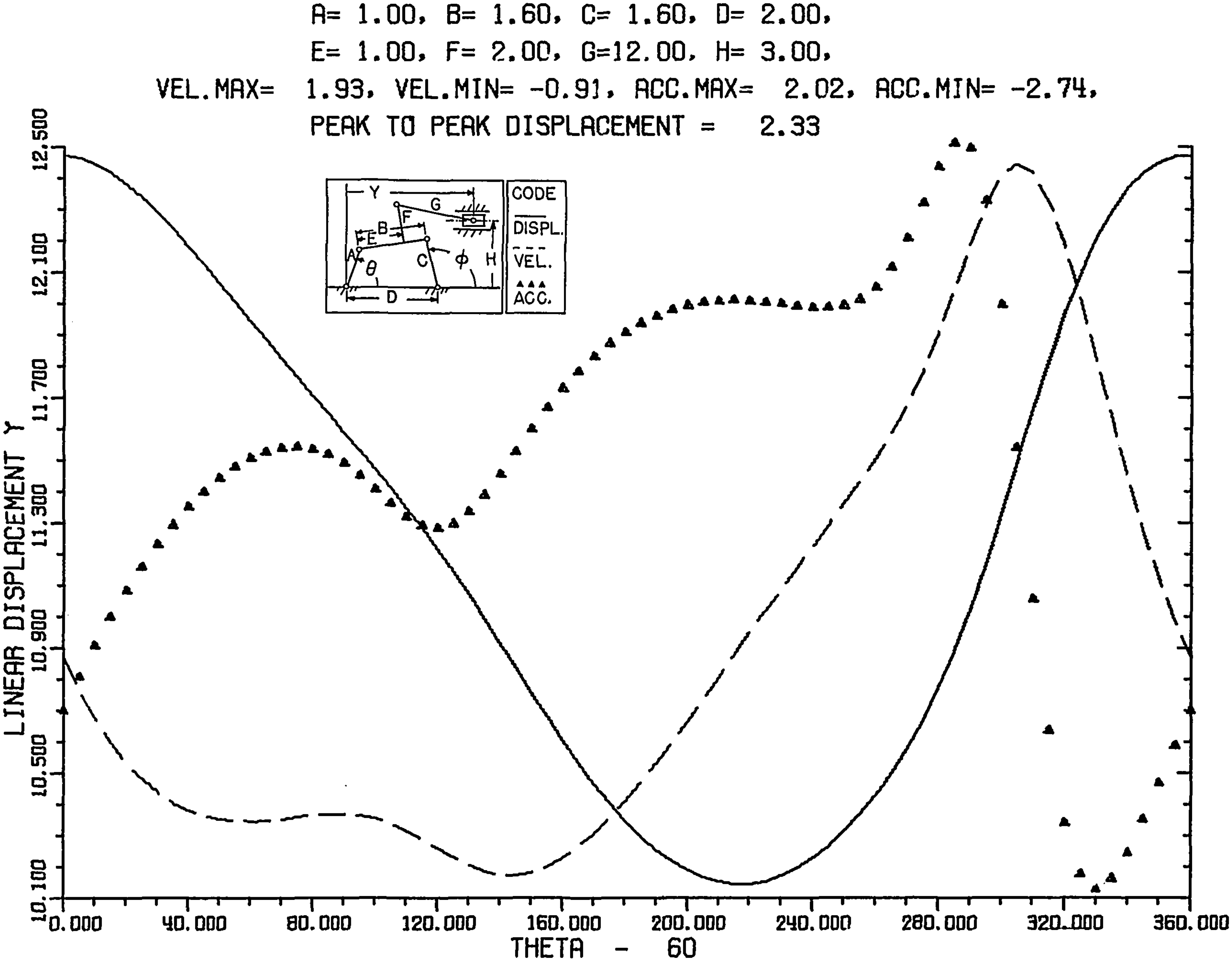

A= 1.00, B= 5.00, C= 1.50, D= 5.00,
E= 1.50, F= 3.00, G= 9.00, H= 3.00,
VEL.MAX= 1.40, VEL.MIN= -0.99, ACC.MAX= 1.04, ACC.MIN= -1.93,
PEAK TO PEAK DISPLACEMENT = 2.26
CODE
DISPL.
VEL.
ACC.
Y
G
F
B
E
A
θ
C
φ
H
D
LINEAR DISPLACEMENT Y
11.000
10.600
10.200
9.800
9.400
9.000
8.600
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 25

PLATE I-12

PLATE I-13


A= 1.00, B= 1.60, C= 1.60, D= 2.00,
E= 1.00, F= 2.00, G=12.00, H= 3.00,
VEL.MAX= 1.93, VEL.MIN= -0.91, ACC.MAX= 2.02, ACC.MIN= -2.74,
PEAK TO PEAK DISPLACEMENT = 2.33
CODE
DISPL.
VEL.
ACC.
Y
G
B
F
E
A
θ
C
φ
H
D
LINEAR DISPLACEMENT Y
12.500
12.100
11.700
11.300
10.900
10.500
10.100
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 60

A= 1.00, B= 1.50, C= 3.00, D= 3.00,
E= 1.00, F= 5.00, G= 9.00, H= 3.00,
VEL.MAX= 1.30, VEL.MIN= -1.40, ACC.MAX= 1.71, ACC.MIN= -1.42,
PEAK TO PEAK DISPLACEMENT = 2.46

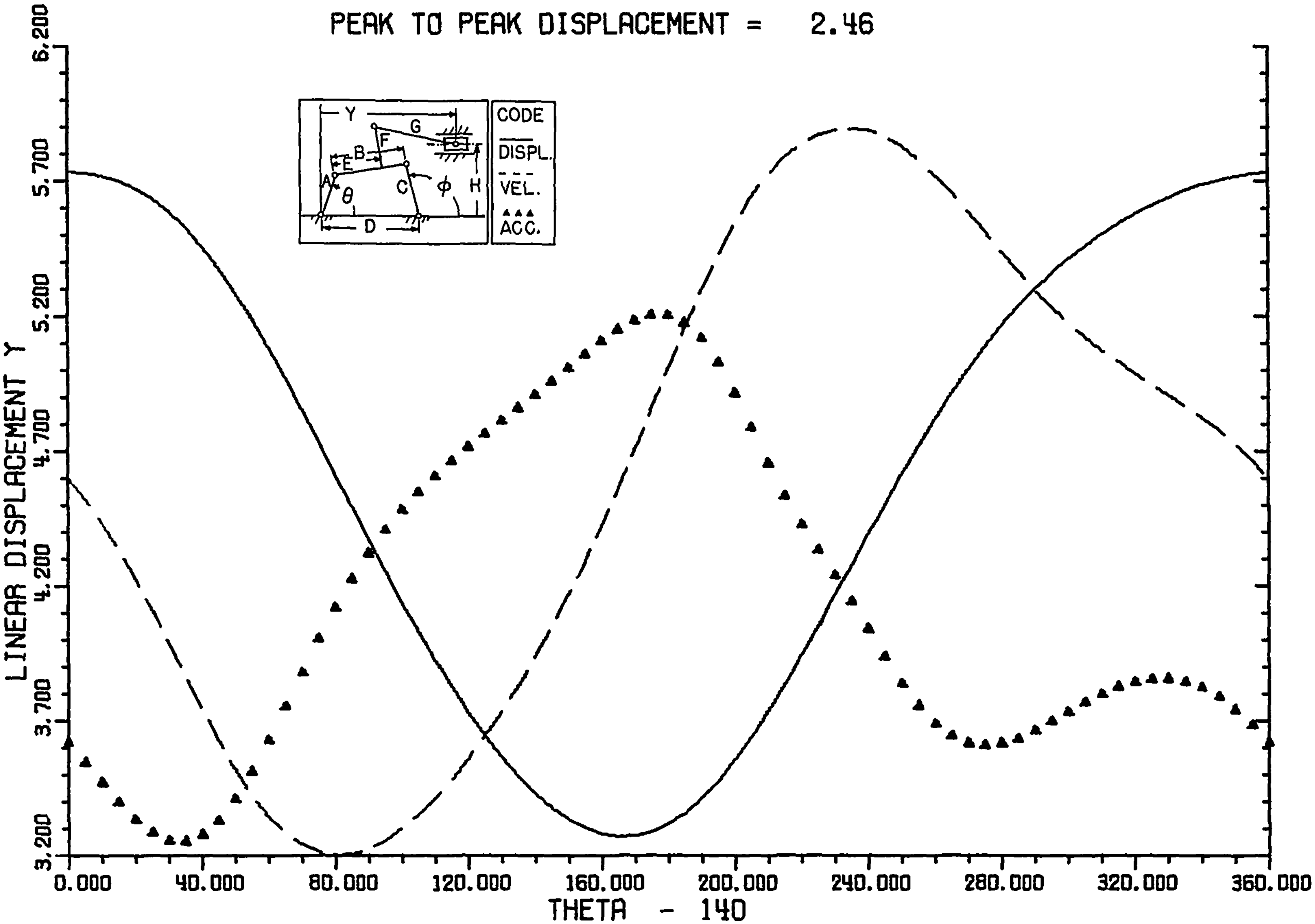


PLATE I-14

A= 1.00, B= 1.40, C= 1.40, D= 1.50,
E= 1.00, F= 3.00, G= 8.00, H= 1.00,
VEL.MAX= 3.12, VEL.MIN= -1.01, ACC.MAX= 7.17, ACC.MIN= -5.76,
PEAK TO PEAK DISPLACEMENT = 2.69

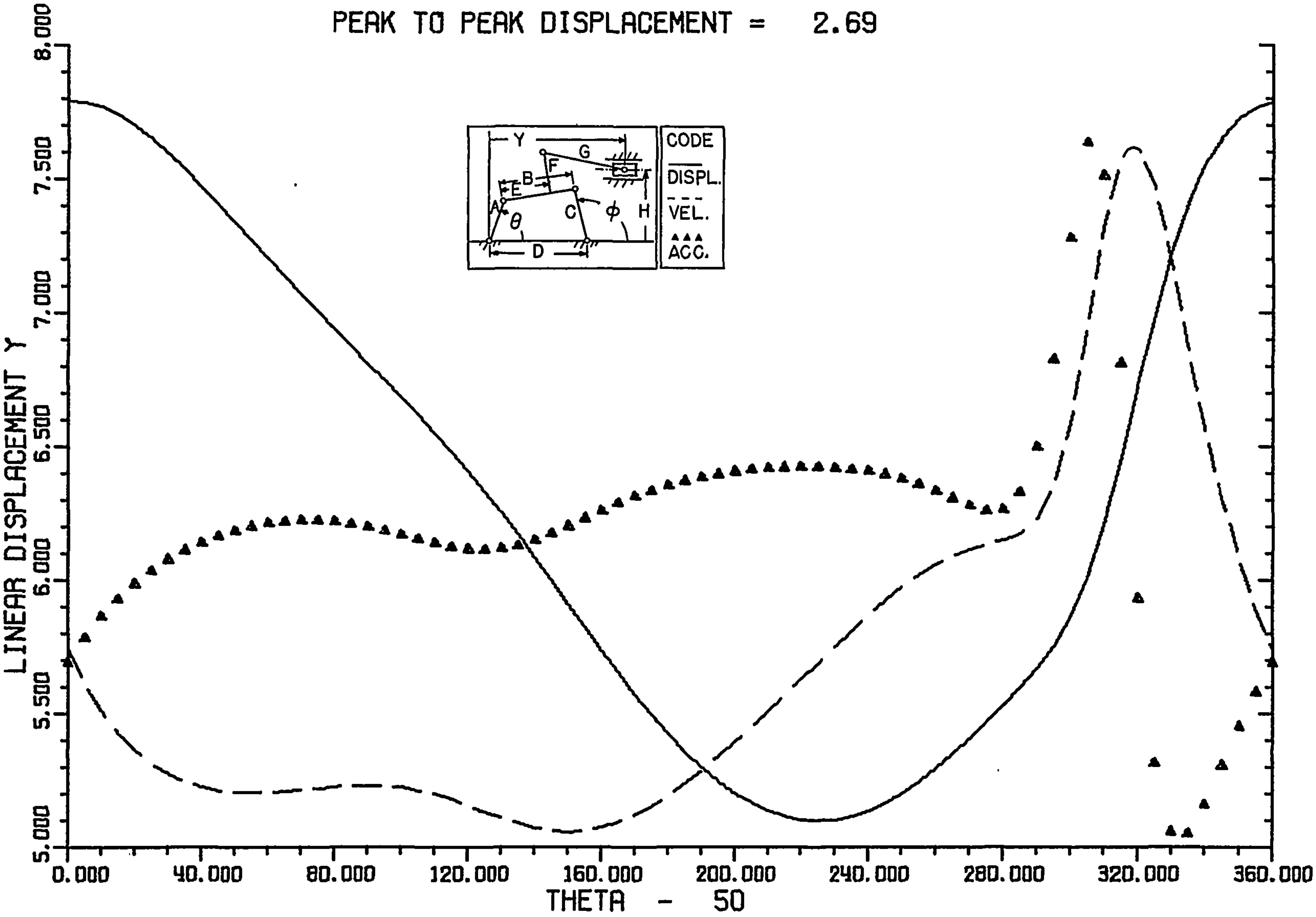


PLATE I-15

A= 1.00, B= 1.70, C= 1.50, D= 2.10,
E= 1.50, F= 4.00, G=10.00, H= 1.00,
VEL.MAX= 2.25, VEL.MIN= -1.74, ACC.MAX= 2.55, ACC.MIN= -2.84,
PEAK TO PEAK DISPLACEMENT = 2.69

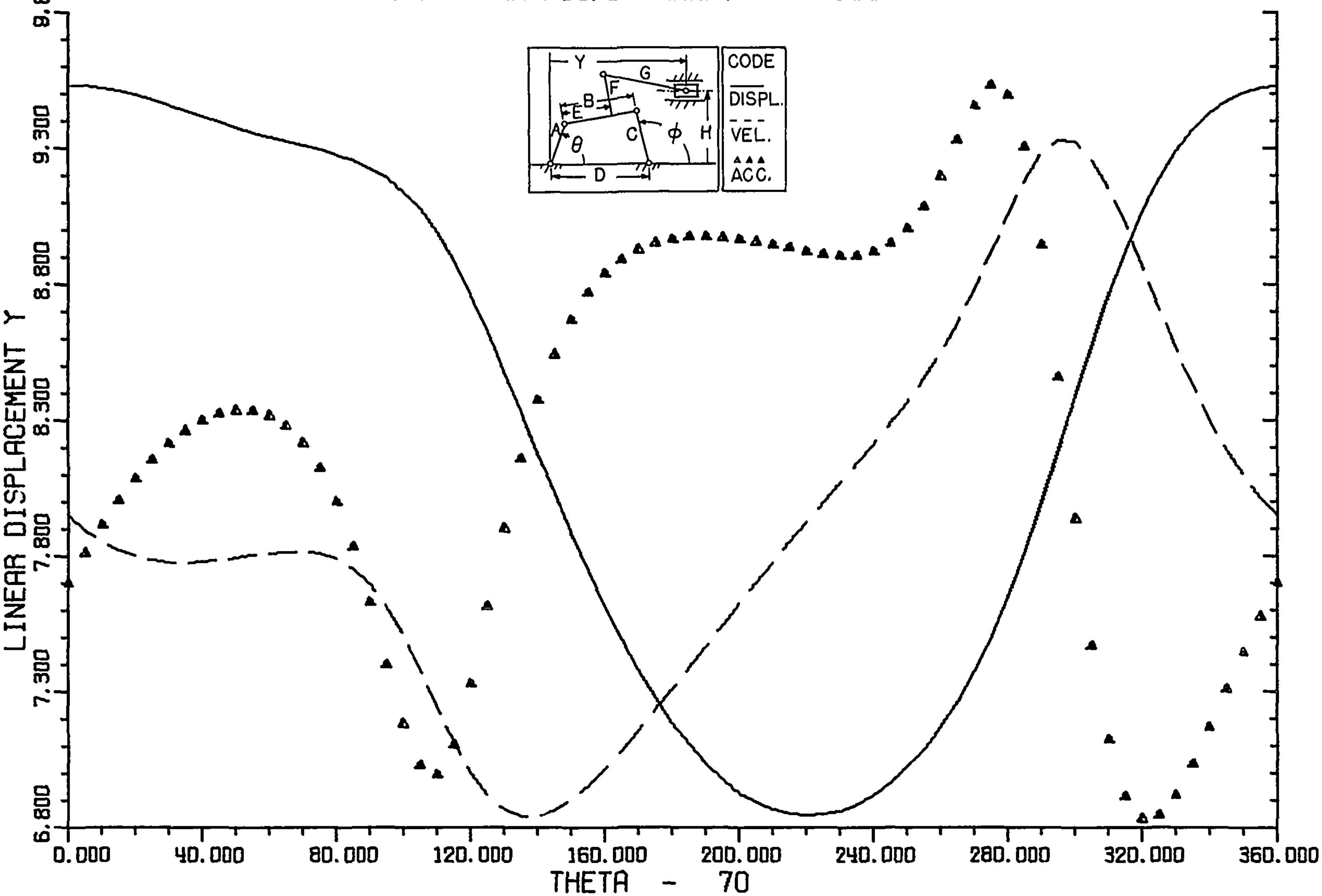


PLATE I-16

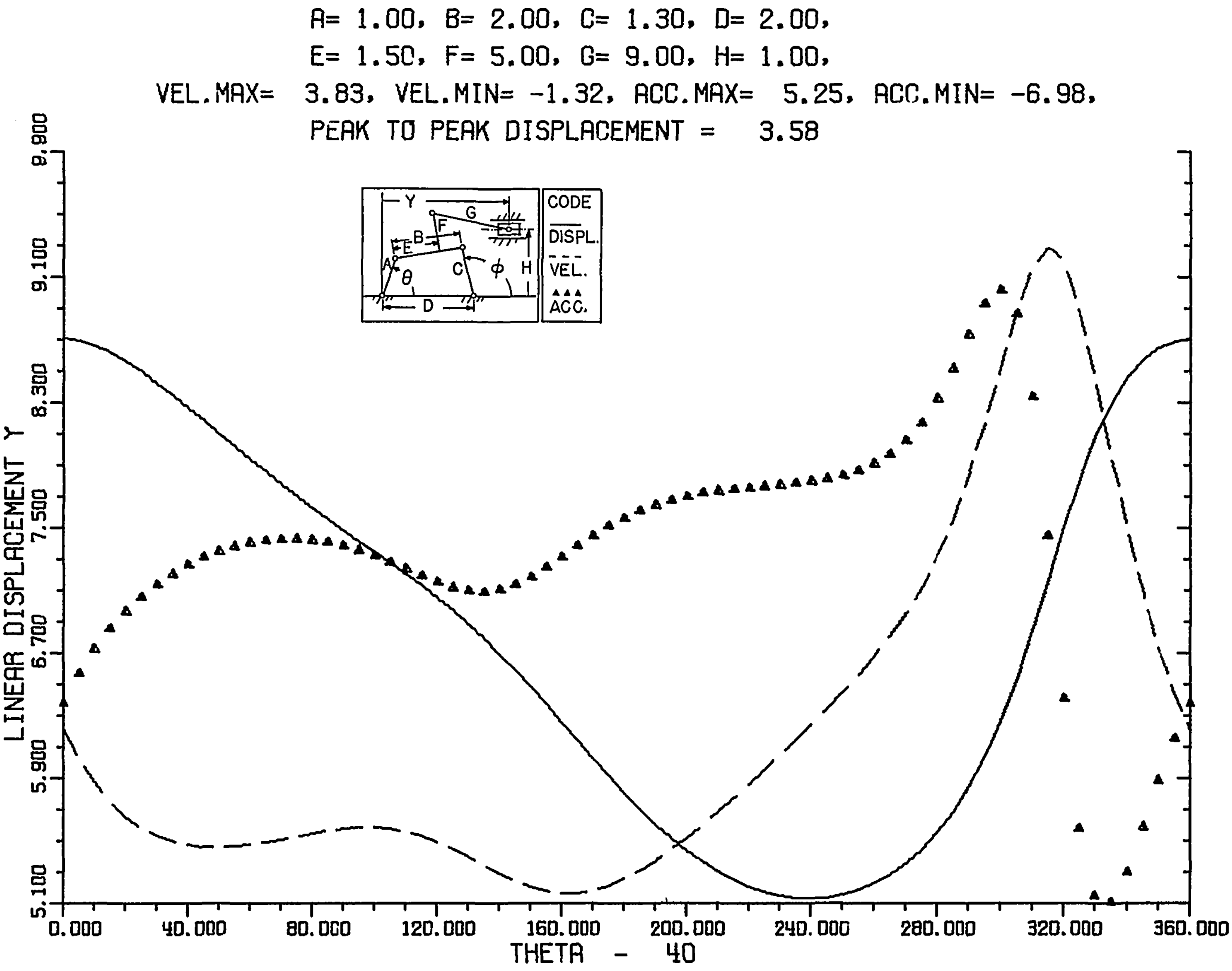

A= 1.00, B= 2.00, C= 1.30, D= 2.00,
E= 1.50, F= 5.00, G= 9.00, H= 1.00,
VEL.MAX= 3.83, VEL.MIN= -1.32, ACC.MAX= 5.25, ACC.MIN= -6.98,
PEAK TO PEAK DISPLACEMENT = 3.58
LINEAR DISPLACEMENT Y
9.900
9.100
8.300
7.500
6.700
5.900
5.100
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 40
Y
G
F
B
E
A
θ
C
φ
H
D
CODE
DISPL.
VEL.
ACC.

PLATE I-17

A= 2.50, B= 7.50, C= 4.00, D= 8.00,
E= 3.00, F= 3.00, G=10.00, H= 2.50,
VEL.MAX= 2.59, VEL.MIN= -2.29, ACC.MAX= 1.66, ACC.MIN= -4.29,
PEAK TO PEAK DISPLACEMENT = 4.46

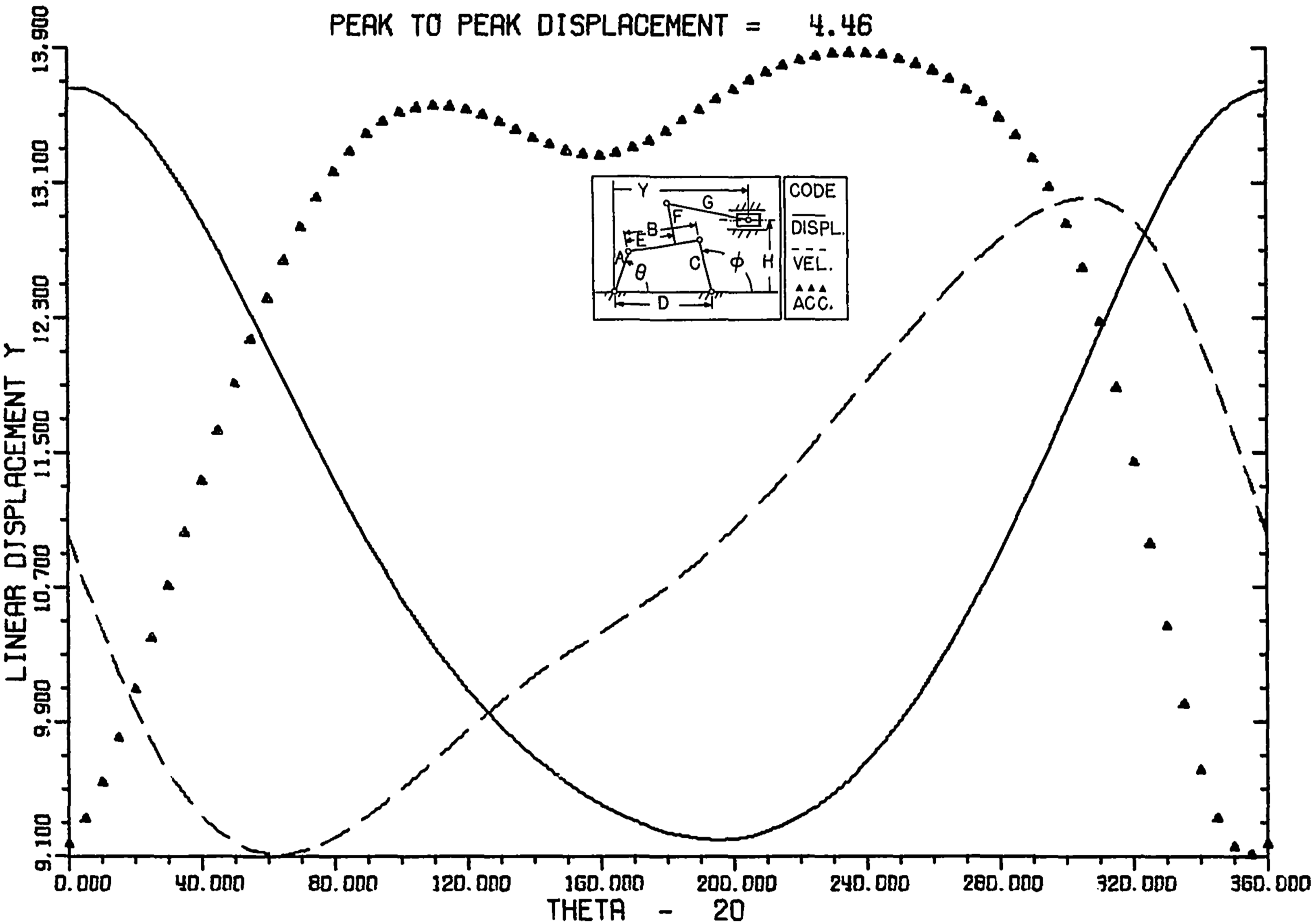


PLATE I-18

A= 1.00, B= 1.25, C= 1.25, D= 1.40,
E= 1.00, F= 4.00, G=10.00, H= 3.00,
VEL.MAX= 6.23, VEL.MIN= -2.27, ACC.MAX= 16.35, ACC.MIN=-11.55,
PEAK TO PEAK DISPLACEMENT = 4.55

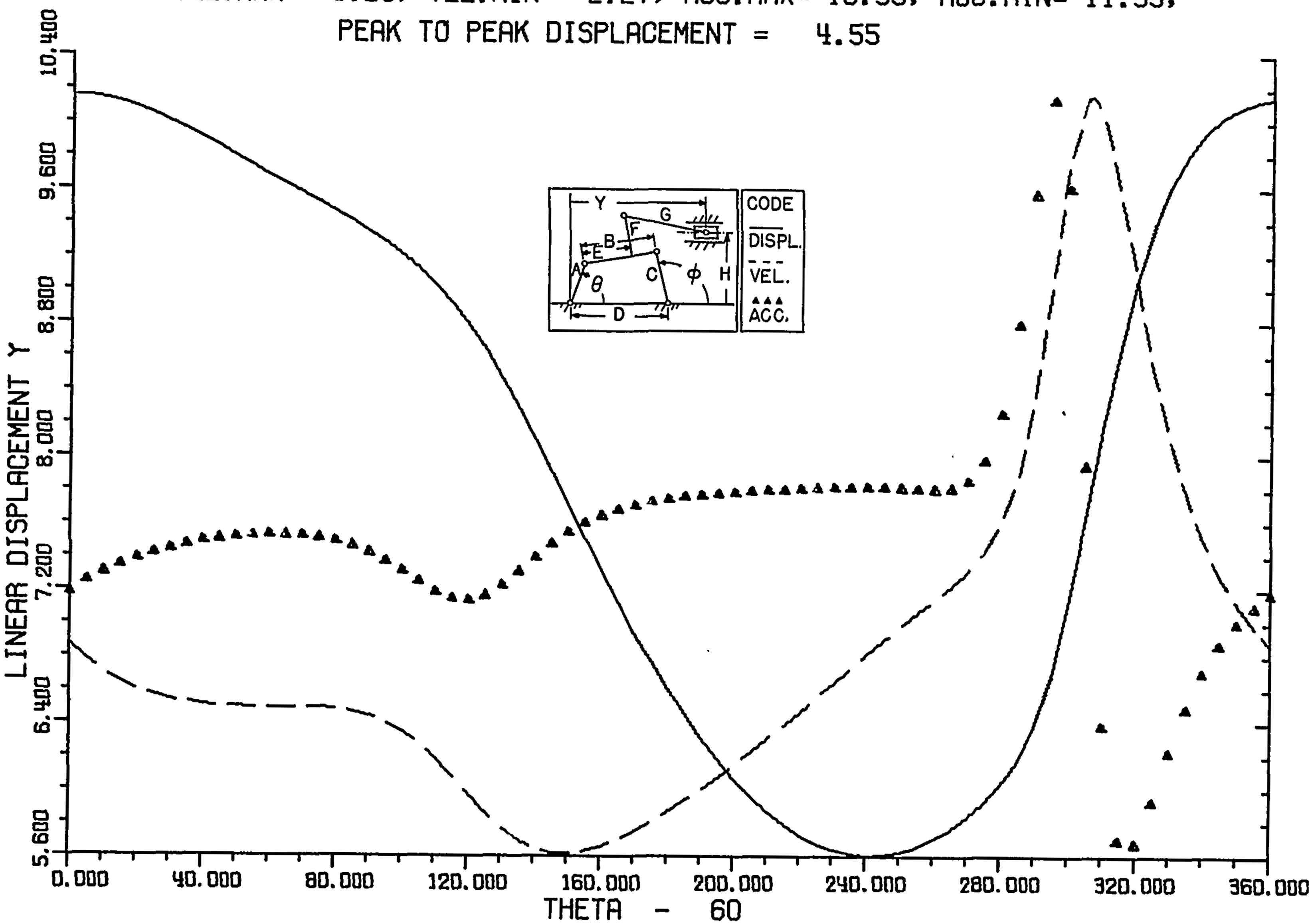


PLATE I-19

A= 2.50, B= 8.50, C= 5.50, D= 8.00,
E= 2.00, F= 2.00, G=10.00, H= 1.50,
VEL.MAX= 2.40, VEL.MIN= -2.52, ACC.MAX= 1.58, ACC.MIN= -3.84,
PEAK TO PEAK DISPLACEMENT = 4.77

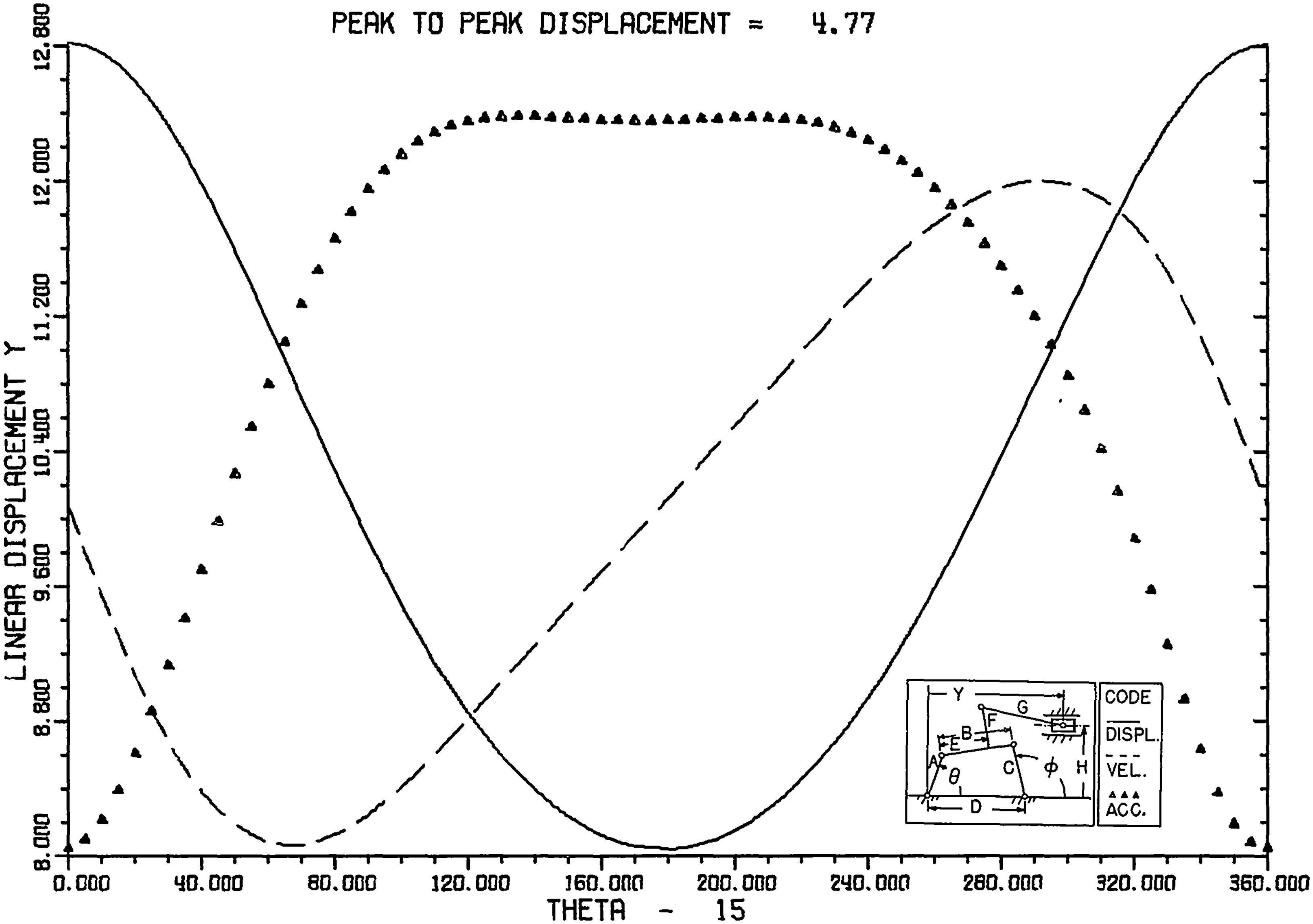
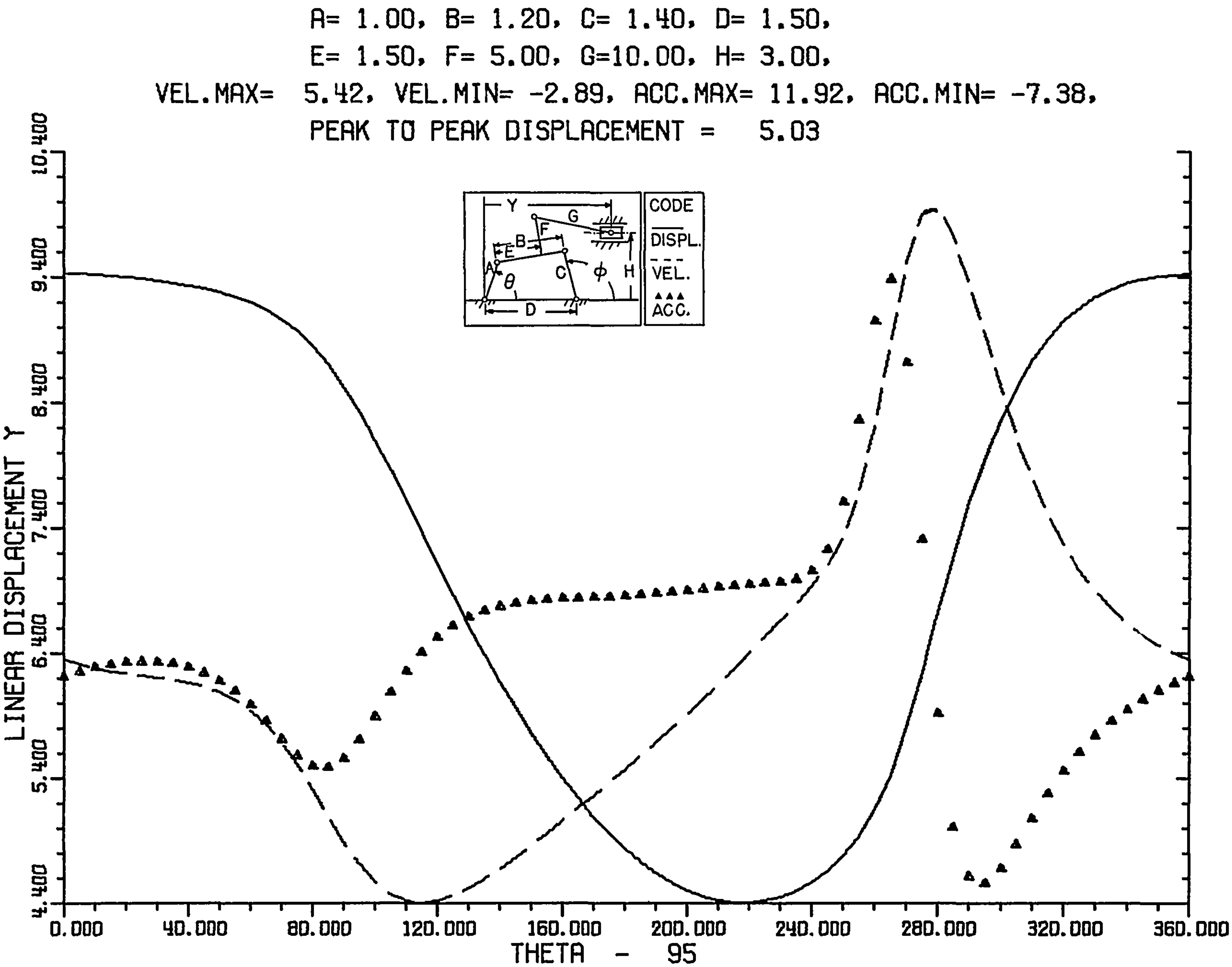


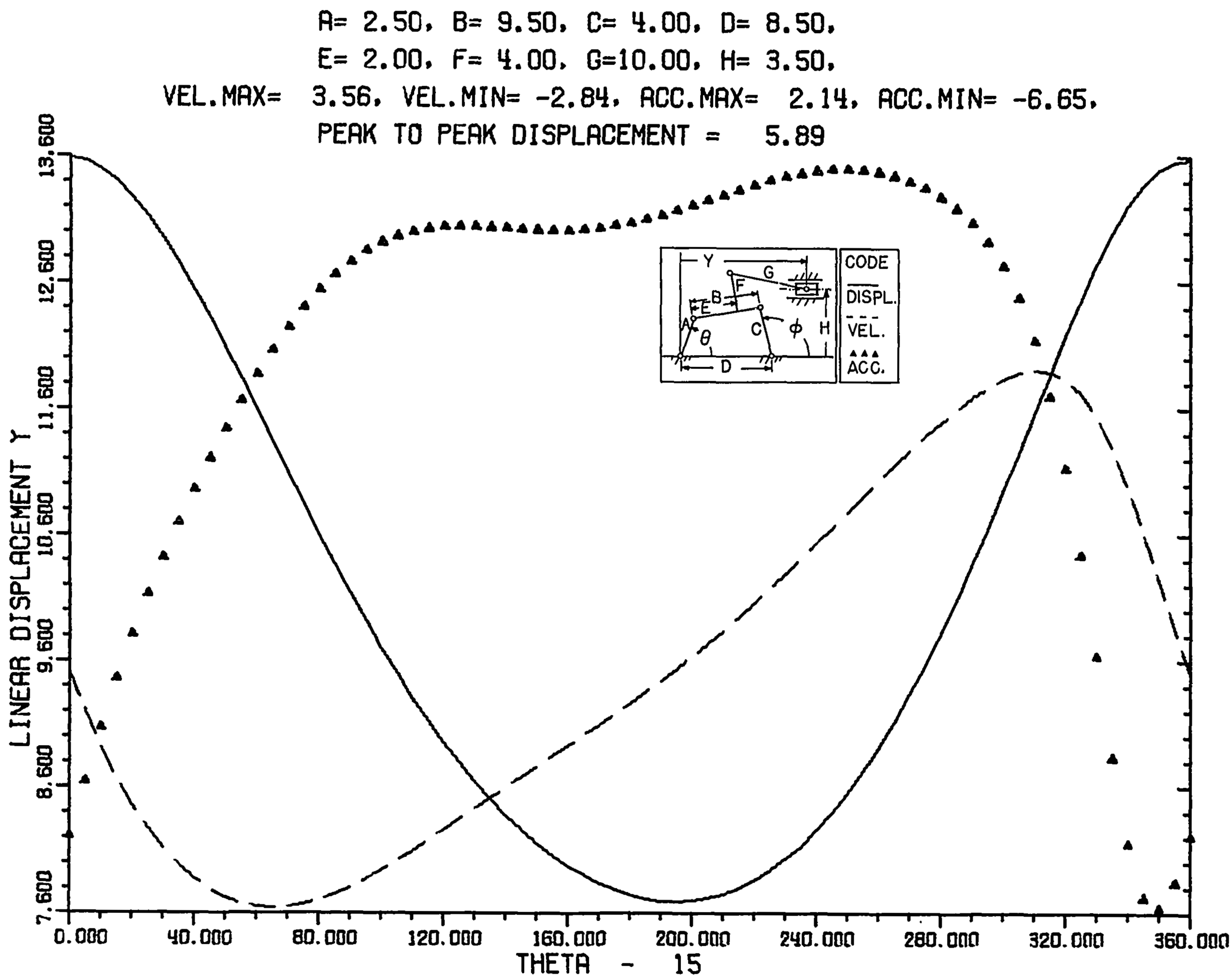
PLATE I-20



A= 1.00, B= 1.20, C= 1.40, D= 1.50,
E= 1.50, F= 5.00, G=10.00, H= 3.00,
VEL.MAX= 5.42, VEL.MIN= -2.89, ACC.MAX= 11.92, ACC.MIN= -7.38,
PEAK TO PEAK DISPLACEMENT = 5.03
LINEAR DISPLACEMENT Y
4.400
5.400
6.400
7.400
8.400
9.400
10.400
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 95
Y
G
B
F
E
A
θ
C
φ
H
D
CODE
DISPL.
VEL.
ACC.

PLATE I-21

PLATE I-22



A= 2.50, B= 9.50, C= 4.00, D= 8.50,
E= 2.00, F= 4.00, G=10.00, H= 3.50,
VEL.MAX= 3.56, VEL.MIN= -2.84, ACC.MAX= 2.14, ACC.MIN= -6.65,
PEAK TO PEAK DISPLACEMENT = 5.89
13.600
12.600
11.600
10.600
9.600
8.600
7.600
LINEAR DISPLACEMENT Y
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 15
Y
G
B
E
F
A
θ
C
ϕ
H
D
CODE
DISPL.
VEL.
ACC.

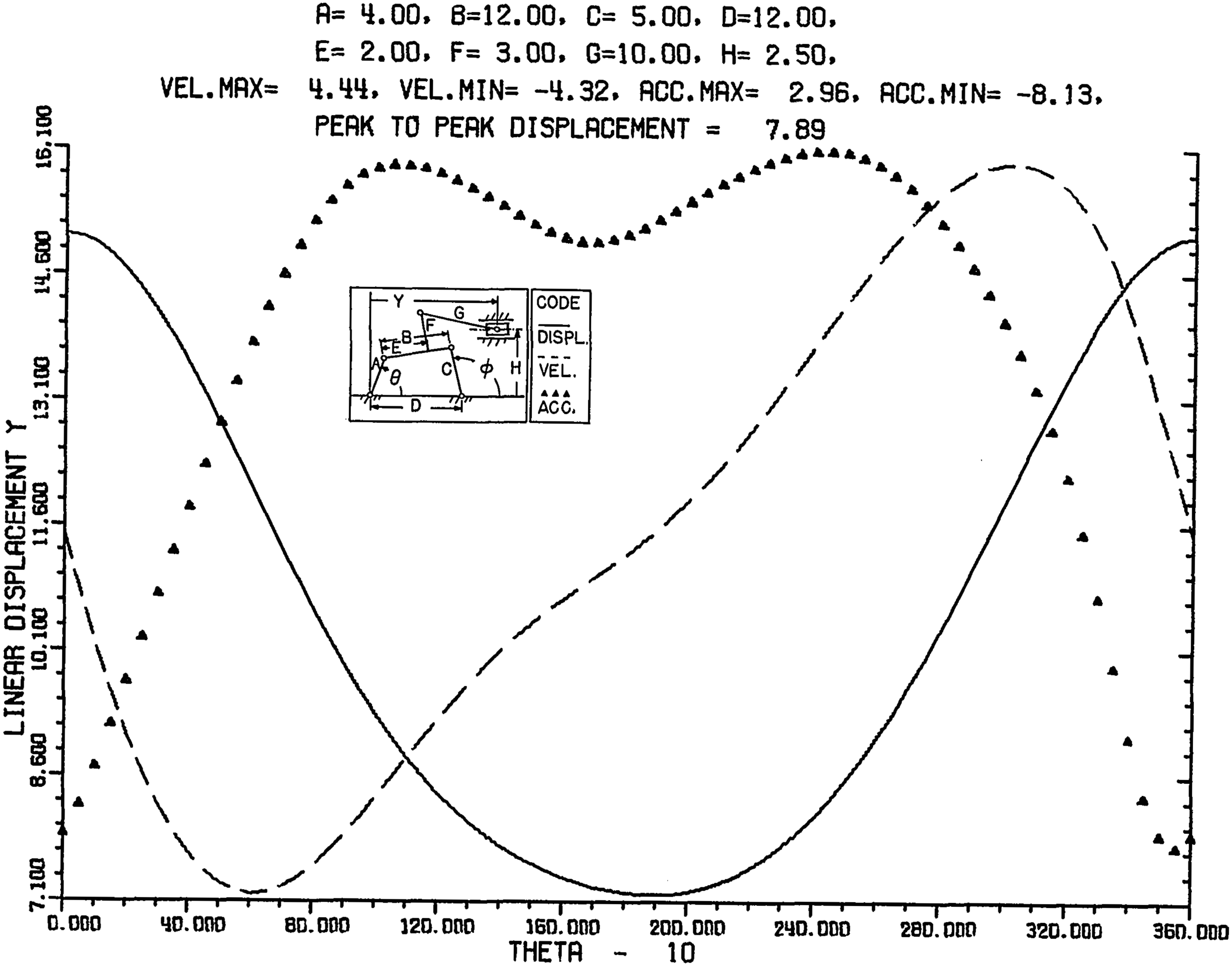

A= 4.00, B=12.00, C= 5.00, D=12.00,
E= 2.00, F= 3.00, G=10.00, H= 2.50,
VEL.MAX= 4.44, VEL.MIN= -4.32, ACC.MAX= 2.96, ACC.MIN= -8.13,
PEAK TO PEAK DISPLACEMENT = 7.89
LINEAR DISPLACEMENT Y
16.100
14.600
13.100
11.600
10.100
8.600
7.100
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 10
Y
G
F
B
E
A
θ
C
ϕ
H
D
CODE
DISPL.
VEL.
ACC.

PLATE I-23

MECHANISM #2

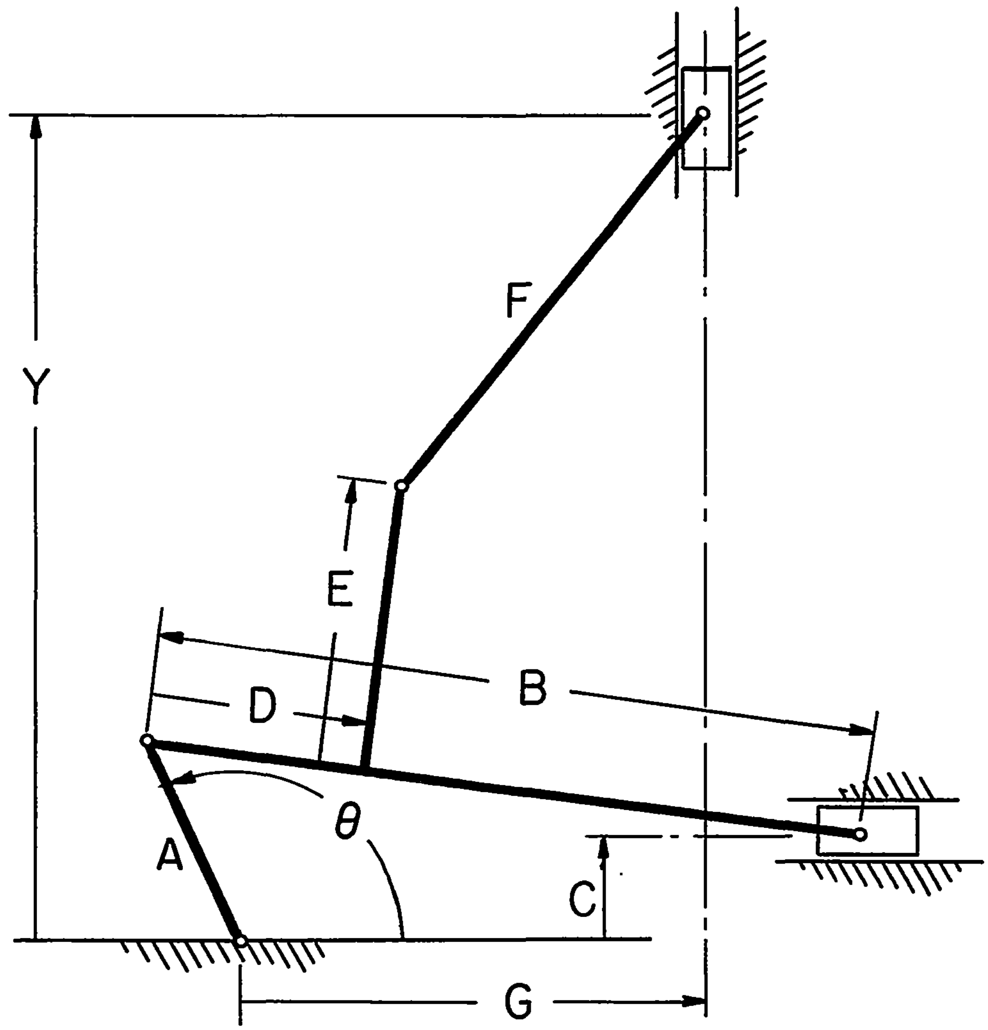


Figure 2-1

Figure 2-1 defines Mechanism #2. It is a slider-crank mechanism with an additional slider linked to a coupler point of the basic mechanism. The input of the mechanism is the angular position, θ, the Greek letter theta, of link A; the output is the linear position of the coupled slider, Y. Upon specifying numerical values for the the lengths, A through G, the output, Y, may be determined for any value of the input, θ.

Each of the graphs for this mechanism shows Y versus θ as a solid line, the derivative of Y with respect to θ versus θ as a dashed line, and the second derivative of Y with respect to θ versus θ as a series of small triangles. Each curve begins with the maximum displacement Y. This has been accomplished by shifting the θ axis. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. The variable θ is presented in the units degrees. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement.

Scales have not been presented for the derivatives but each graph heading includes the maximum and minimum of both the velocity and acceleration. The units for displacement, velocity, and acceleration will correspond to that chosen for the quantities A, B, C, et cetera. For example if the link lengths are specified in inches then the velocity, $dY/d\theta$, will be in inches per radian. A more conventional engineering unit for velocity may be obtained as:

$$\frac{dY}{dt} = \frac{dY}{d\theta} \times \frac{d\theta}{dt} \tag{2-1}$$

in which

$$\frac{d\theta}{dt} = \text{rpm} \times 2\pi \times \frac{1}{60} \qquad \left(\frac{\text{deg}}{\text{sec}} = \frac{\text{rev}}{\text{min}} \times \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{\text{sec}}\right).$$

With this modification Eq. 2-1 may be rewritten:

$$\frac{dY}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{dY}{d\theta}, \qquad \frac{\text{inches}}{\text{second}}. \tag{2-2}$$

Expressed in words, Eq. 2-2 indicates that the velocity of the slider (inches/second) is obtained by multiplying the angular speed of link A (revolutions per minute) by $\pi/30$ and then multiplying this product by $dY/d\theta$ (inches/radian). Values for this latter term may be obtained from the heading of a graph (VEL. MAX or VEL. MIN).

The acceleration of the slider may be written:

$$\frac{d^2Y}{dt^2} = \frac{d}{dt}\left[\frac{dY}{d\theta}\ \frac{d\theta}{dt}\right]$$

$$= \frac{d}{d\theta}\left[\frac{dY}{d\theta}\right]\left[\frac{d\theta}{dt}\right]^2 + \left[\frac{dY}{d\theta}\right]\left[\frac{d^2\theta}{dt^2}\right]$$

$$= \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{d\theta}{dt}\right]^2 + \left[\frac{dY}{d\theta}\right]\left[\frac{d^2\theta}{dt^2}\right]. \qquad (2\text{-}3)$$

If the angular speed of link A remains constant then the angular acceleration of link A, $d^2\theta/dt^2$, is zero. The expression for the linear acceleration of the slider, Eq. 2-3, with link A turning with constant speed simplifies to

$$\frac{d^2Y}{dt^2} = \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{d\theta}{dt}\right]^2$$

$$= \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{\pi}{30} \times \text{rpm}\right]^2, \quad \frac{\text{inches}}{\text{second}^2}. \qquad (2\text{-}4)$$

Values for $d^2Y/d\theta^2$ may be obtained from a graph (the series of small triangles) or the extreme values may be obtained from the heading of a graph (ACC. MAX or ACC. MIN).

Referring to Fig. 2-2 the equations relating the output to the input may be derived. Looking at the basic slider-crank mechanism defined by the links A, B, and C, as projected onto a vertical line:

$$A \sin\theta = B \sin\gamma + C. \qquad (2\text{-}5)$$

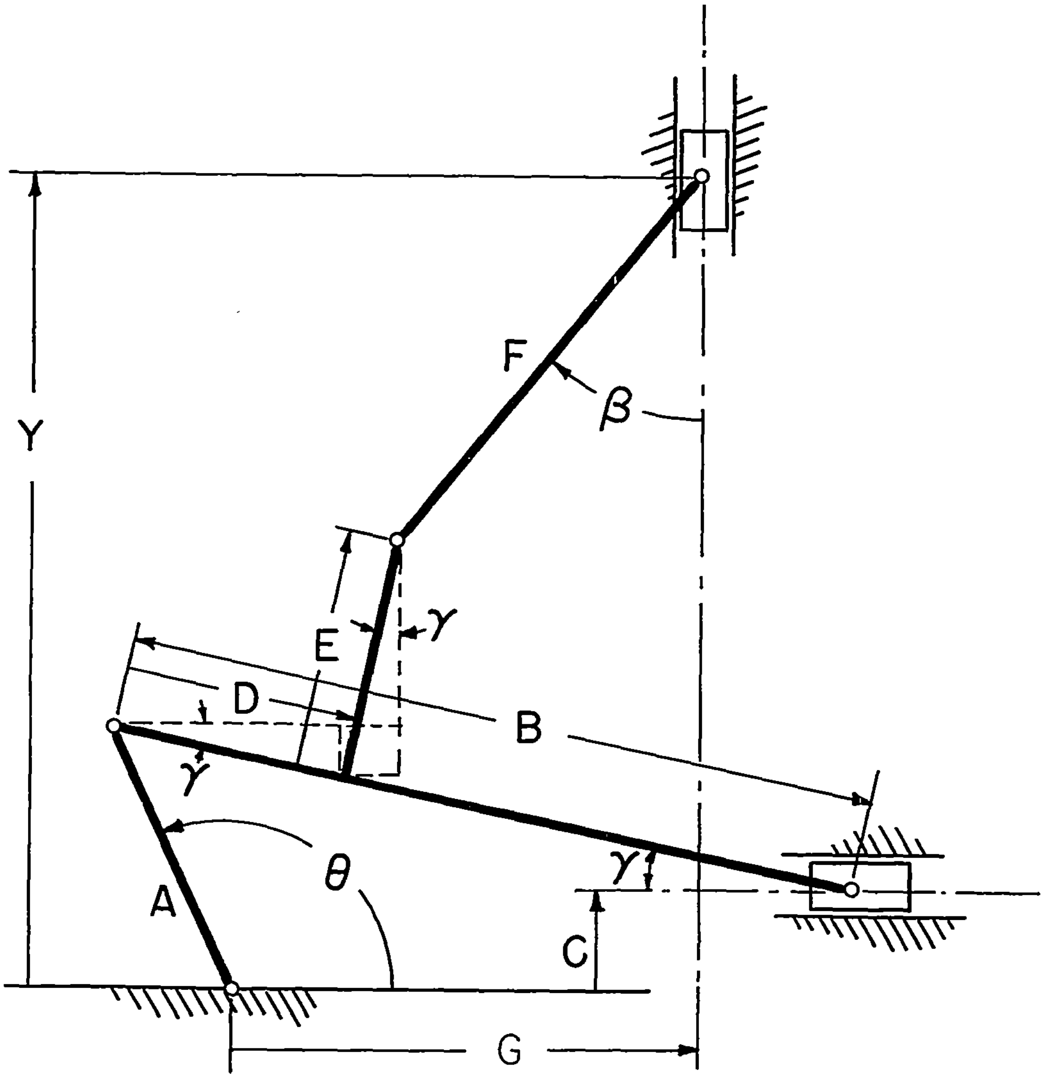


Figure 2-2

Looking at the complete mechanism as projected onto a horizontal line

$$A \cos \theta + D \cos \gamma + F \sin \beta + E \sin \gamma = G \cdot \qquad (2\text{-}6)$$

Given values for A, B, and C and for a specific value, θ, the angle γ may be determined from Eq. 2-5. With all of the link lengths specified and knowing γ then the angle β can be calculated from Eq. 2-6. With this information, the output, Y, can be written as: .

$$Y = A \sin \theta + E \cos \gamma - D \sin \gamma + F \cos \beta . \qquad (2\text{-}7)$$

The accompanying graphs show Y as a function of θ for selected values of the link lengths. The relationships of Eqs. 2-5, 2-6, and 2-7 afford the evaluation of the displacement which is shown as the solid line on the graphs.

Linear Velocity

The linear velocity of the slider, that has been linked to a basic slider-crank mechanism, may be determined by differentiating Eq. 2-7 with respect to θ and then substituting this expression into Eq. 2-2. Performing the differentiation produces:

$$\frac{dY}{d\theta} = A \cos\theta - (E \sin\gamma + D \cos\gamma)\frac{d\gamma}{d\theta} - F \sin\beta \frac{d\beta}{d\theta} . \qquad (2\text{-}8)$$

The value of $d\gamma/d\theta$ may be determined by differentiating Eq. 2-5 with respect to θ as:

$$A \cos\theta = B \cos\gamma \frac{d\gamma}{d\theta}$$

or

$$\frac{d\gamma}{d\theta} = \frac{A \cos\theta}{B \cos\gamma} . \qquad (2\text{-}9)$$

Differentiating Eq. 2-6 with respect to θ and transposing will yield:

$$\frac{d\beta}{d\theta} = \frac{A \sin\theta + (D \sin\gamma - E \cos\gamma)\frac{d\gamma}{d\theta}}{F \cos\beta} . \qquad (2\text{-}10)$$

With Eqs. 2-9 and 2-10 the expression for the velocity, Eq. 2-8, may be evaluated. This with Eq. 2-2 determines the linear velocity of the linked slider.

Linear Acceleration

The linear acceleration of the slider may be determined, once $d^2Y/d\theta^2$ is known, by means of Eq. 2-4. Differentiating Eq. 2-8 with respect to θ will result in:

$$\frac{d^2Y}{d\theta^2} = -A\sin\theta - (E\sin\gamma + D\cos\gamma)\frac{d^2\gamma}{d\theta^2} - (E\cos\gamma - D\sin\gamma)\left[\frac{d\gamma}{d\theta}\right]^2 - F\cos\beta\left[\frac{d\beta}{d\theta}\right]^2 - F\sin\beta\,\frac{d^2\beta}{d\theta^2}. \tag{2-11}$$

The terms that are known in this equation include A, D, E, F, and appropriate values for θ. Also, γ, $d\gamma/d\theta$, and $d\beta/d\theta$ may be calculated by equations previously noted. Two terms remain to be determined. One of these may be obtained by differentiating Eq. 2-9 with respect to θ:

$$\frac{d^2\gamma}{d\theta^2} = \frac{B\sin\gamma\left[\frac{d\gamma}{d\theta}\right]^2 - A\sin\theta}{B\cos\gamma}. \tag{2-12}$$

Eq. 2-10 may be differentiated with respect to θ to yield:

$$\frac{d^2\beta}{d\theta^2} = \left\{A\cos\theta + (D\sin\gamma - E\cos\gamma)\frac{d^2\gamma}{d\theta^2} + (D\cos\gamma + E\sin\gamma)\left[\frac{d\gamma}{d\theta}\right]^2 + F\sin\beta\left[\frac{d\beta}{d\theta}\right]^2\right\}\Big/\left[(\cos\beta)(F)\right]. \tag{2-13}$$

To evaluate Eq. 2-13 requires the value of $d^2\gamma/d\theta^2$ and this may be obtained from Eq. 2-12. With the equations developed the expression for $d^2Y/d\theta^2$ as given by Eq. 2-11 may be evaluated. The linear acceleration of the slider, d^2Y/dt^2 may in turn be evaluated using Eq. 2-4. The curves given by the series of small triangles depict $d^2Y/d\theta^2$ for several selections of the dimensions which define this Mechanism #2.

A= 4.00, B= 6.00, C= 1.00, D= 7.00,
E= 1.00, F=14.00, G= 5.00,
VEL.MAX= 0.70, VEL.MIN= -1.78, ACC.MAX= 2.97, ACC.MIN= -3.28,
PEAK TO PEAK DISPLACEMENT = 1.73

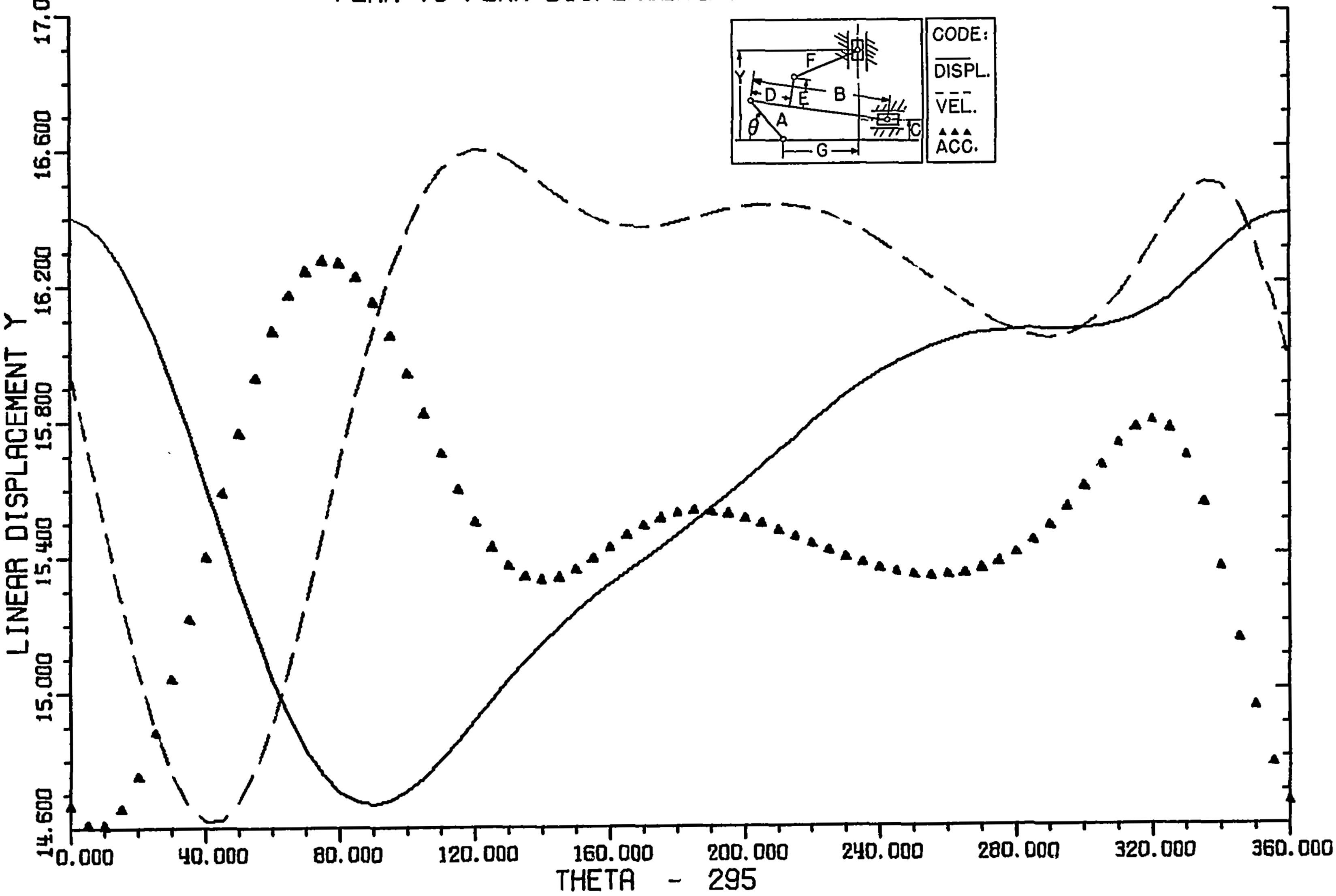


PLATE 2-1

A= 4.00, B= 6.00, C= 1.00, D= 7.00,
E= 4.00, F=13.00, G= 5.00,
VEL.MAX= 2.32, VEL.MIN= -1.85, ACC.MAX= 6.30, ACC.MIN= -5.06,
PEAK TO PEAK DISPLACEMENT = 1.79

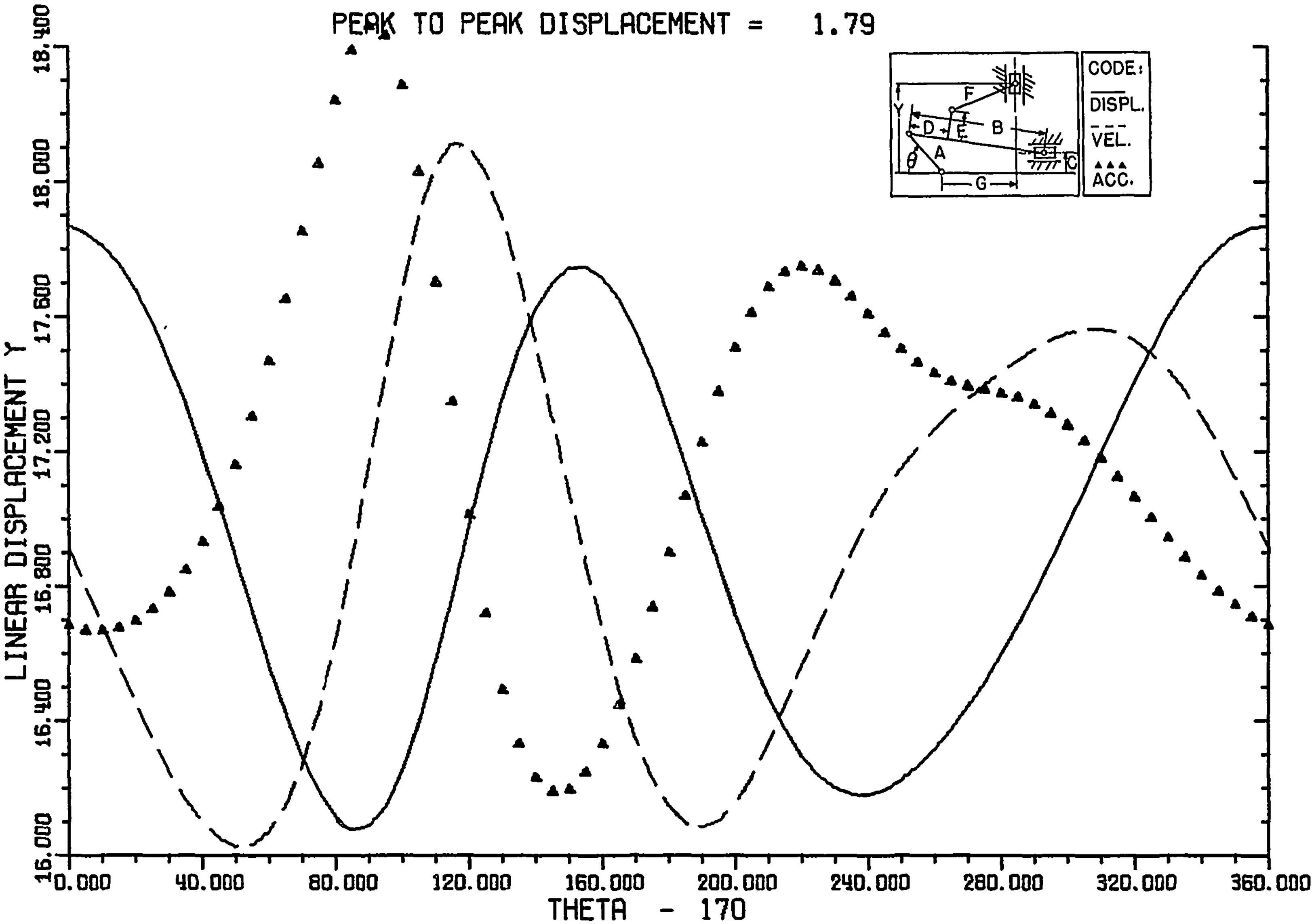


PLATE 2-2

A= 1.00, B= 7.00, C= 3.00, D= 2.00,
E= 3.00, F=15.00, G= 5.00,
VEL.MAX= 1.12, VEL.MIN= -1.17, ACC.MAX= 1.24, ACC.MIN= -1.07,
PEAK TO PEAK DISPLACEMENT = 2.29

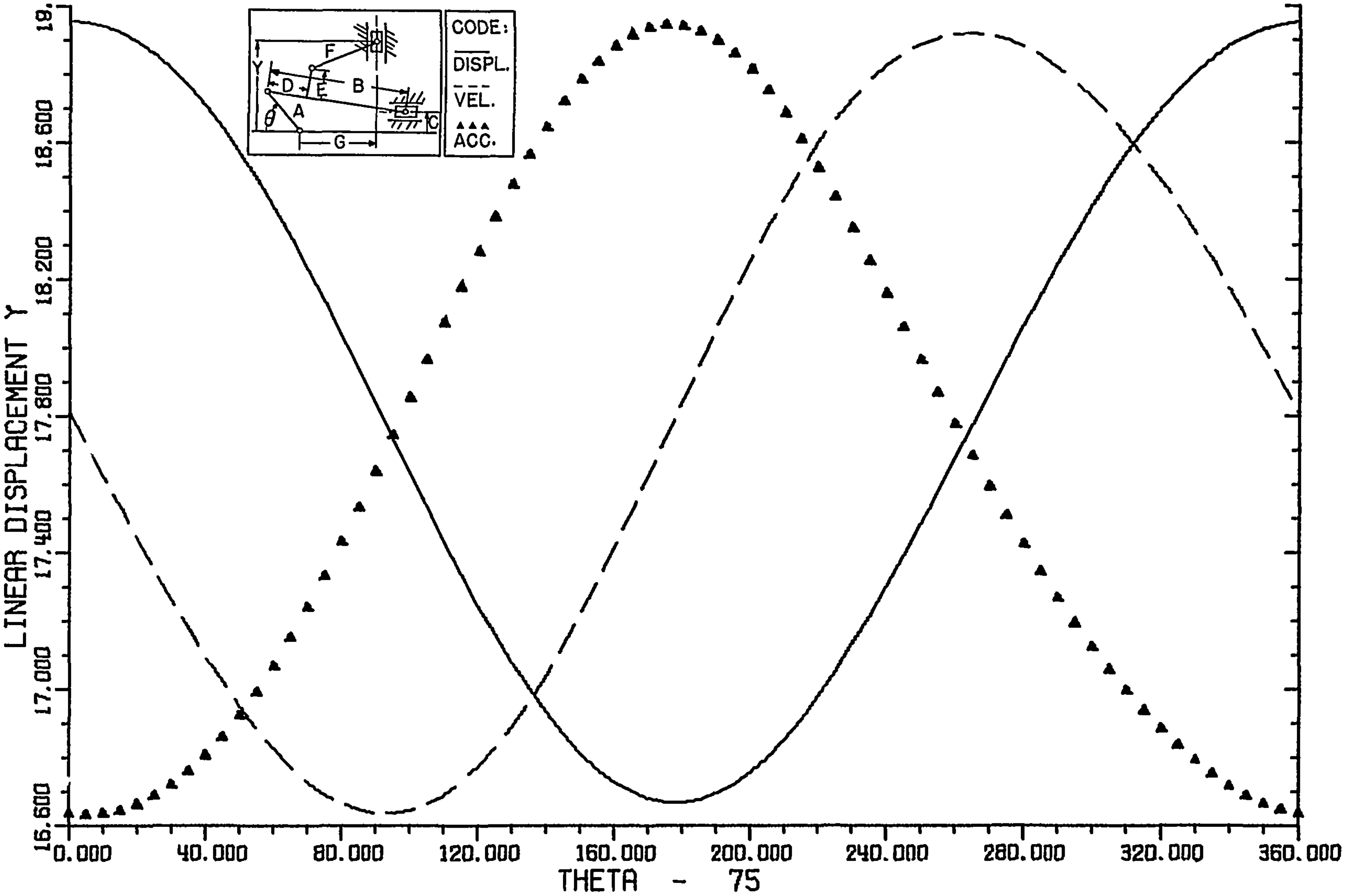


PLATE 2-3

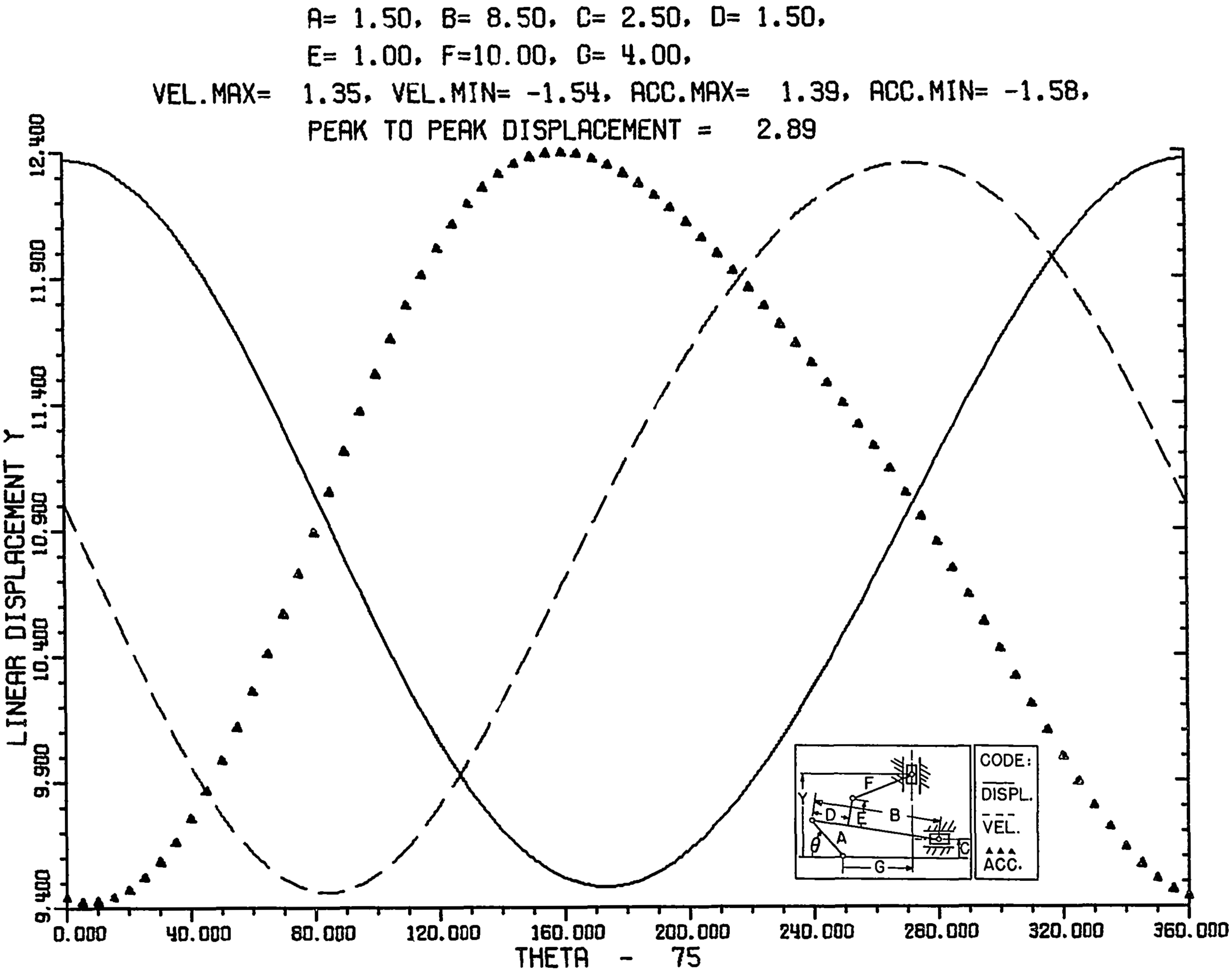

A= 1.50, B= 8.50, C= 2.50, D= 1.50,
E= 1.00, F=10.00, G= 4.00,
VEL.MAX= 1.35, VEL.MIN= -1.54, ACC.MAX= 1.39, ACC.MIN= -1.58,
PEAK TO PEAK DISPLACEMENT = 2.89
LINEAR DISPLACEMENT Y
12.400
11.900
11.400
10.900
10.400
9.900
9.400
9.
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 75
F
Y
D
E
B
A
θ
C
G
CODE:
DISPL.
VEL.
ACC.

PLATE 2-4

A= 2.00, B= 9.00, C= 4.00, D= 5.00,
E= 3.00, F=11.00, G= 6.00,
VEL.MAX= 1.54, VEL.MIN= -1.86, ACC.MAX= 2.21, ACC.MIN= -1.58,
PEAK TO PEAK DISPLACEMENT = 3.40

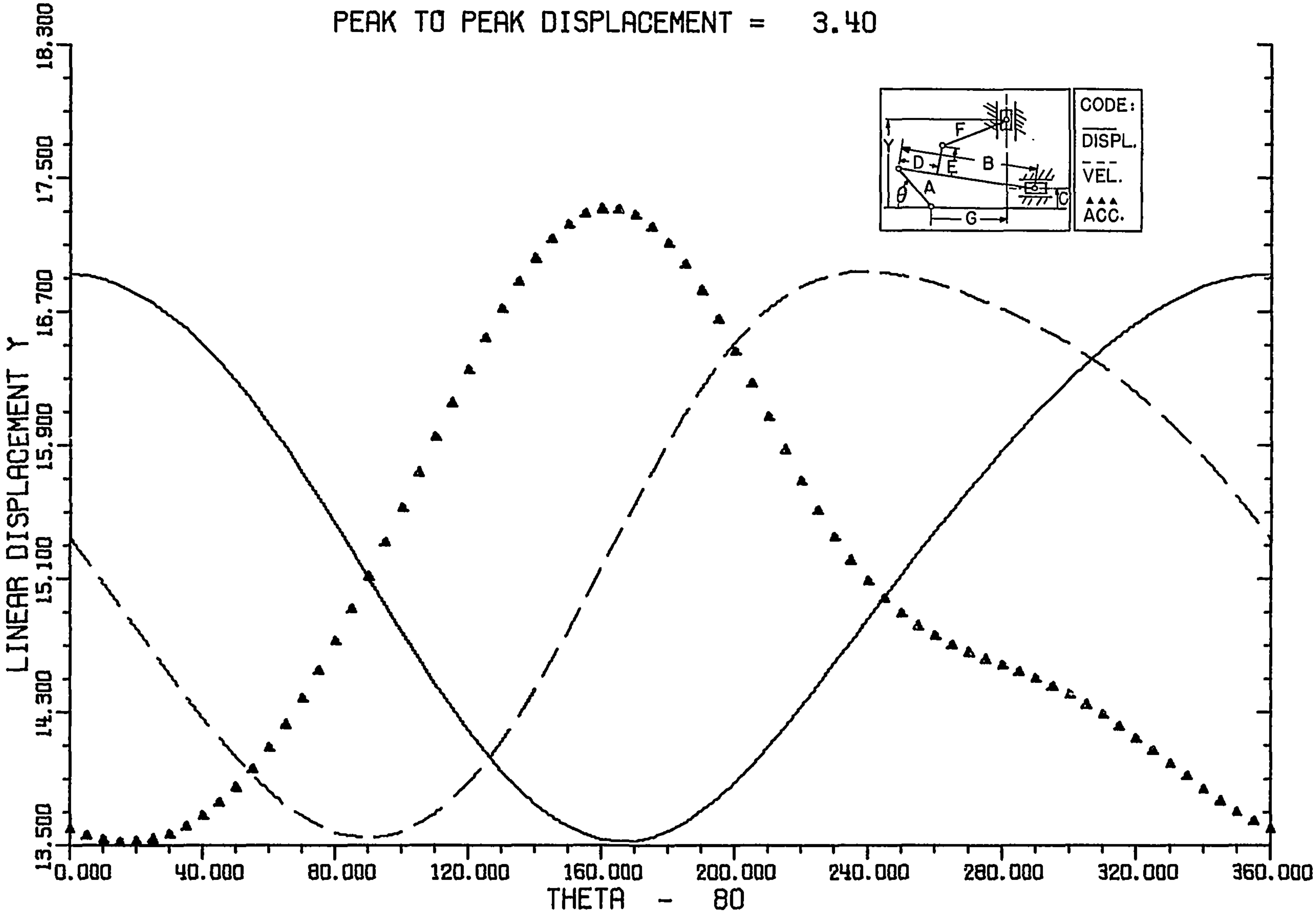


PLATE 2-5

A= 4.00, B=12.00, C= 4.00, D= 8.00,
E= 4.00, F=16.00, G= 5.00,
VEL.MAX= 1.65, VEL.MIN= -2.23, ACC.MAX= 2.61, ACC.MIN= -2.47,
PEAK TO PEAK DISPLACEMENT = 3.66

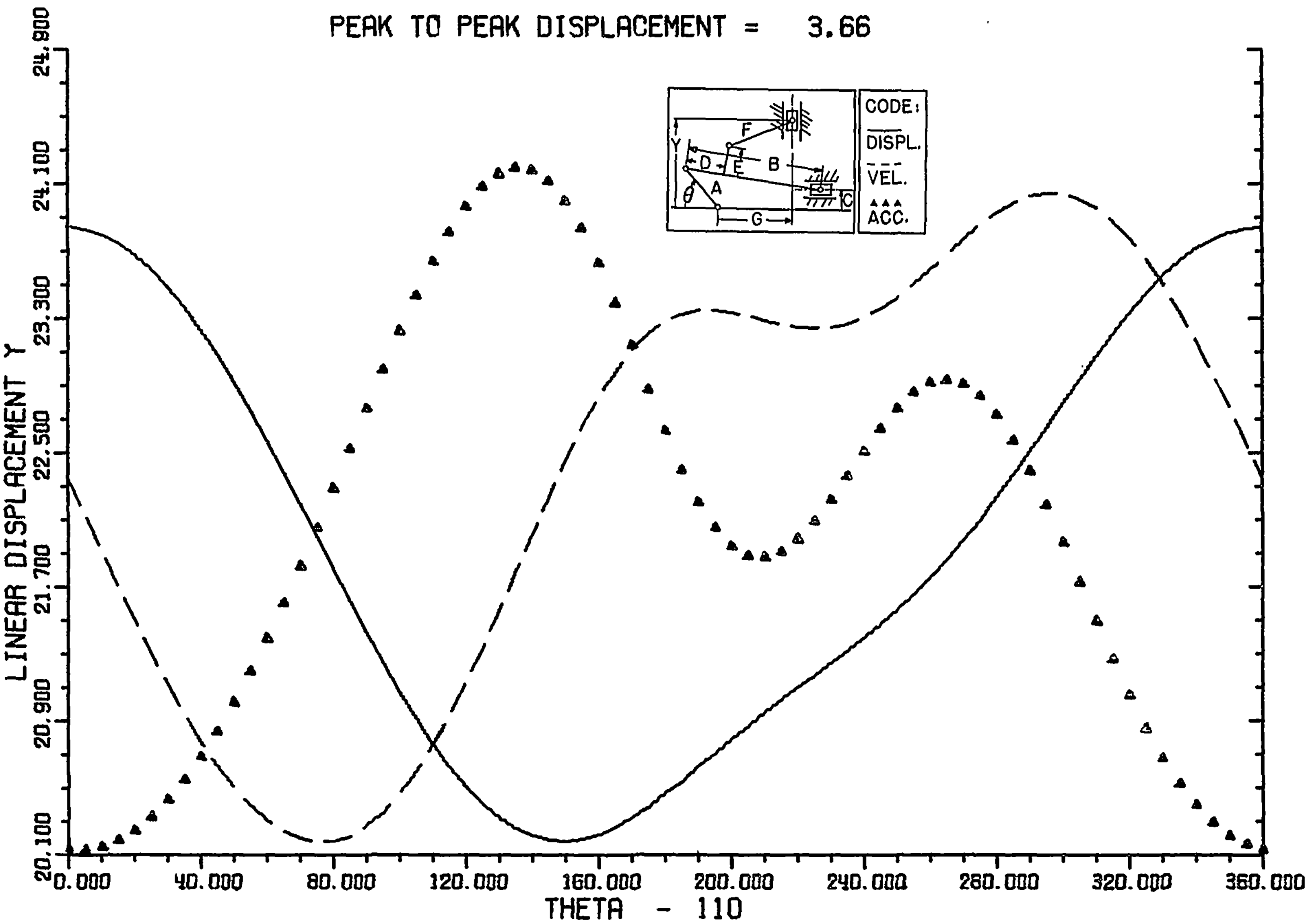


PLATE 2-6

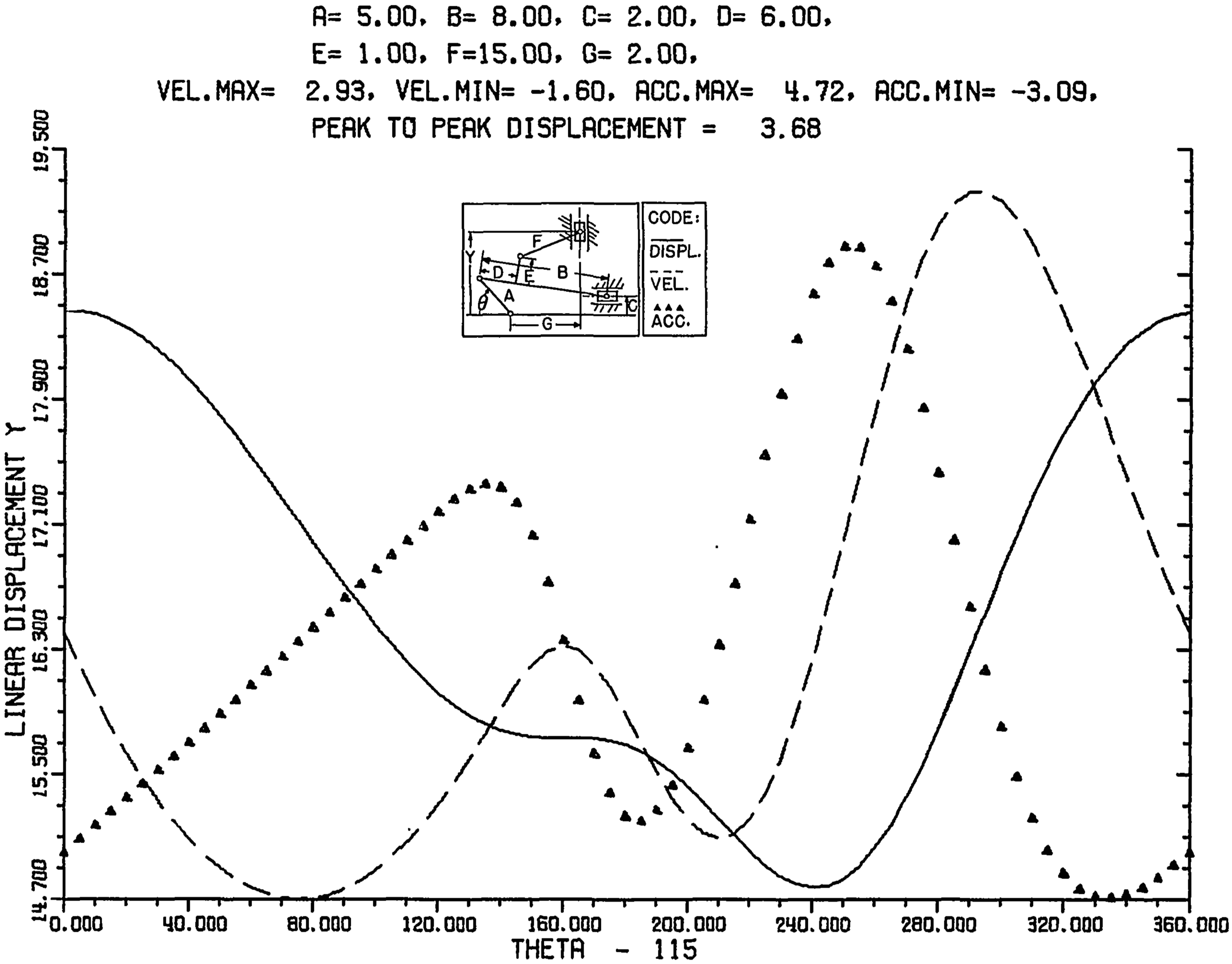

A= 5.00, B= 8.00, C= 2.00, D= 6.00,
E= 1.00, F=15.00, G= 2.00,
VEL.MAX= 2.93, VEL.MIN= -1.60, ACC.MAX= 4.72, ACC.MIN= -3.09,
PEAK TO PEAK DISPLACEMENT = 3.68
LINEAR DISPLACEMENT Y
19.500
18.700
17.900
17.100
16.300
15.500
14.700
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 115
CODE:
DISPL.
VEL.
ACC.
F
Y
D
E
B
A
θ
C
G

PLATE 2-7

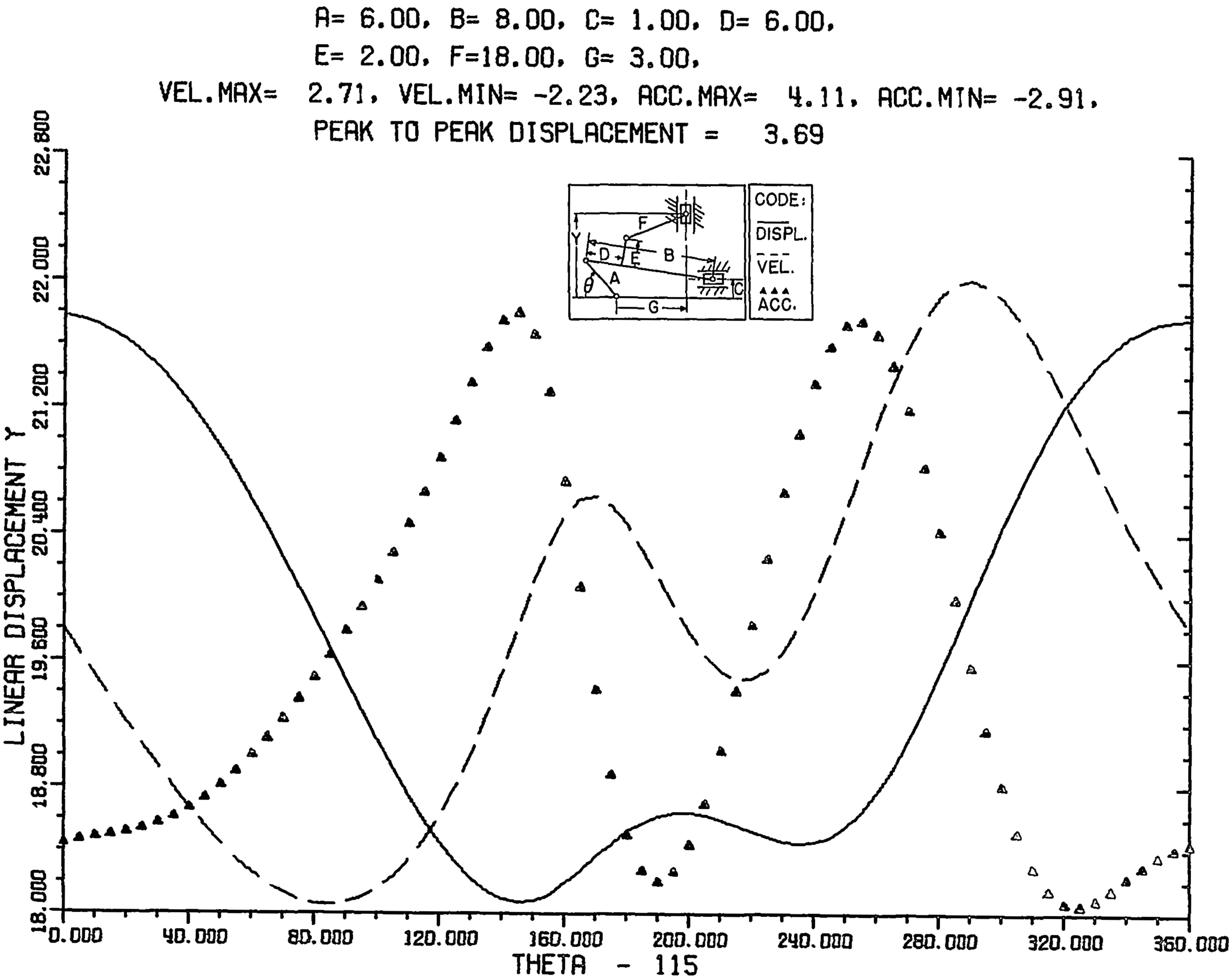

A= 6.00, B= 8.00, C= 1.00, D= 6.00,
E= 2.00, F=18.00, G= 3.00,
VEL.MAX= 2.71, VEL.MIN= -2.23, ACC.MAX= 4.11, ACC.MIN= -2.91,
PEAK TO PEAK DISPLACEMENT = 3.69
LINEAR DISPLACEMENT Y
22.800
22.000
21.200
20.400
19.600
18.800
18.000
CODE:
DISPL.
VEL.
ACC.
F
Y
D
E
B
A
θ
C
G
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 115

PLATE 2-8

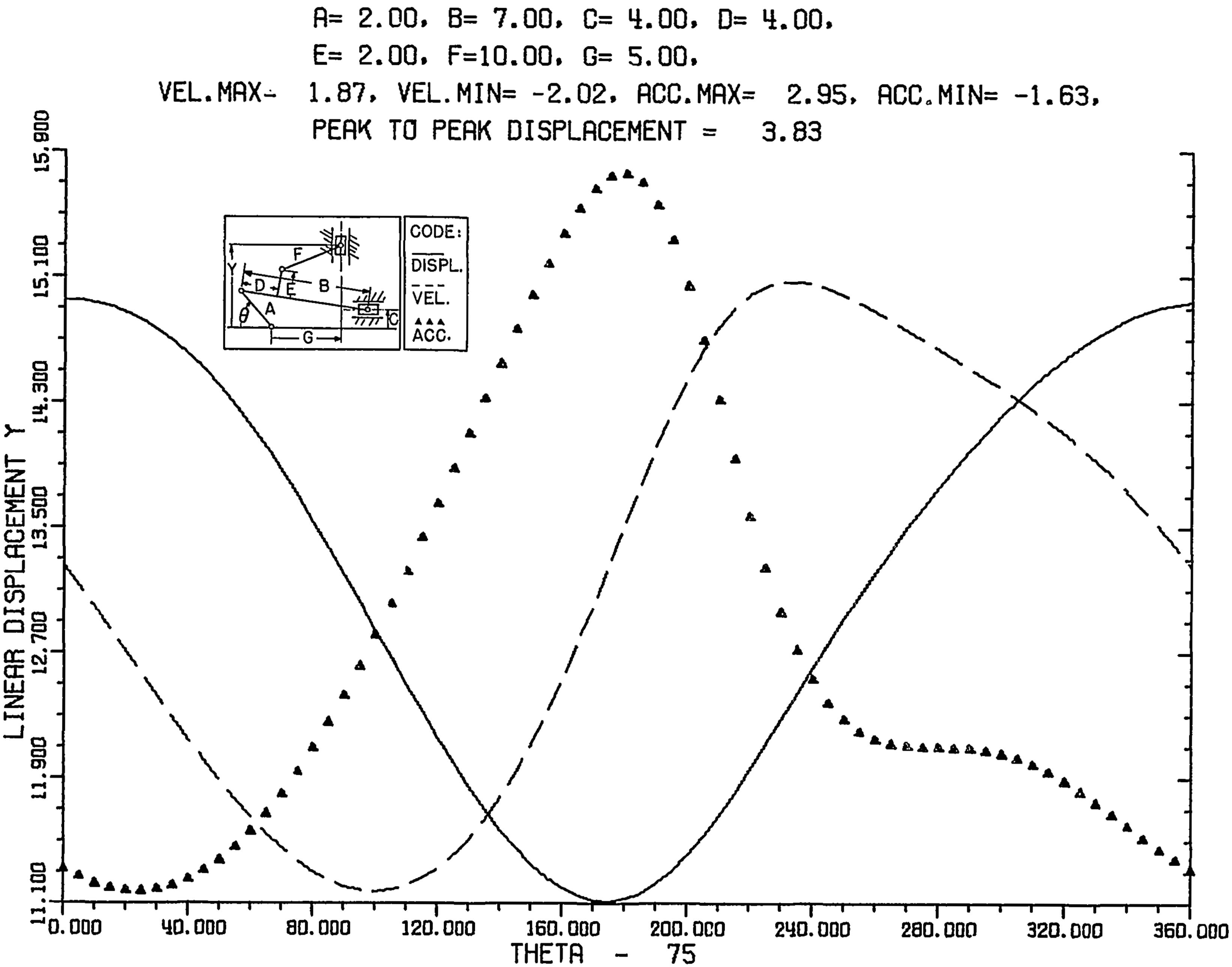

A= 2.00, B= 7.00, C= 4.00, D= 4.00,
E= 2.00, F=10.00, G= 5.00,
VEL.MAX= 1.87, VEL.MIN= -2.02, ACC.MAX= 2.95, ACC.MIN= -1.63,
PEAK TO PEAK DISPLACEMENT = 3.83
LINEAR DISPLACEMENT Y
15.900
15.100
14.300
13.500
12.700
11.900
11.100
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 75
CODE:
DISPL.
VEL.
ACC.
Y
F
D
E
B
A
θ
C
G

PLATE 2-9

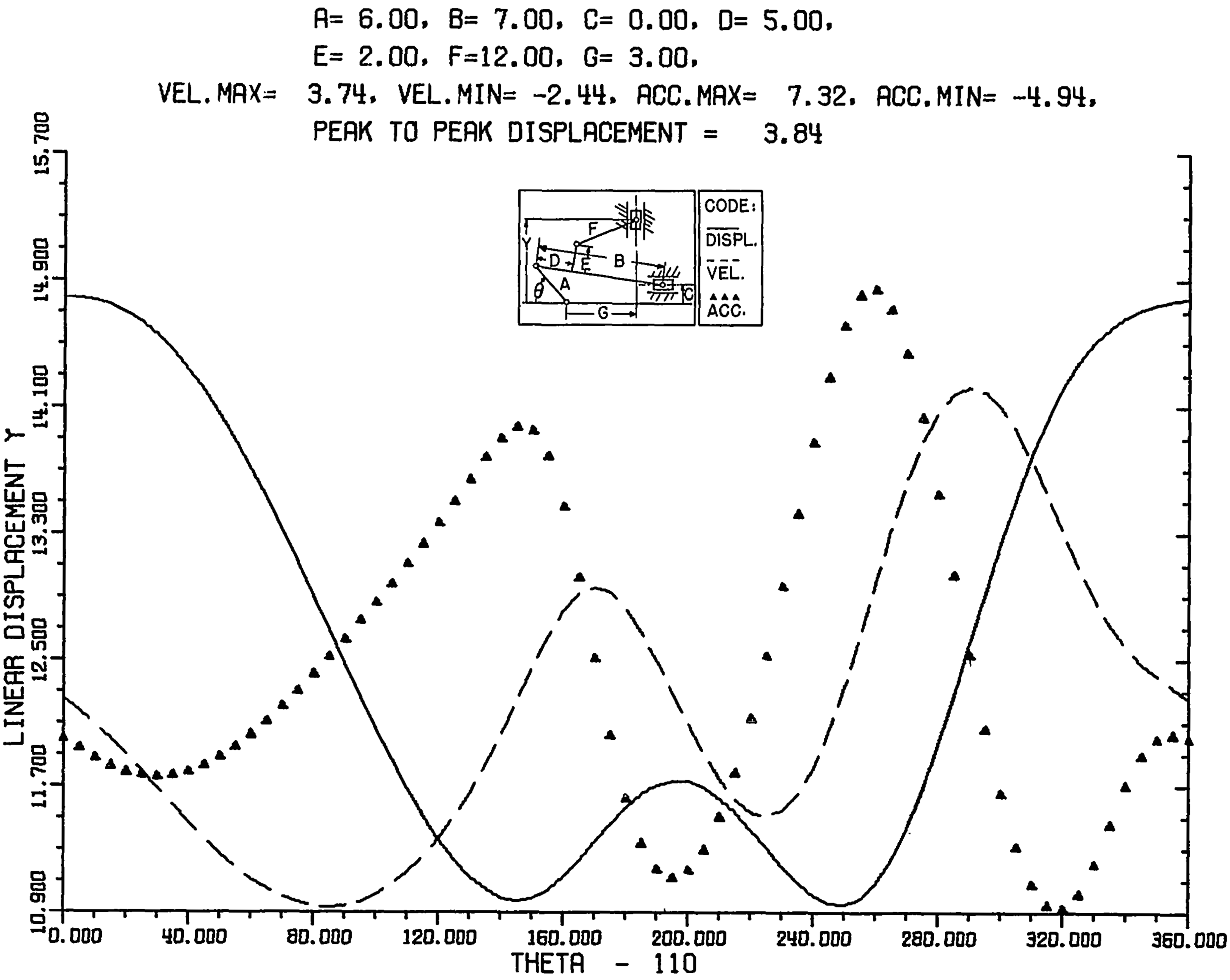

A= 6.00, B= 7.00, C= 0.00, D= 5.00,
E= 2.00, F=12.00, G= 3.00,
VEL.MAX= 3.74, VEL.MIN= -2.44, ACC.MAX= 7.32, ACC.MIN= -4.94,
PEAK TO PEAK DISPLACEMENT = 3.84
CODE:
DISPL.
VEL.
ACC.
Y
F
D
E
B
A
θ
G
C
LINEAR DISPLACEMENT Y
15.700
14.900
14.100
13.300
12.500
11.700
10.900
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 110

PLATE 2-10

A= 1.00, B= 8.00, C= 6.00, D= 4.00,
E= 4.00, F=12.00, G= 7.00,
VEL.MAX= 2.18, VEL.MIN= -2.12, ACC.MAX= 2.88, ACC.MIN= -1.63,
PEAK TO PEAK DISPLACEMENT = 4.25

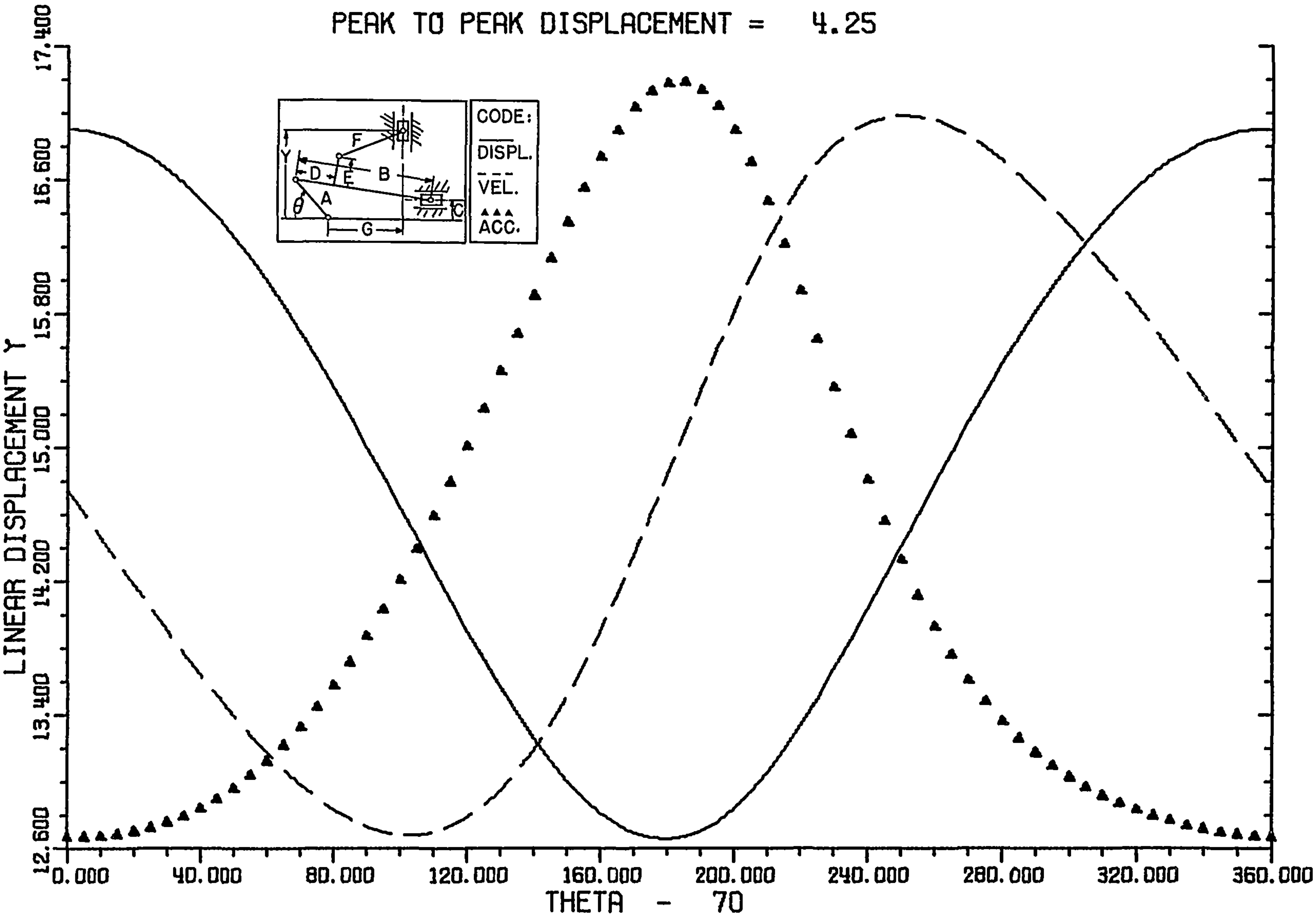


PLATE 2-11

A= 7.00, B= 8.00, C= 0.00, D= 6.00,
E= 3.00, F=13.00, C= 4.00,
VEL.MAX= 4.00, VEL.MIN= -3.22, ACC.MAX= 9.44, ACC.MIN= 0.84,
PEAK TO PEAK DISPLACEMENT = 4.56

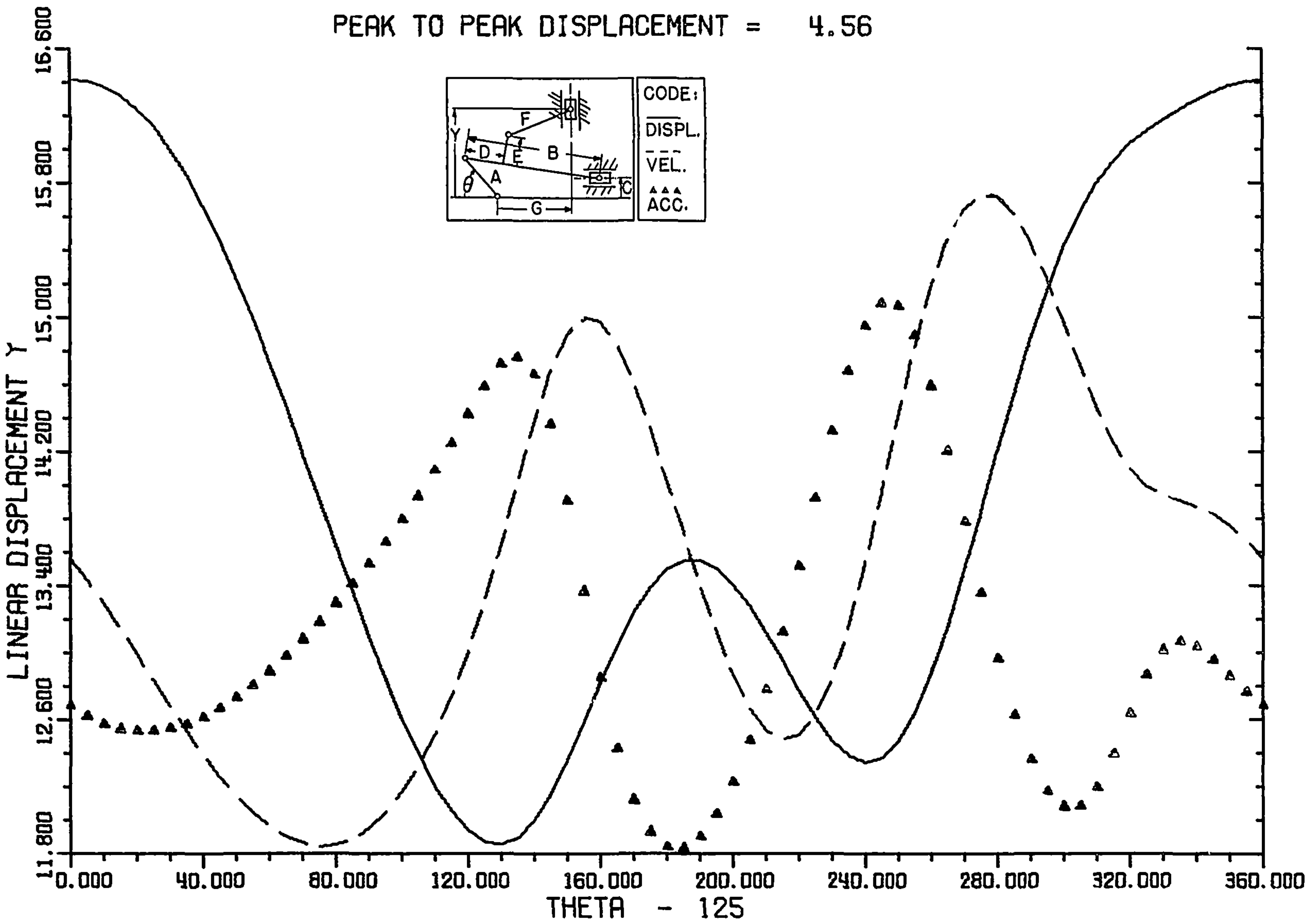


PLATE 2-12

A= 4.00, B= 6.00, C= 1.00, D= 5.00,
E= 5.00, F=14.00, G= 5.00,
VEL.MAX= 3.98, VEL.MIN= -3.62, ACC.MAX= 9.16, ACC.MIN= -4.29,
PEAK TO PEAK DISPLACEMENT = 4.84

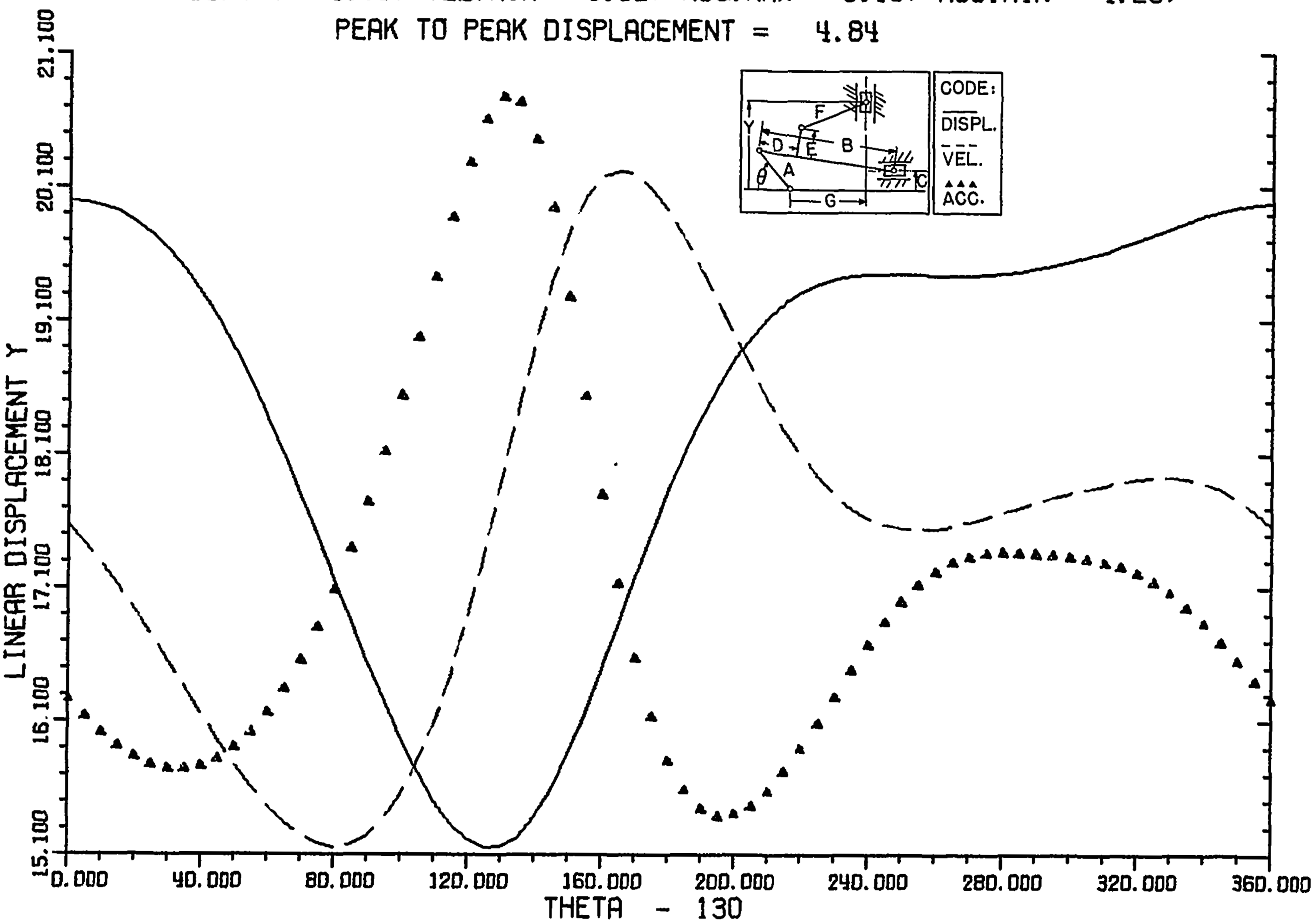


PLATE 2-13

A= 2.50, B= 6.50, C= 2.00, D= 1.50,
E= 2.00, F=11.00, G= 3.00,
VEL.MAX= 2.19, VEL.MIN= -2.68, ACC.MAX= 2.76, ACC.MIN= -2.54,
PEAK TO PEAK DISPLACEMENT = 4.96

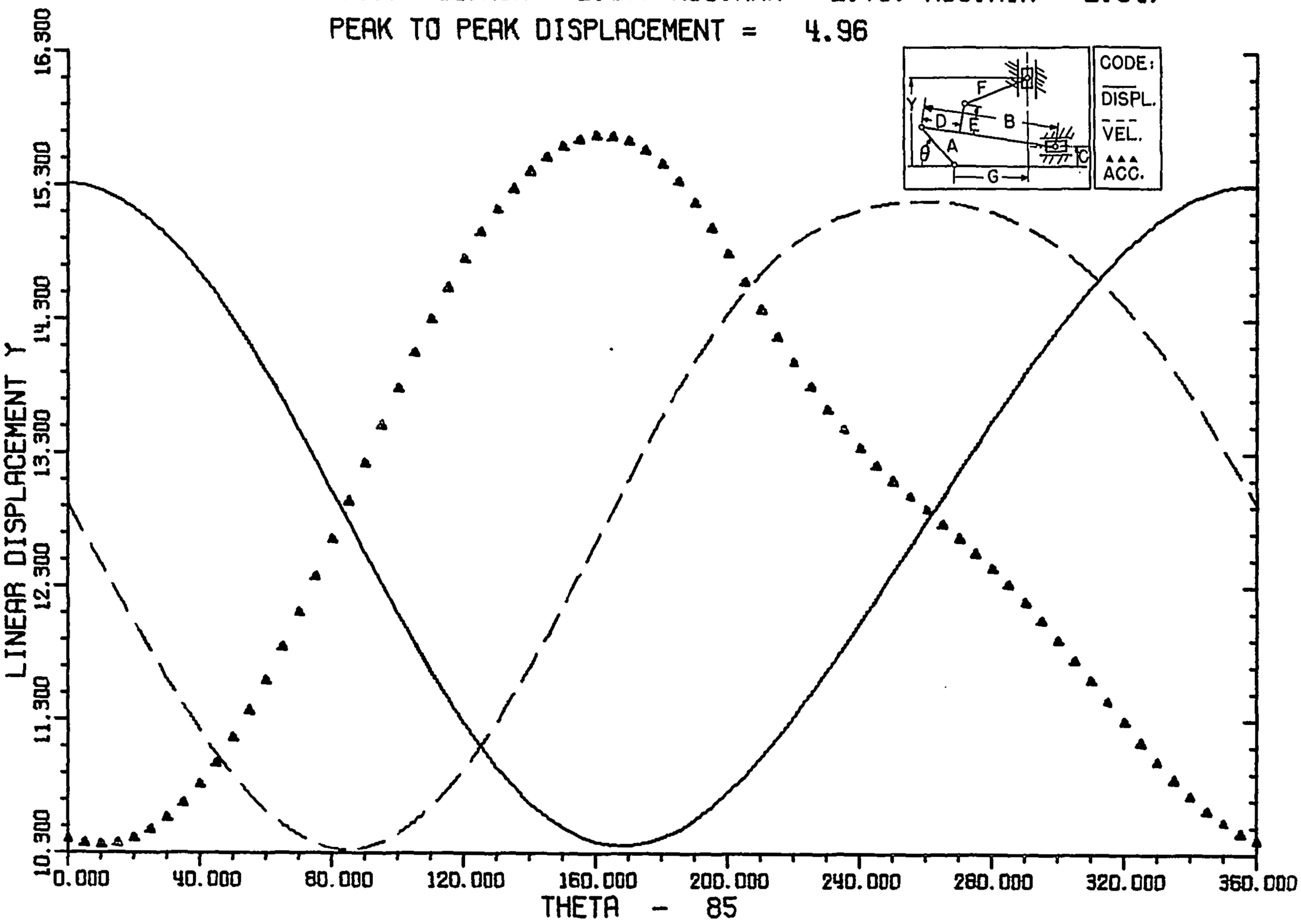


PLATE 2-14

A= 5.00, B= 7.00, C= 1.00, D= 8.00,
E= 3.00, F-15.00, G= 2.00,
VEL.MAX= 2.65, VEL.MIN= -4.24, ACC.MAX= 8.47, ACC.MIN= -6.03,
PEAK TO PEAK DISPLACEMENT = 5.06

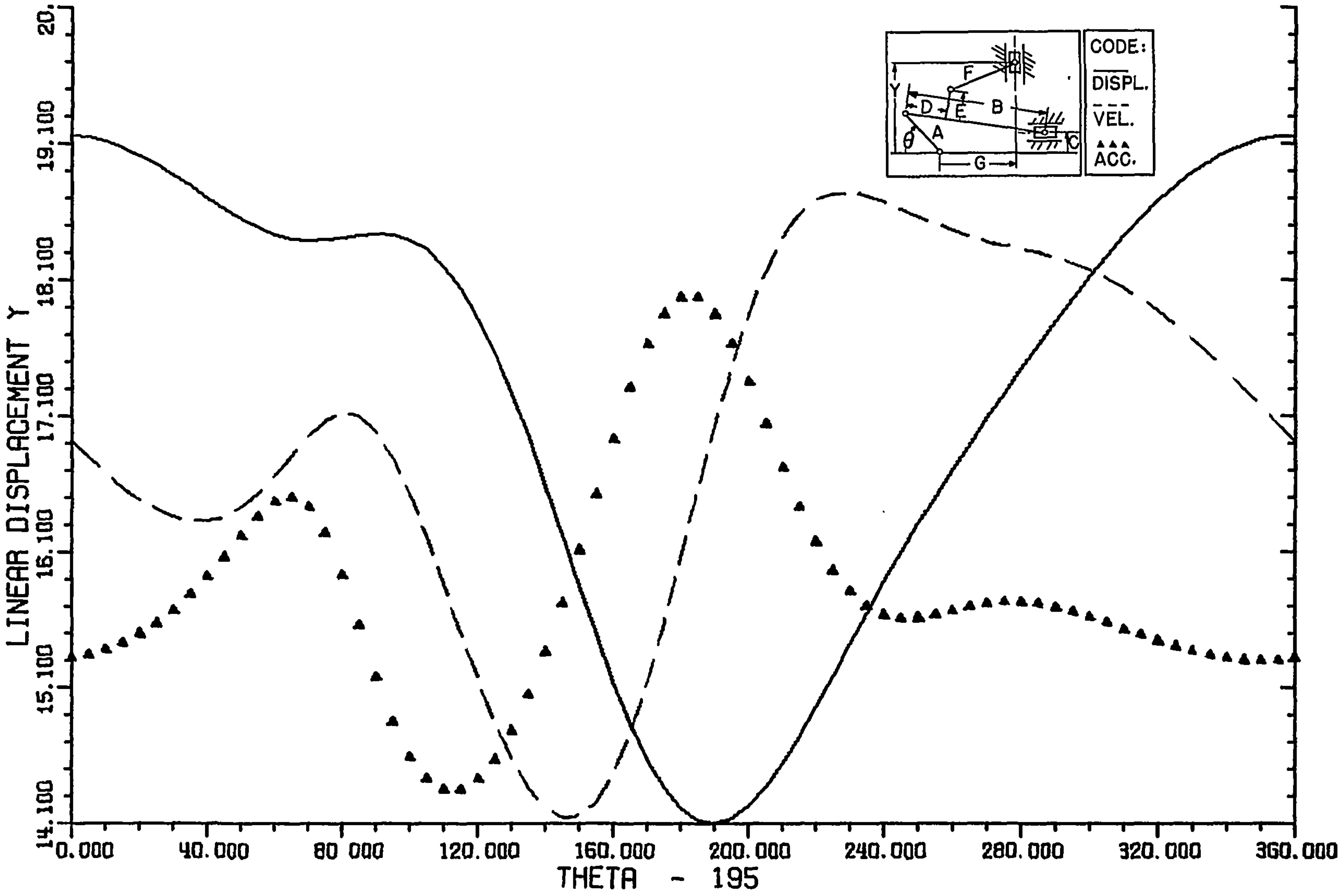


PLATE 2-15

A= 3.00, B= 6.00, C= 2.00, D= 4.00,
E= 1.00, F= 7.00, G= 6.00,
VEL.MAX= 2.82, VEL.MIN= -2.90, ACC.MAX= 4.55, ACC.MIN= -2.40,
PEAK TO PEAK DISPLACEMENT = 5.11

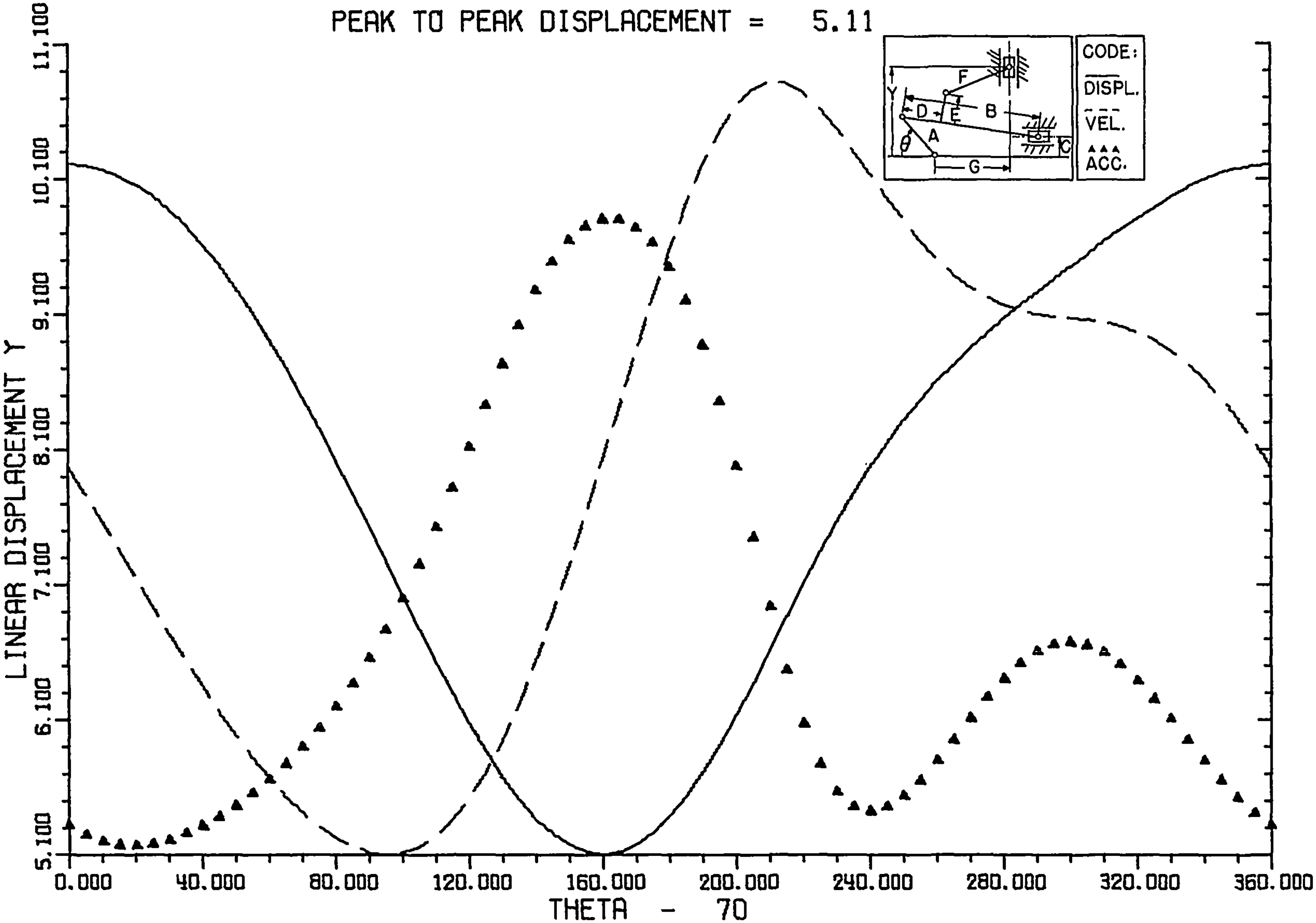


PLATE 2-16

A= 5.00, B= 8.00, C= 2.00, D= 4.00,
E= 1.00, F=10.00, G= 2.00,
VEL.MAX= 4.08, VEL.MIN= -3.15, ACC.MAX= 5.33, ACC.MIN= -5.00,
PEAK TO PEAK DISPLACEMENT = 5.43

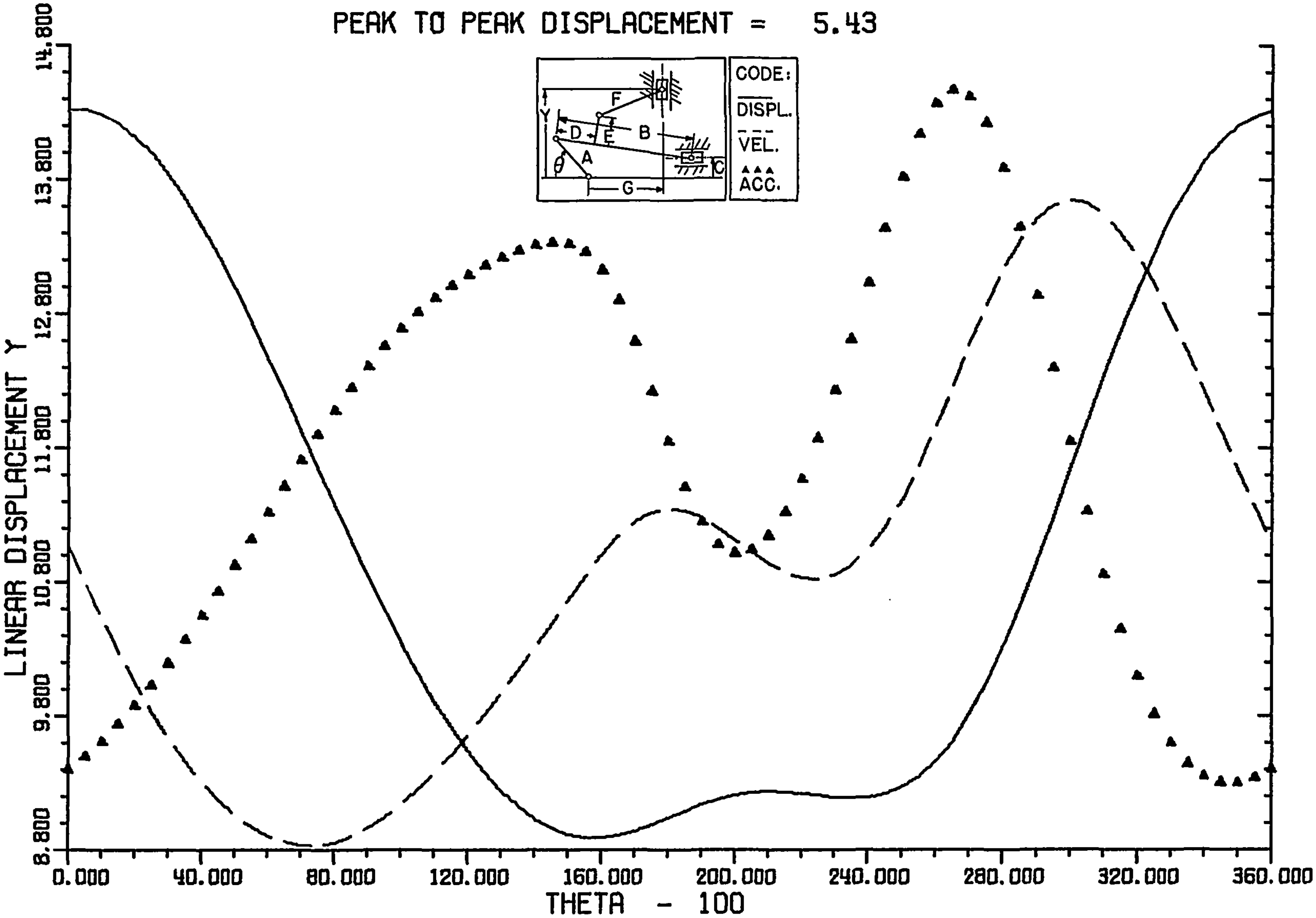


PLATE 2-17

A= 3.00, B= 9.00, C= 4.00, D= 4.00,
E= 2.00, F= 4.00, G= 3.00,
VEL.MAX= 2.53, VEL.MIN= -3.97, ACC.MAX= 6.98, ACC.MIN= -4.13,
PEAK TO PEAK DISPLACEMENT = 5.49

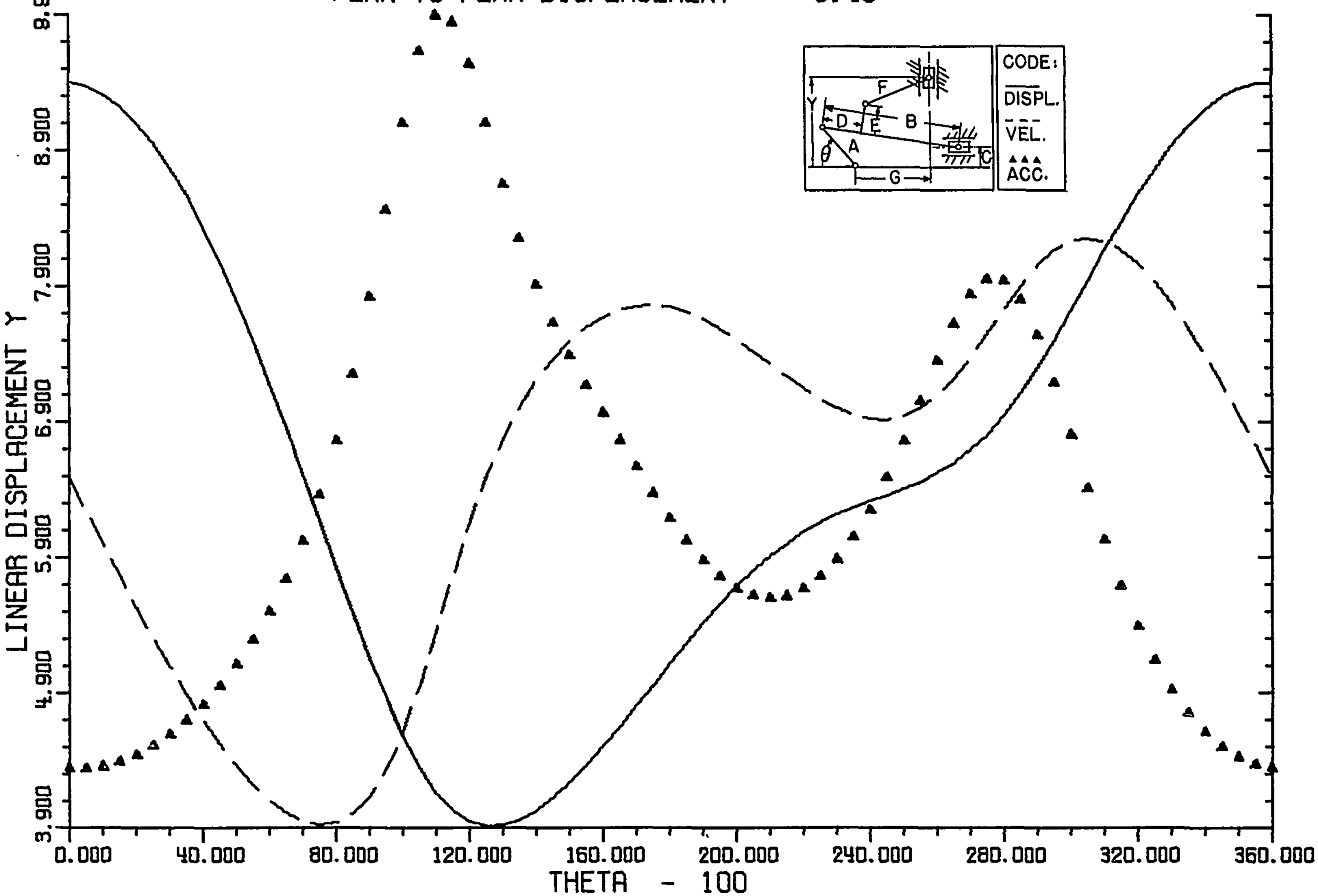
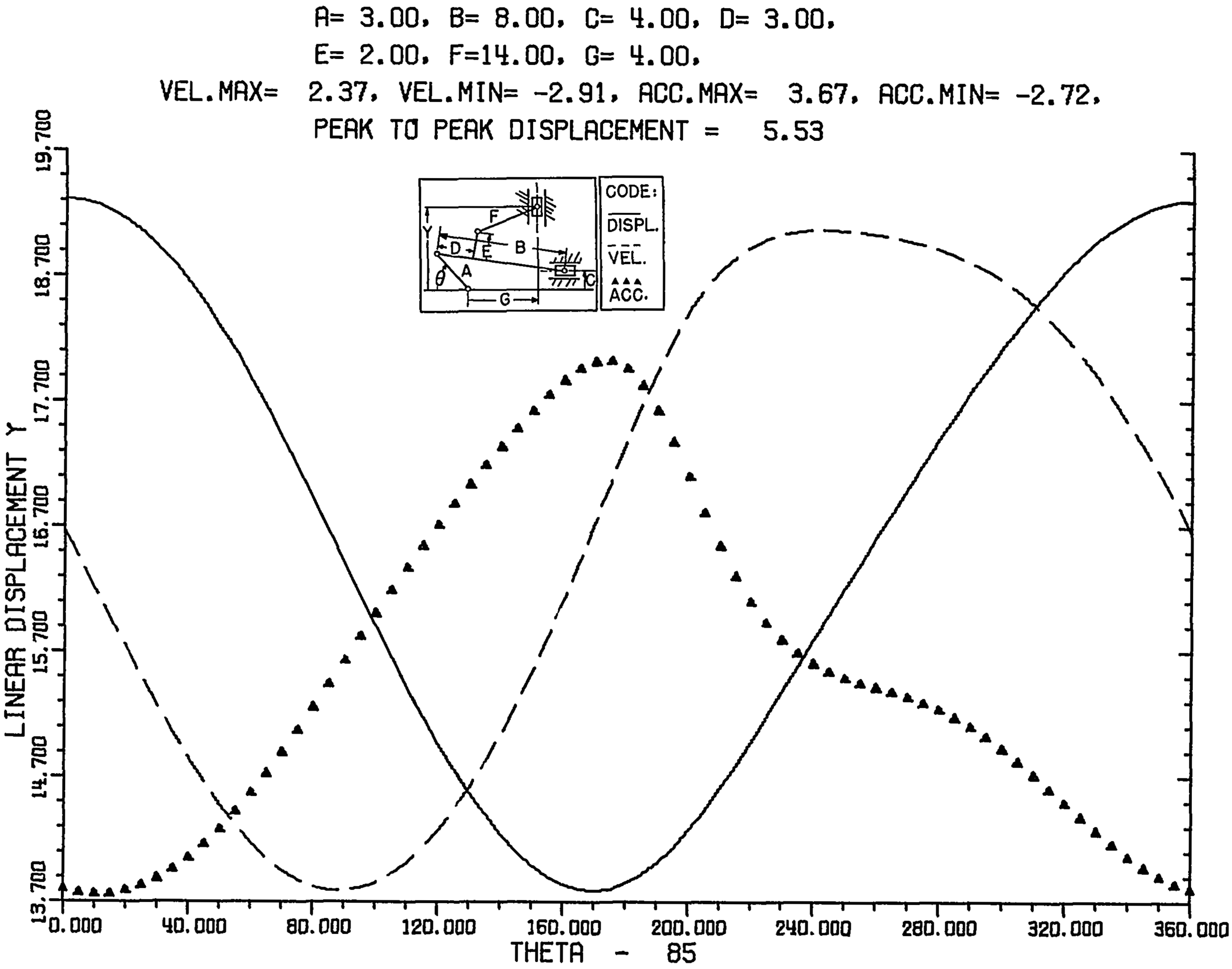


PLATE 2-18



A= 3.00, B= 8.00, C= 4.00, D= 3.00,
E= 2.00, F=14.00, G= 4.00,
VEL.MAX= 2.37, VEL.MIN= -2.91, ACC.MAX= 3.67, ACC.MIN= -2.72,
PEAK TO PEAK DISPLACEMENT = 5.53
LINEAR DISPLACEMENT Y
19.700
18.700
17.700
16.700
15.700
14.700
13.700
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 85
CODE:
DISPL.
VEL.
ACC.
Y
F
D
E
B
A
θ
C
G

PLATE 2-19

A= 7.00, B=12.00, C= 3.00, D= 8.00,
E= 3.00, F=17.00, G= 4.00,
VEL.MAX= 4.26, VEL.MIN= -3.55, ACC.MAX= 6.08, ACC.MIN= -4.95,
PEAK TO PEAK DISPLACEMENT = 5.82

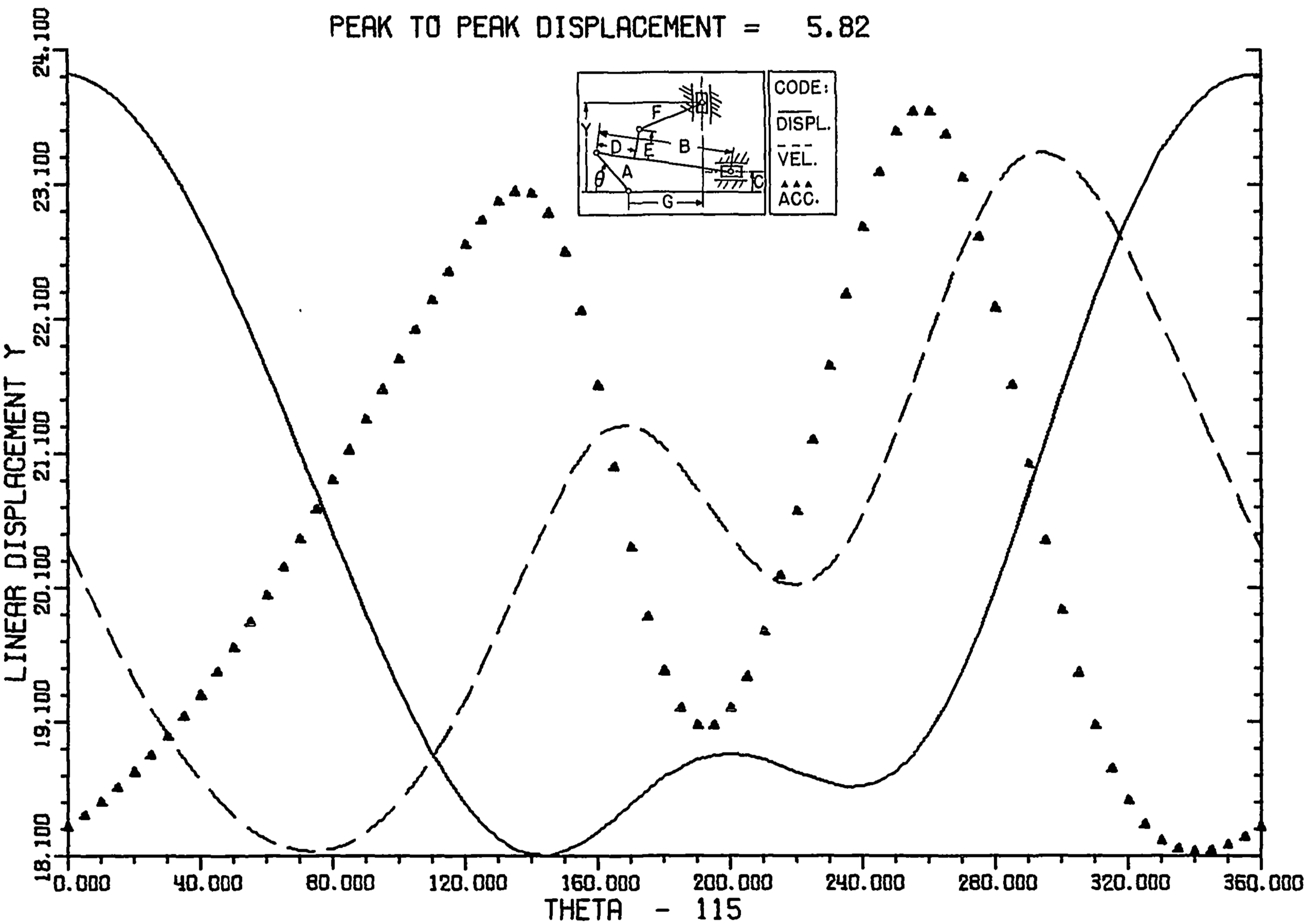


PLATE 2-20

A=-2.00, B=10.00, C= 7.00, D= 4.00,
E= 3.00, F=11.00, G= 6.00,
VEL.MAX= 2.88, VEL.MIN= -3.05, ACC.MAX= 4.11, ACC.MIN= -2.42,
PEAK TO PEAK DISPLACEMENT = 5.88

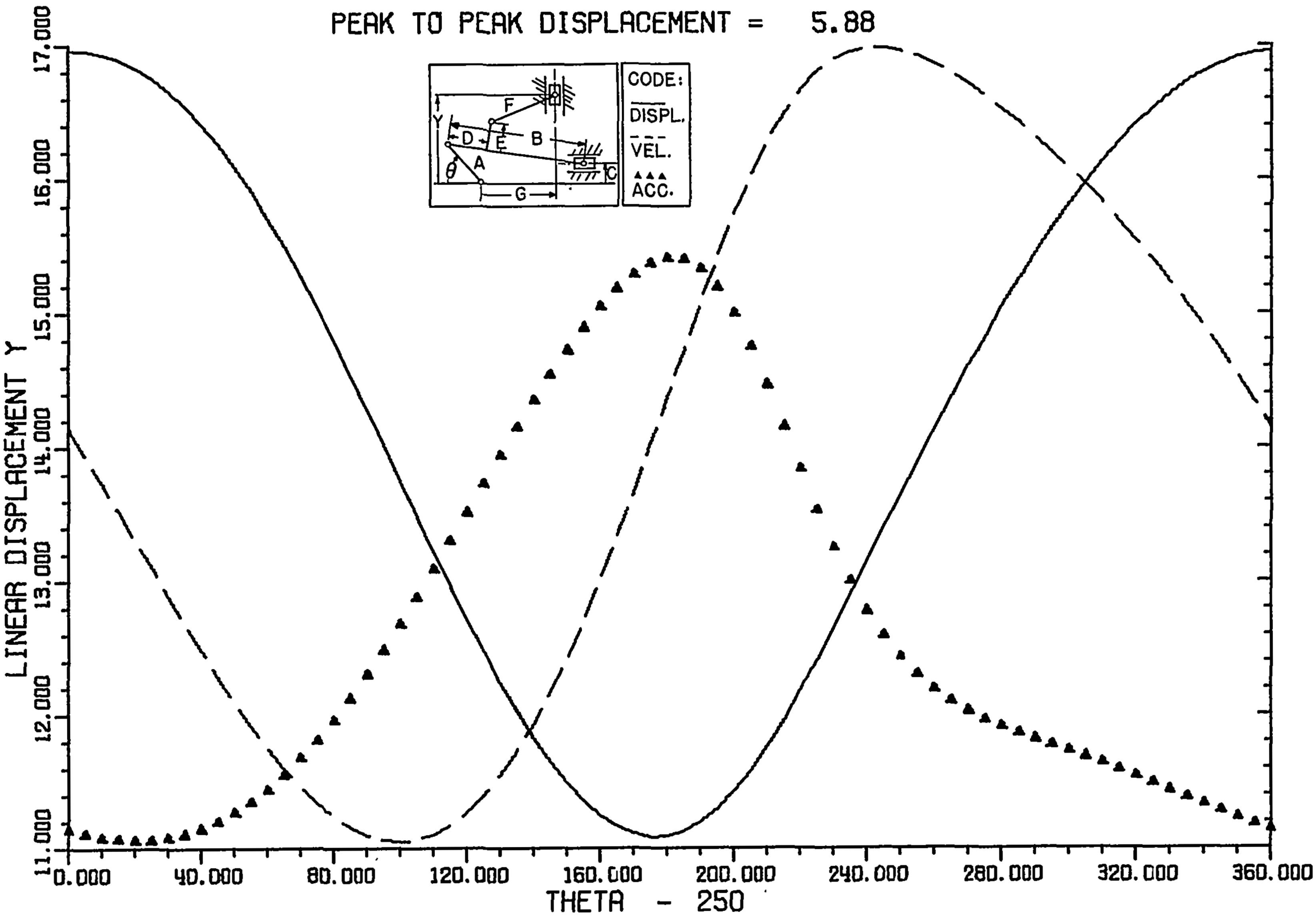


PLATE 2-21

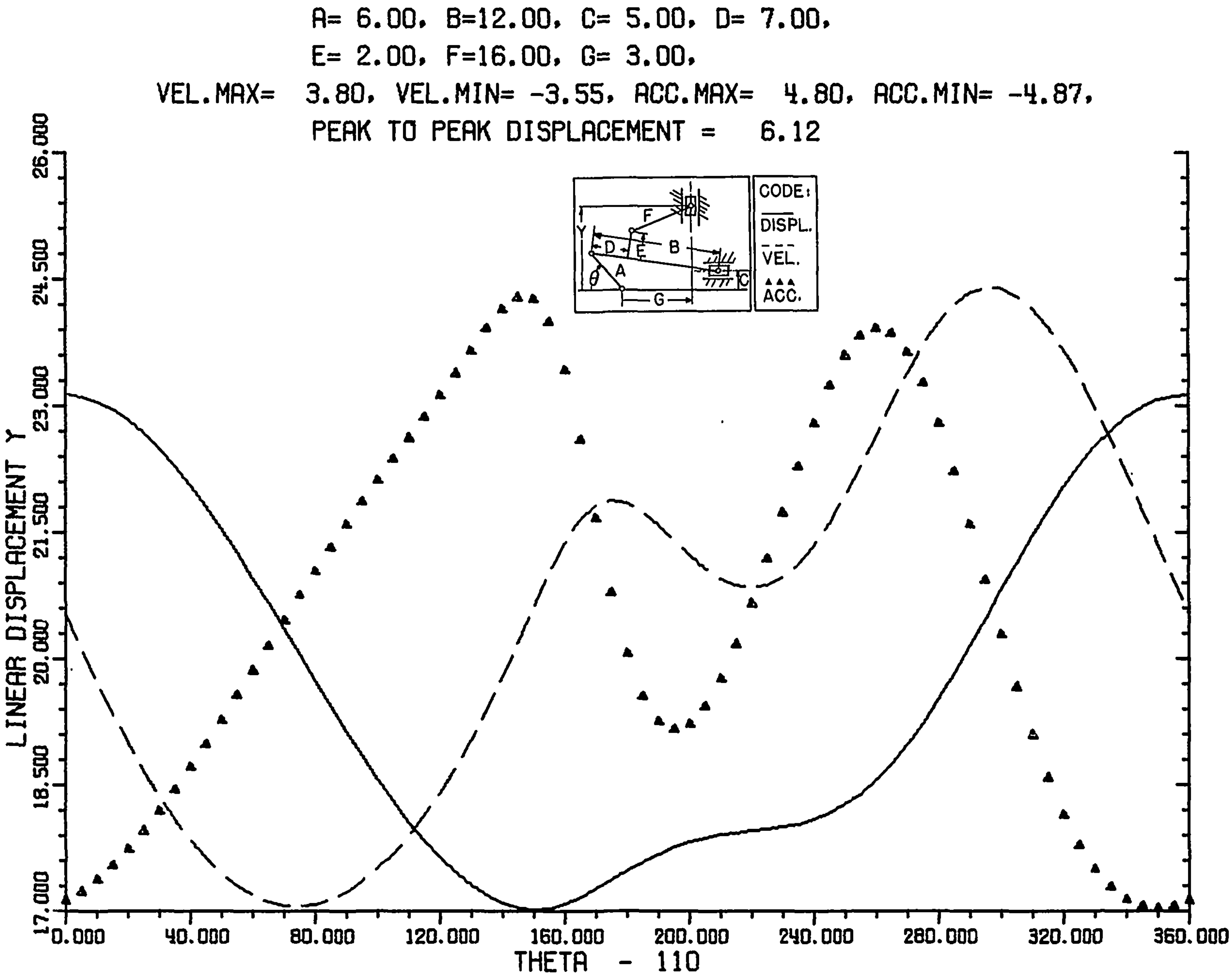

A= 6.00, B=12.00, C= 5.00, D= 7.00,
E= 2.00, F=16.00, G= 3.00,
VEL.MAX= 3.80, VEL.MIN= -3.55, ACC.MAX= 4.80, ACC.MIN= -4.87,
PEAK TO PEAK DISPLACEMENT = 6.12
LINEAR DISPLACEMENT Y
THETA - 110
CODE:
DISPL.
VEL.
ACC.
26.000
24.500
23.000
21.500
20.000
18.500
17.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 2-22

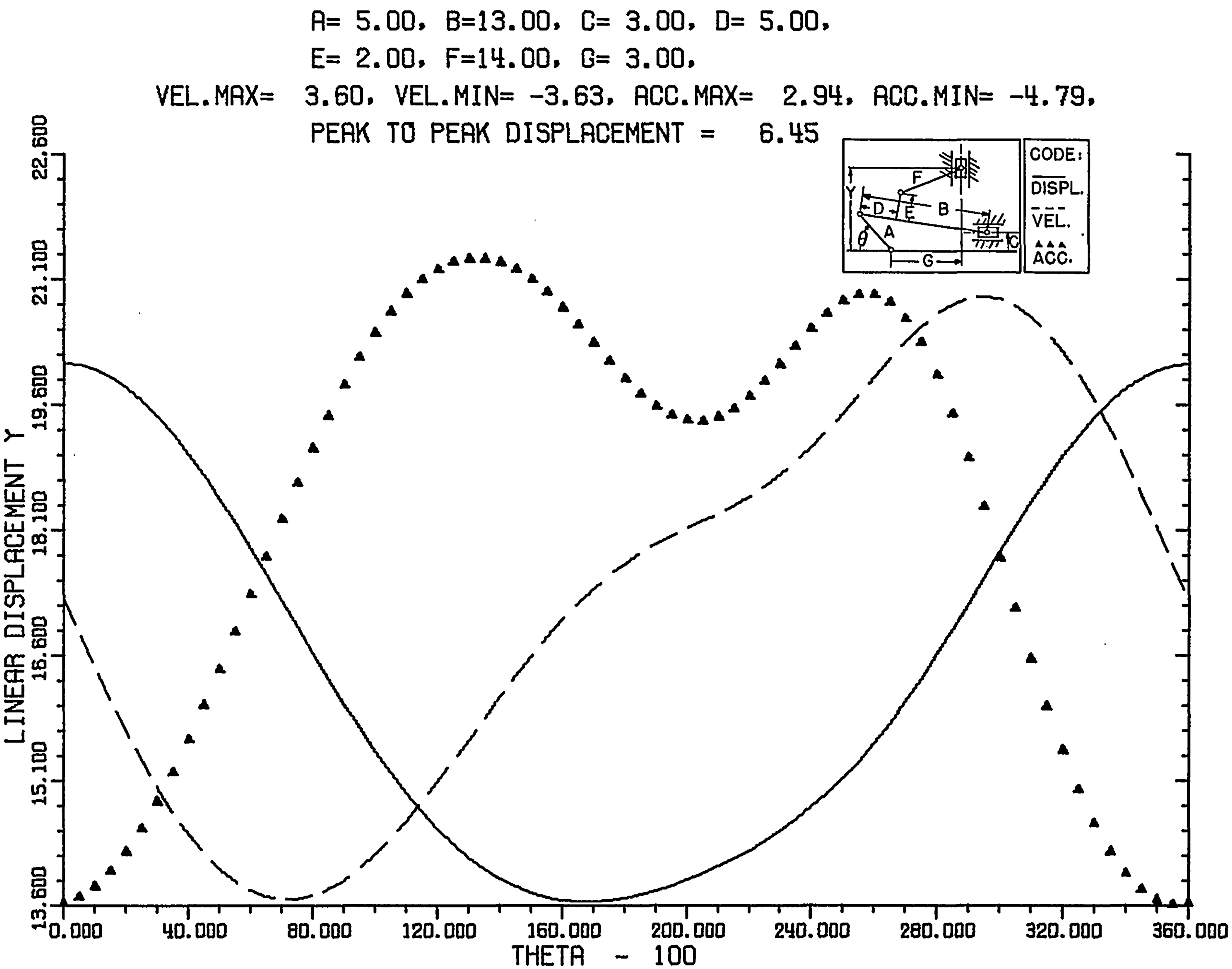

A= 5.00, B=13.00, C= 3.00, D= 5.00,
E= 2.00, F=14.00, G= 3.00,
VEL.MAX= 3.60, VEL.MIN= -3.63, ACC.MAX= 2.94, ACC.MIN= -4.79,
PEAK TO PEAK DISPLACEMENT = 6.45
CODE:
DISPL.
VEL.
ACC.
LINEAR DISPLACEMENT Y
THETA - 100
13.600
15.100
16.600
18.100
19.600
21.100
22.600
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 2-23

A= 3.50, B= 8.50, C= 3.00, D= 2.50,
E= 3.00, F=10.00, G= 2.00,
VEL.MAX= 2.52, VEL.MIN= -3.71, ACC.MAX= 3.96, ACC.MIN= -3.74,
PEAK TO PEAK DISPLACEMENT = 6.49

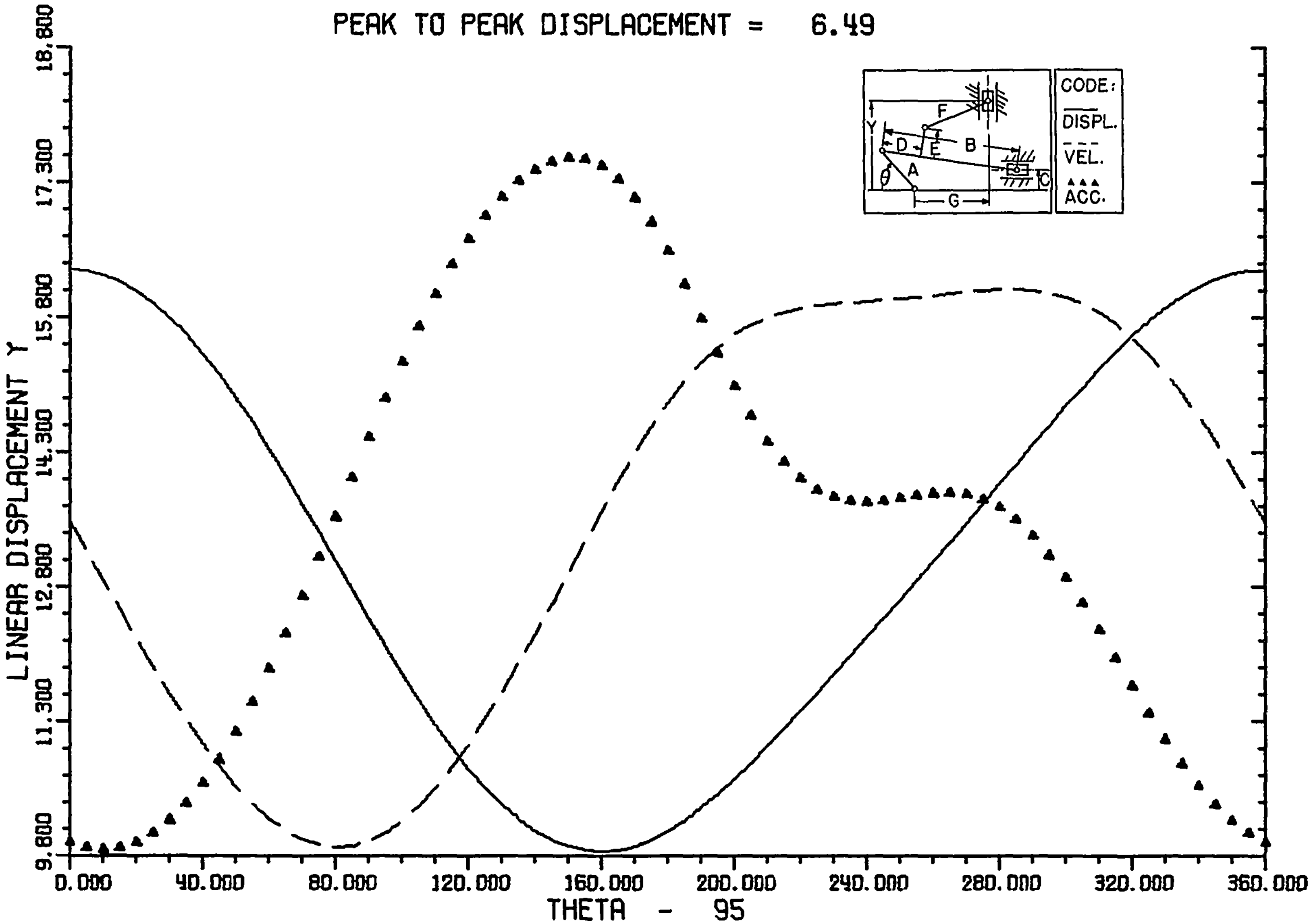


PLATE 2-24

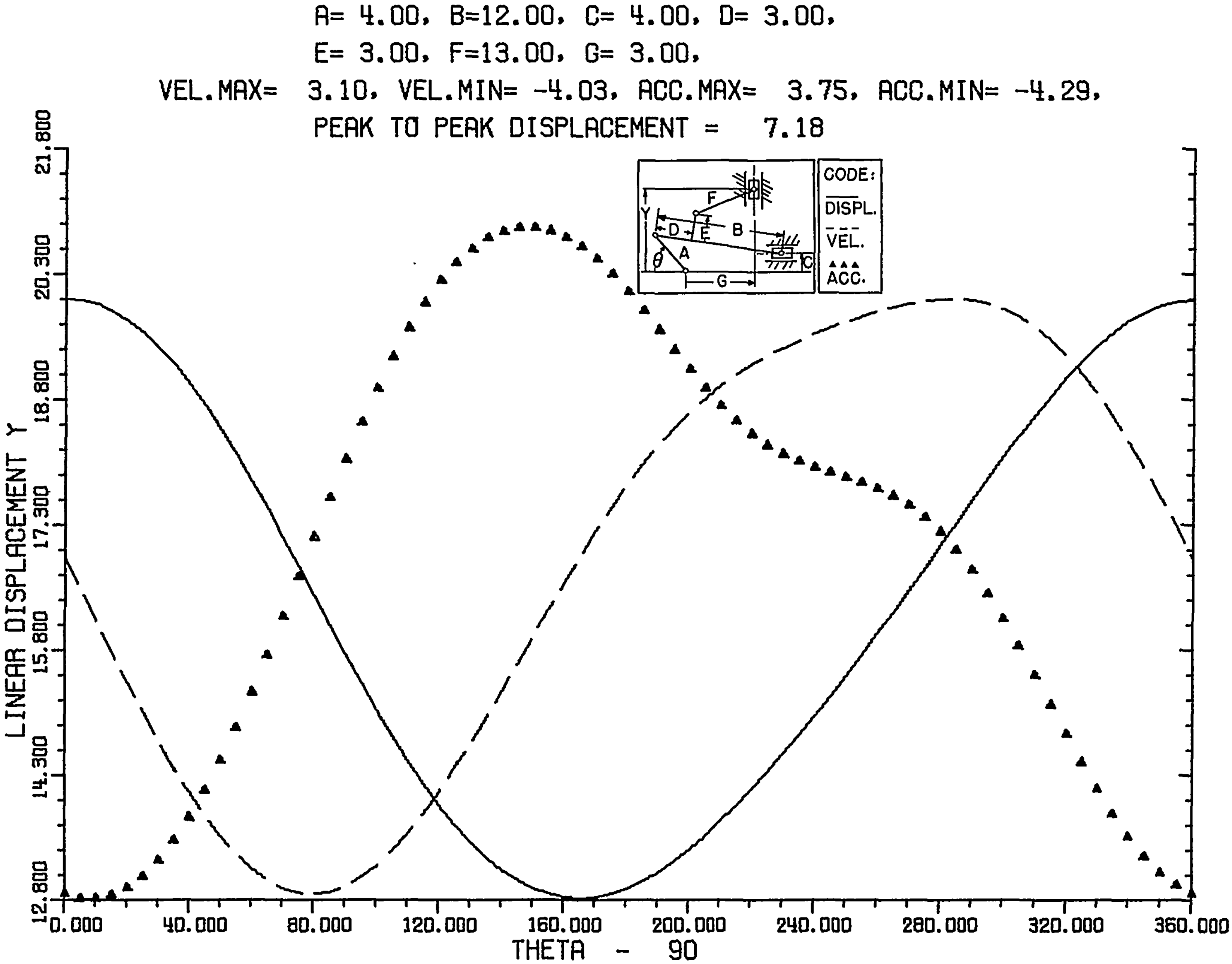

A= 4.00, B=12.00, C= 4.00, D= 3.00,
E= 3.00, F=13.00, G= 3.00,
VEL.MAX= 3.10, VEL.MIN= -4.03, ACC.MAX= 3.75, ACC.MIN= -4.29,
PEAK TO PEAK DISPLACEMENT = 7.18
CODE:
DISPL.
VEL.
ACC.
Y
F
D
E
B
A
θ
C
G
LINEAR DISPLACEMENT Y
21.800
20.300
18.800
17.300
15.800
14.300
12.800
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 90

PLATE 2-25

A= 5.00, B= 7.00, C= 1.00, D= 8.00,
E= 2.00, F=12.00, G= 2.00,
VEL.MAX= 4.76, VEL.MIN= -6.62, ACC.MAX= 18.74, ACC.MIN= -7.70,
PEAK TO PEAK DISPLACEMENT = 7.27

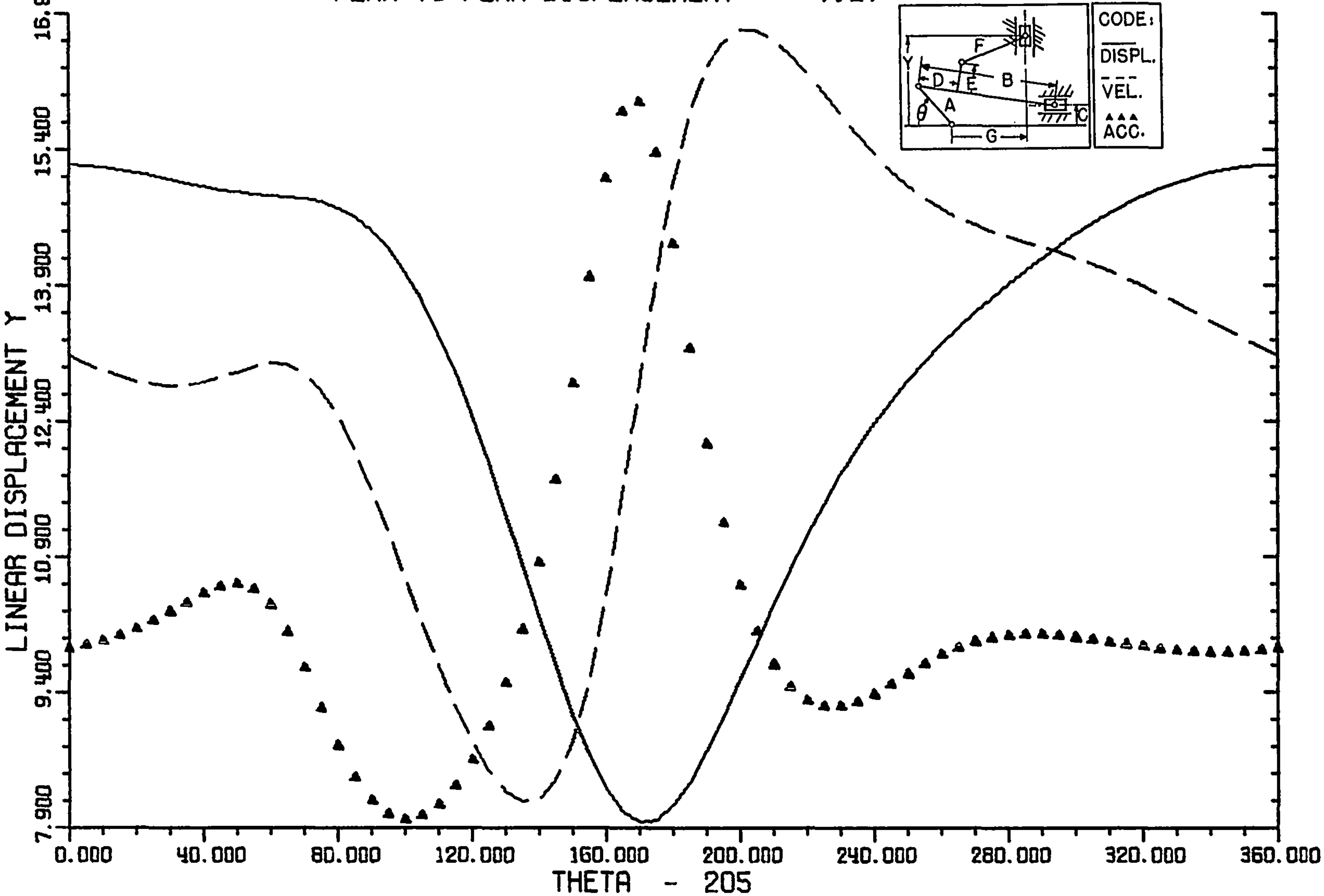


PLATE 2-26

A= 7.00, B=13.00, C= 3.00, D= 7.00,
E= 3.00, F=17.00, G= 4.00,
VEL.MAX= 4.48, VEL.MIN= -4.37, ACC.MAX= 4.71, ACC.MIN= -5.77,
PEAK TO PEAK DISPLACEMENT = 7.40

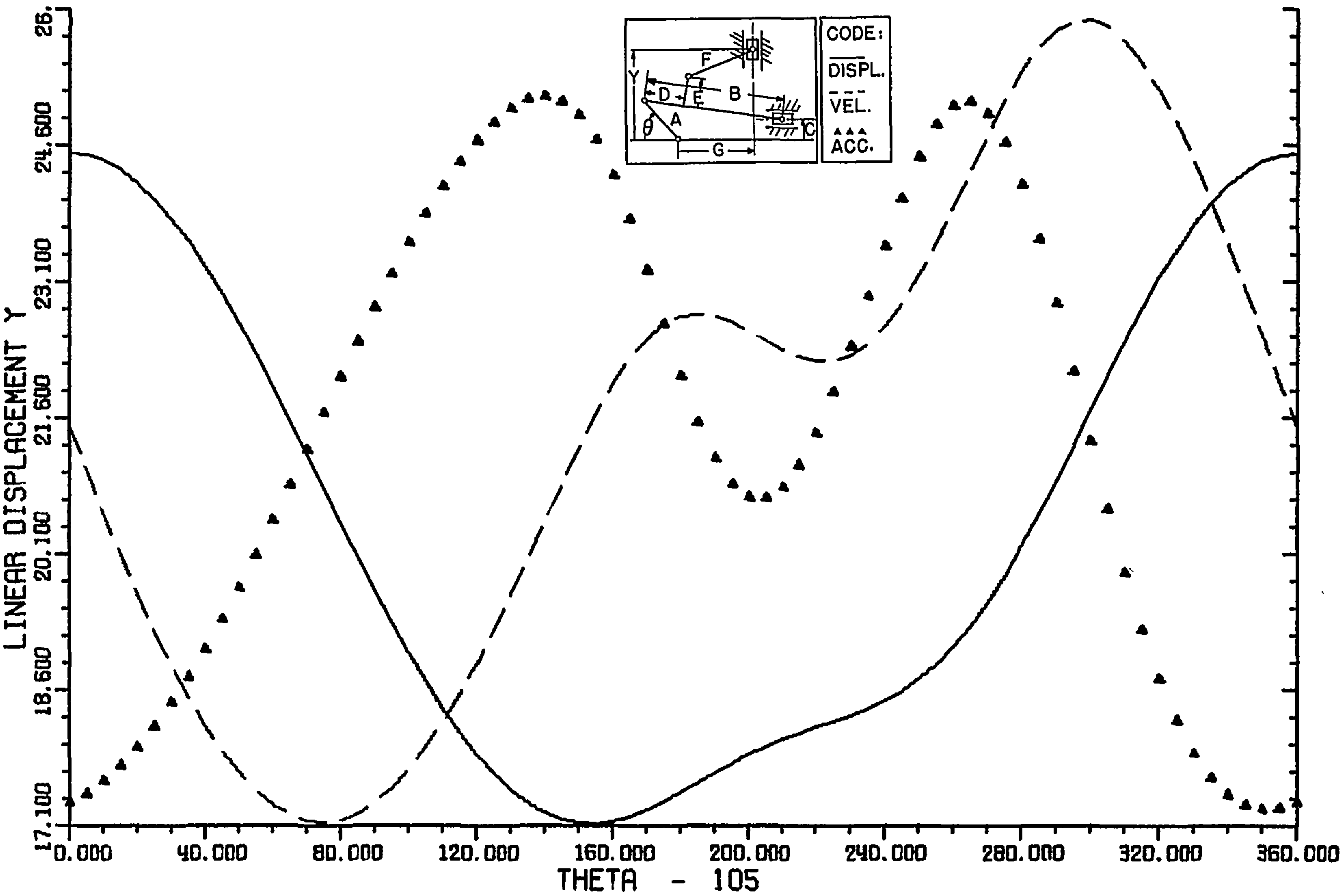


PLATE 2-27

A= 4.00, B=12.00, C= 5.00, D= 4.00,
E= 4.00, F=18.00, G= 5.00,
VEL.MAX= 3.29, VEL.MIN= -4.13, ACC.MAX= 4.62, ACC.MIN= -3.84,
PEAK TO PEAK DISPLACEMENT = 7.63

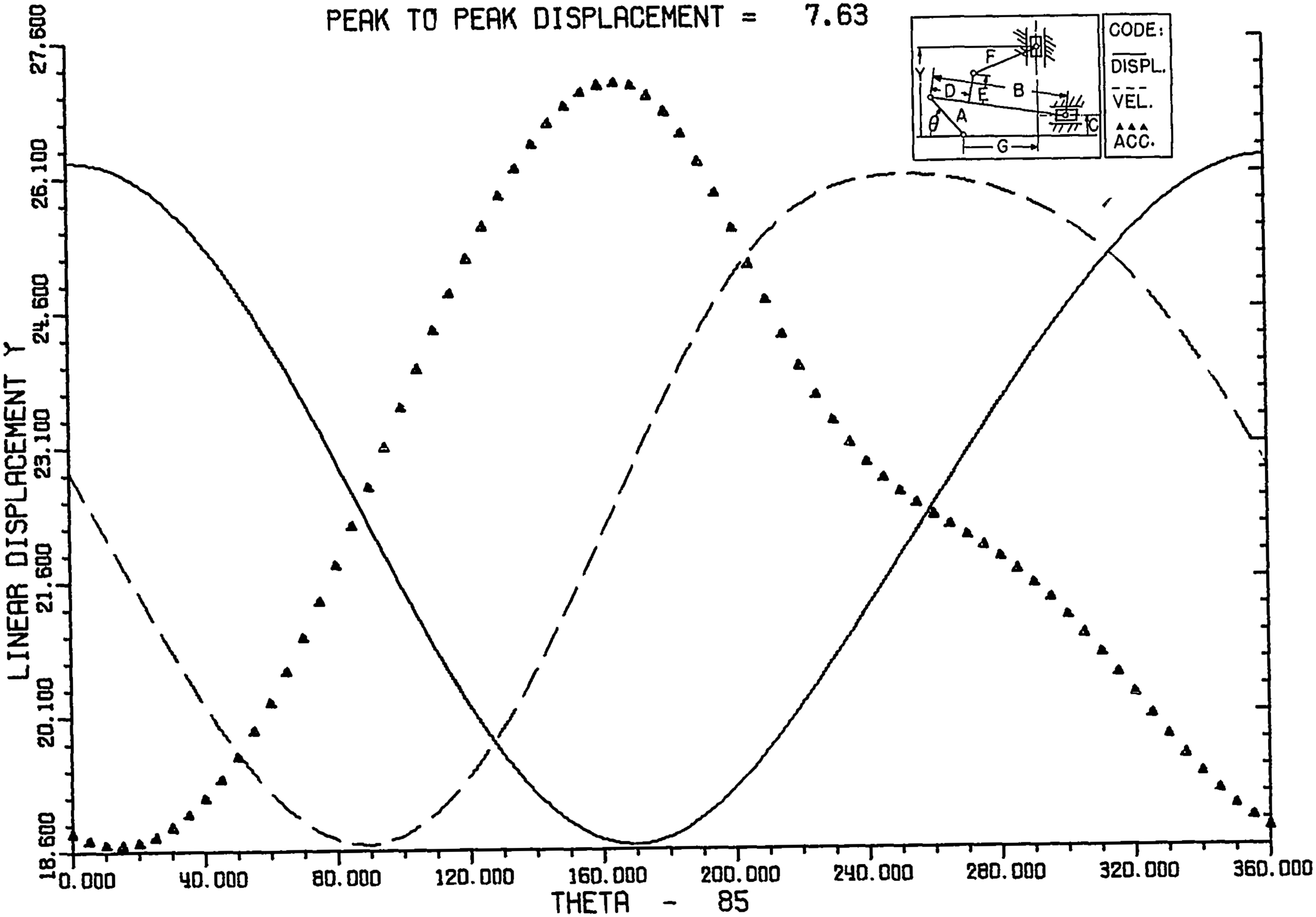


PLATE 2-28

A= 6.00, B=14.00, C= 4.00, D= 5.00,
E= 1.00, F=17.00, G= 3.00,
VEL.MAX= 4.52, VEL.MIN= -4.38, ACC.MAX= 3.39, ACC.MIN= -5.96,
PEAK TO PEAK DISPLACEMENT = 7.92

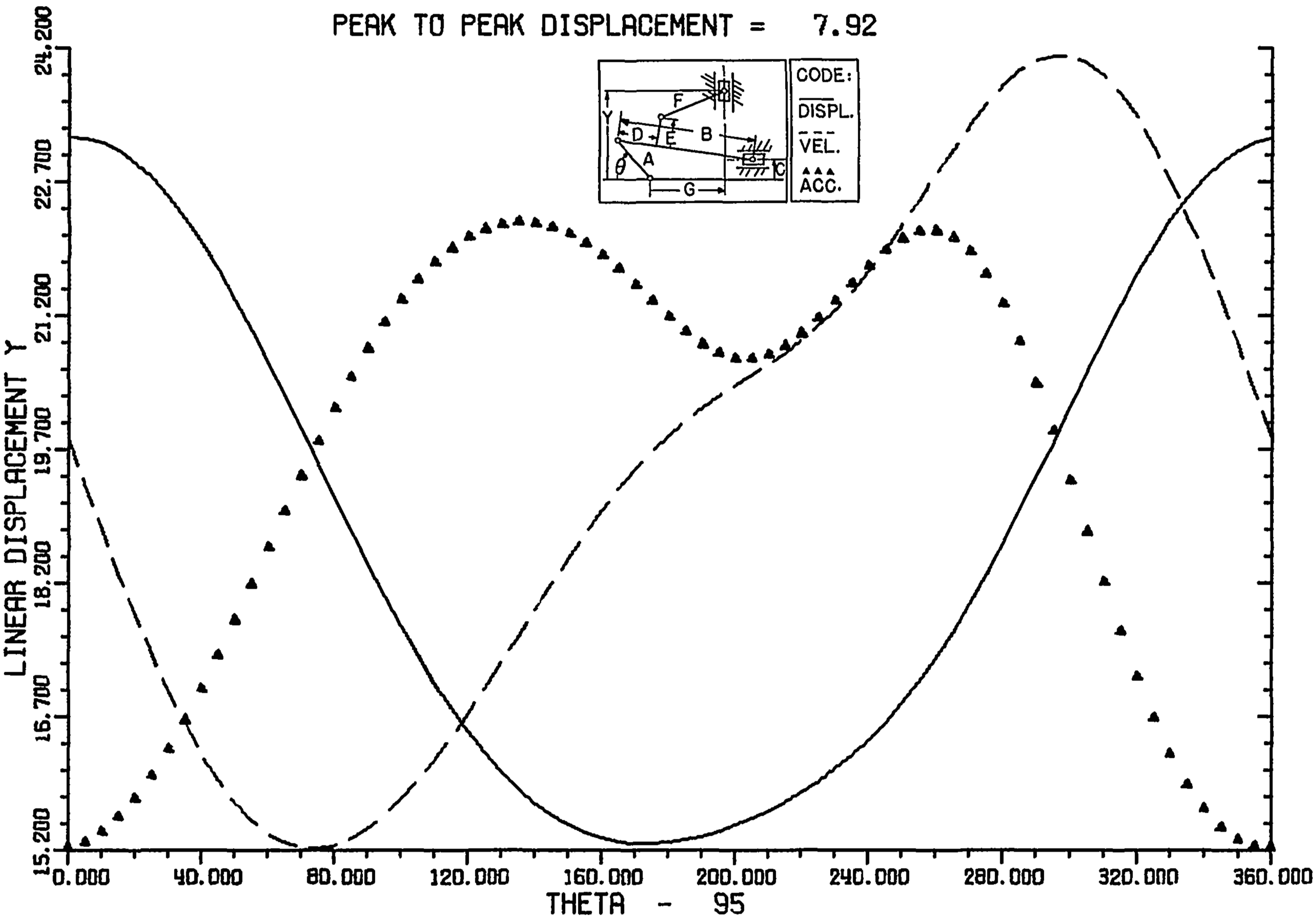
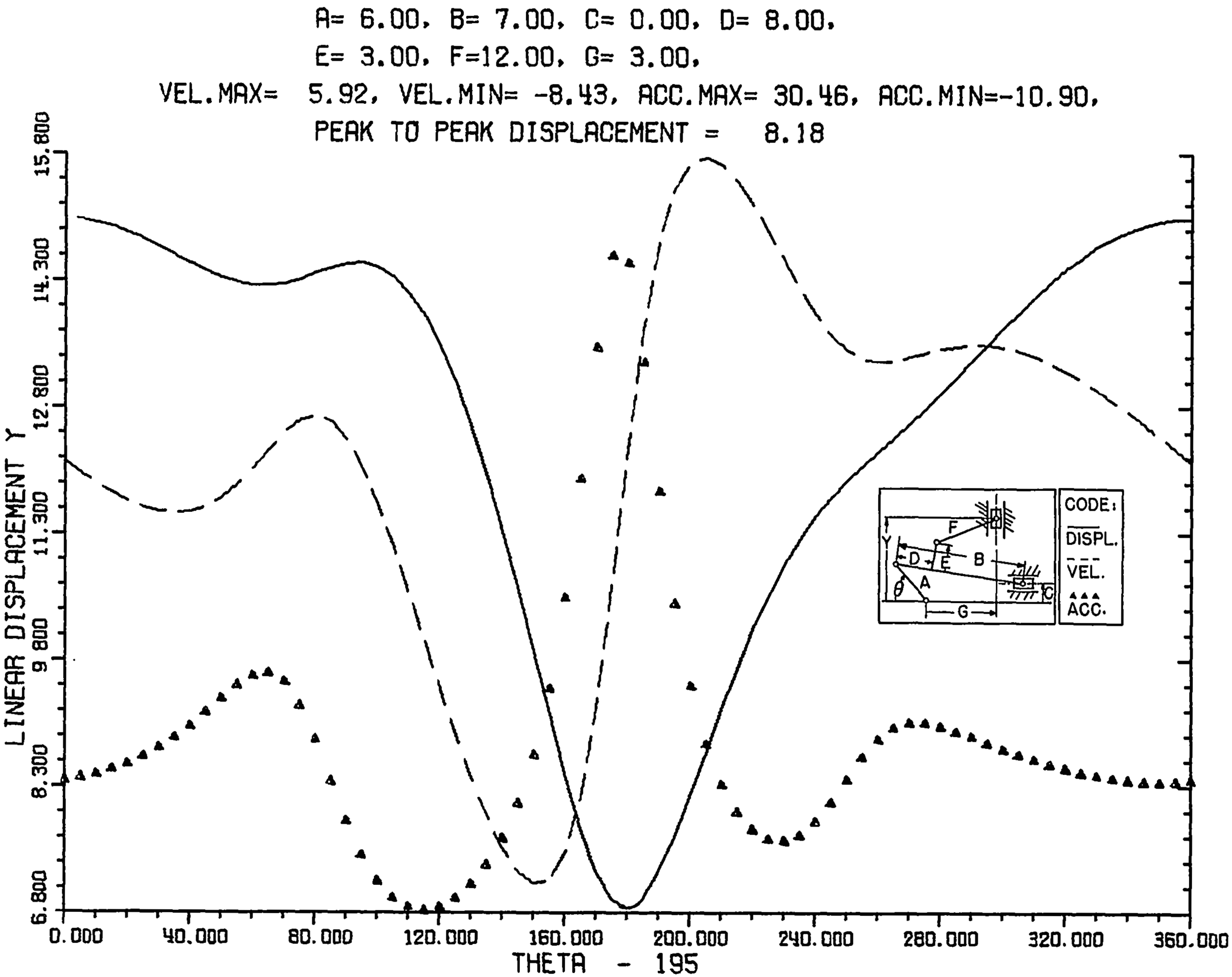


PLATE 2-29



A= 6.00, B= 7.00, C= 0.00, D= 8.00,
E= 3.00, F=12.00, G= 3.00,
VEL.MAX= 5.92, VEL.MIN= -8.43, ACC.MAX= 30.46, ACC.MIN=-10.90,
PEAK TO PEAK DISPLACEMENT = 8.18
LINEAR DISPLACEMENT Y
15.800
14.300
12.800
11.300
9.800
8.300
6.800
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 195
CODE:
DISPL.
VEL.
ACC.
F
Y
D
E
B
A
θ
C
G

PLATE 2-30

A= 3.00, B=11.00, C= 5.00, D= 3.00,
E= 4.00, F=12.00, G= 7.00,
VEL.MAX= 3.82, VEL.MIN= -4.80, ACC.MAX= 5.58, ACC.MIN= -3.90,
PEAK TO PEAK DISPLACEMENT = 8.52

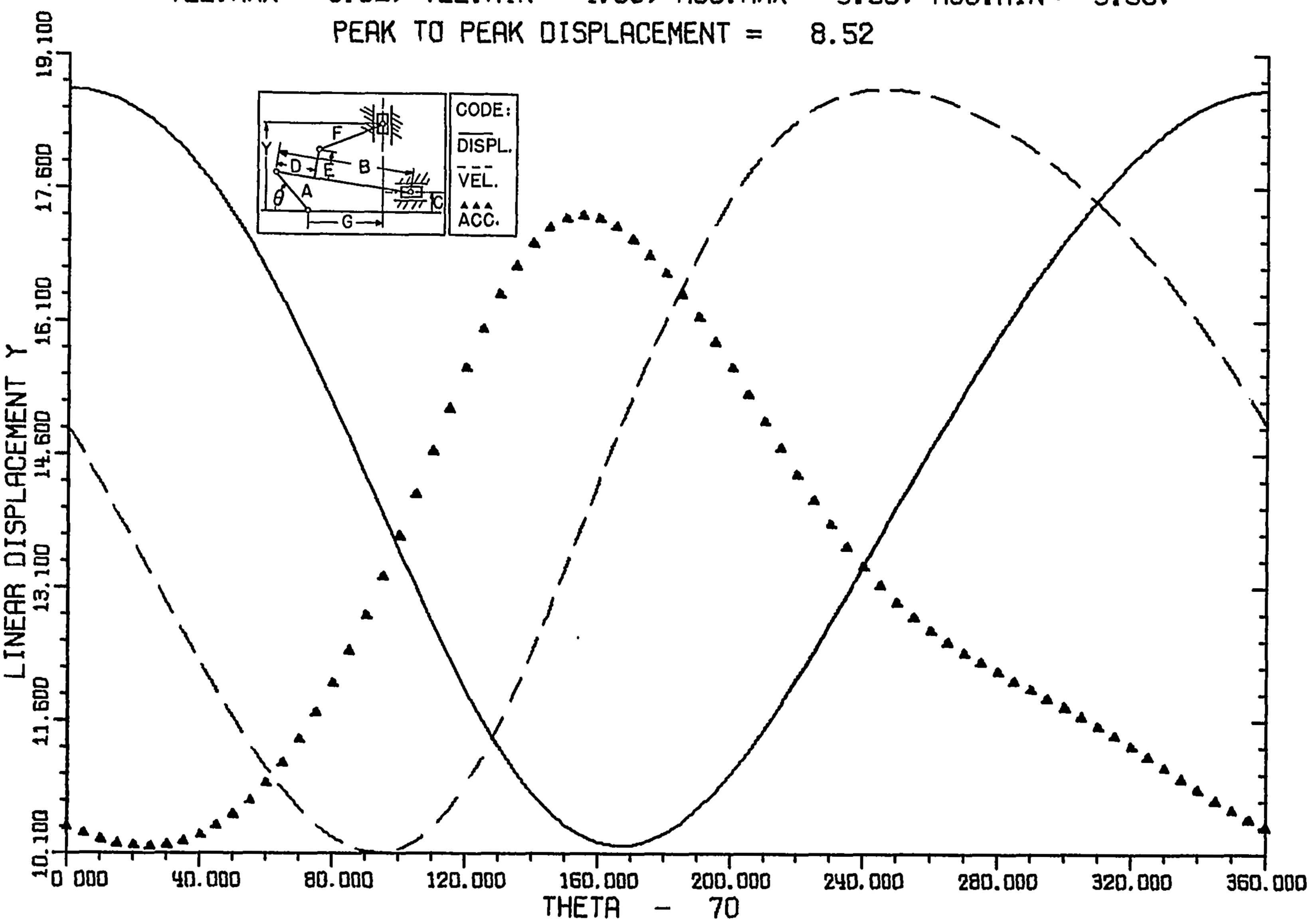


PLATE 2-31

A= 4.00, B=13.00, C= 7.00, D= 3.00,
E= 4.00, F=14.00, G= 3.00,
VEL.MAX= 3.75, VEL.MIN= -4.91, ACC.MAX= 5.26, ACC.MIN= -4.71,
PEAK TO PEAK DISPLACEMENT = 8.94

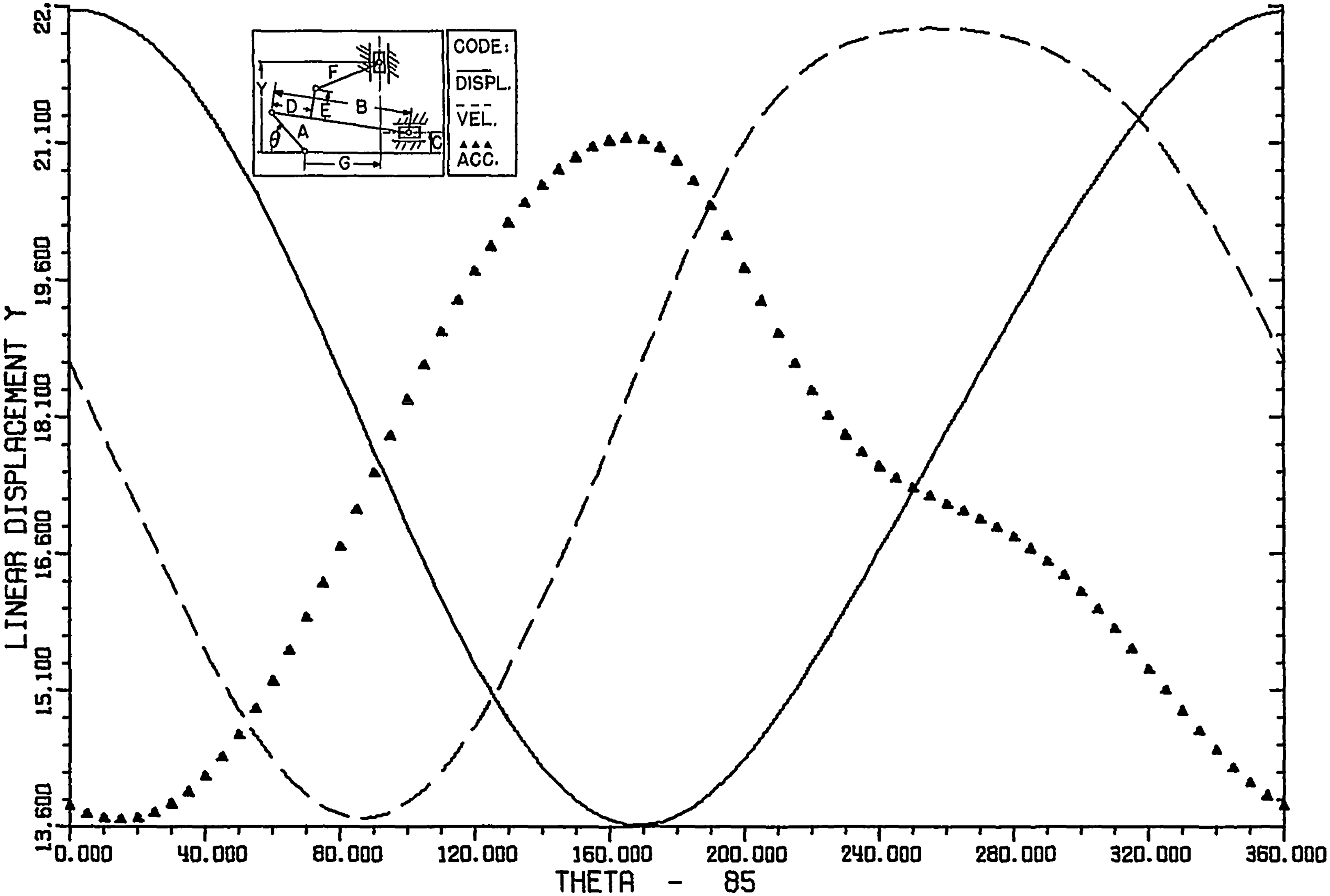


PLATE 2-32

A= 7.00, B=15.00, C= 4.00, D= 6.00,
E= 3.00, F=14.00, G= 4.00,
VEL.MAX= 5.38, VEL.MIN= -5.74, ACC.MAX= 5.39, ACC.MIN= -7.52,
PEAK TO PEAK DISPLACEMENT = 9.51

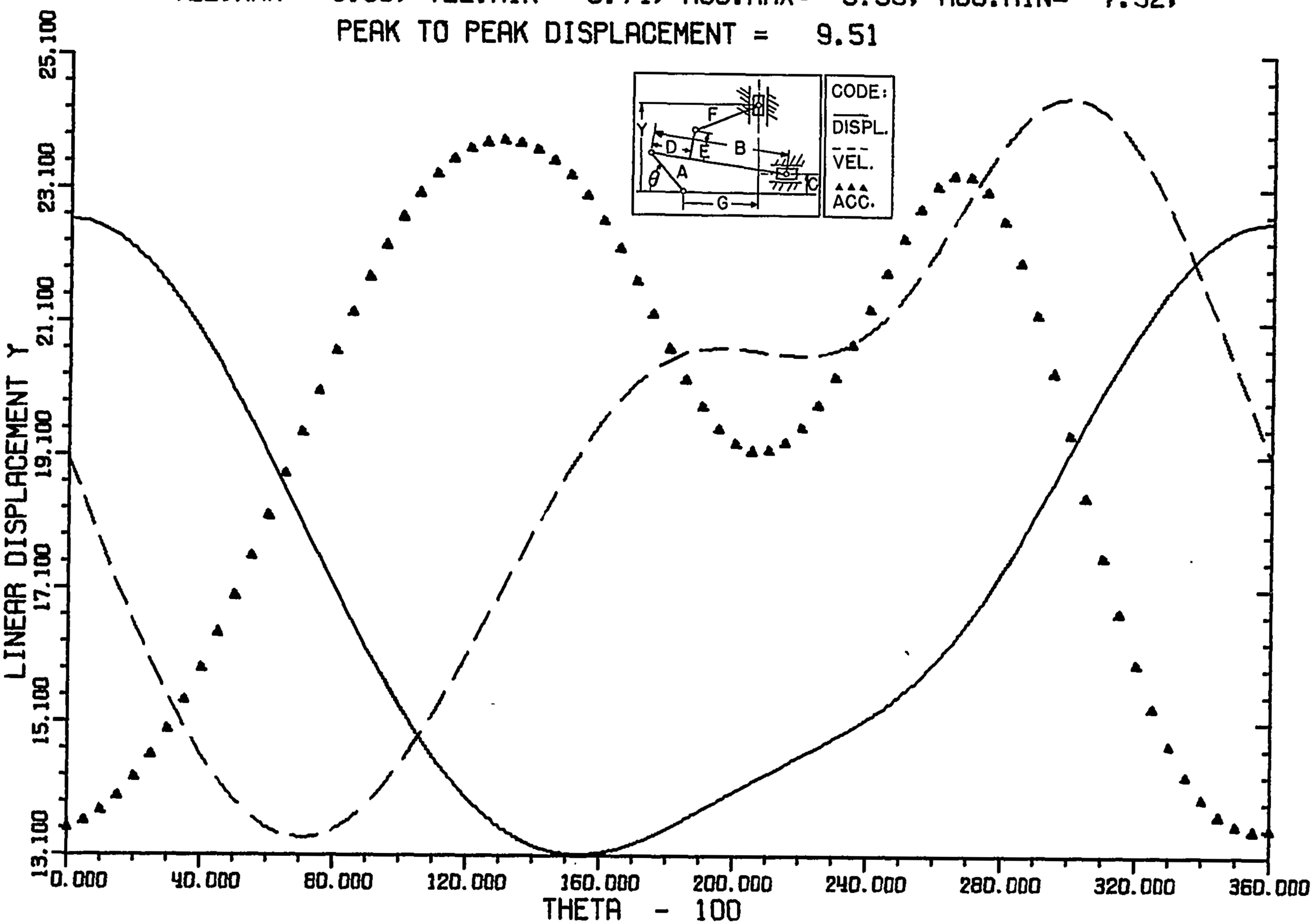


PLATE 2-33

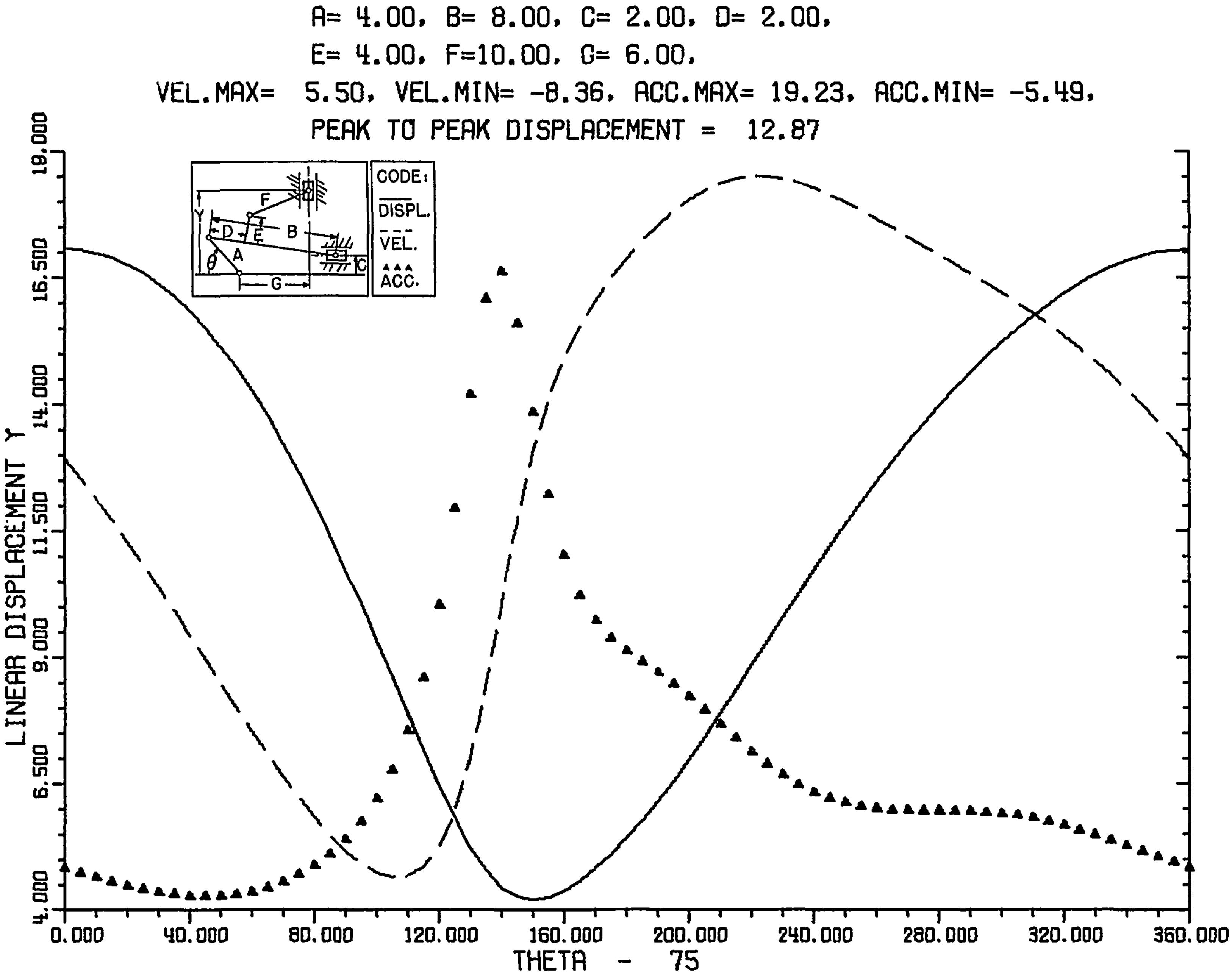

A= 4.00, B= 8.00, C= 2.00, D= 2.00,
E= 4.00, F=10.00, G= 6.00,
VEL.MAX= 5.50, VEL.MIN= -8.36, ACC.MAX= 19.23, ACC.MIN= -5.49,
PEAK TO PEAK DISPLACEMENT = 12.87
CODE:
DISPL.
VEL.
ACC.
Y
F
D
E
B
A
θ
C
G
LINEAR DISPLACEMENT Y
19.000
16.500
14.000
11.500
9.000
6.500
4.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 75

PLATE 2-34

A= 7.00, B=13.00, C= 4.00, D= 4.00,
E= 4.00, F=13.00, G= 4.00,
VEL.MAX= 5.08, VEL.MIN= -8.11, ACC.MAX= 8.49, ACC.MIN= -8.33,
PEAK TO PEAK DISPLACEMENT = 13.23

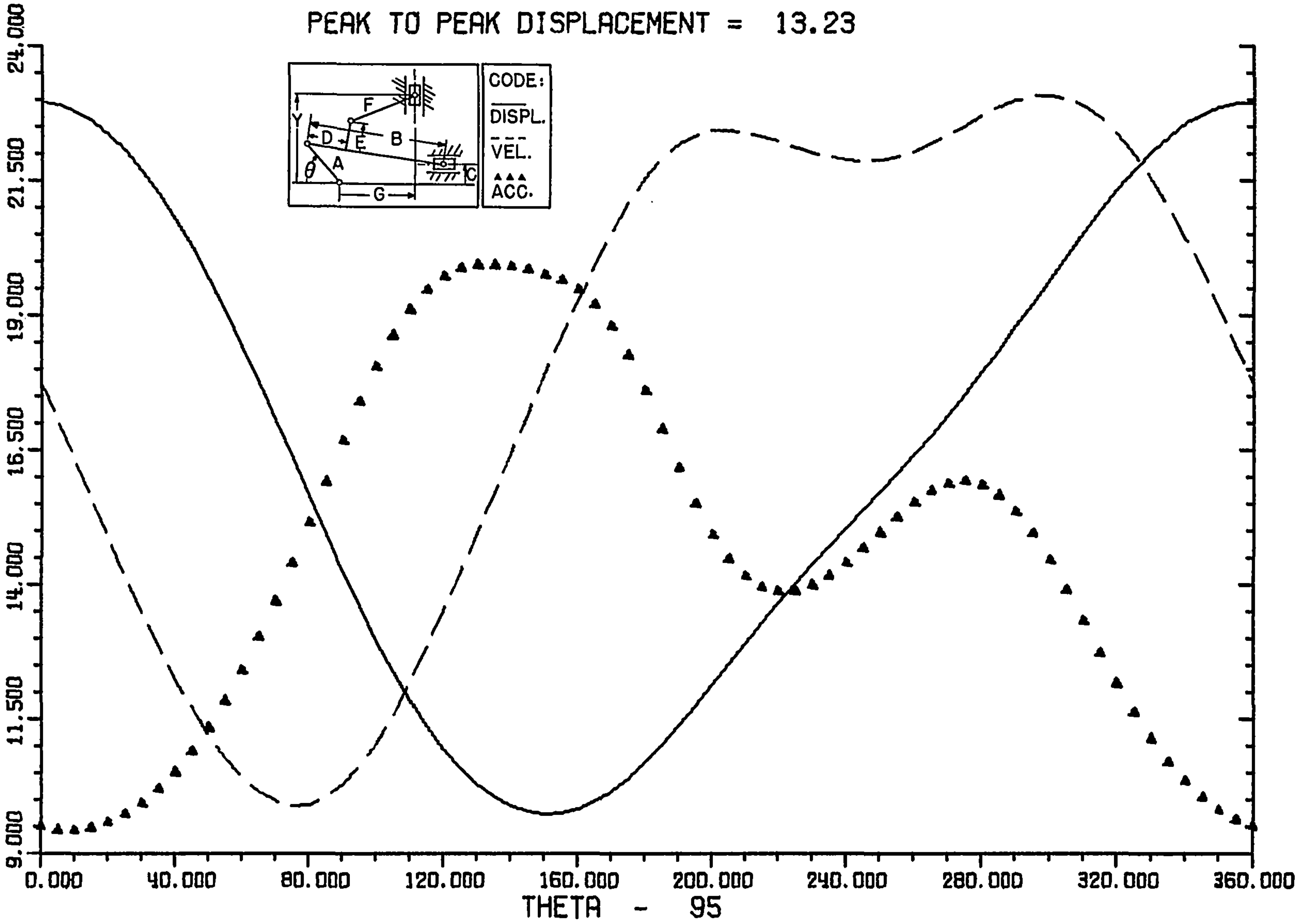


PLATE 2-35

MECHANISM #3

MECHANISM #3

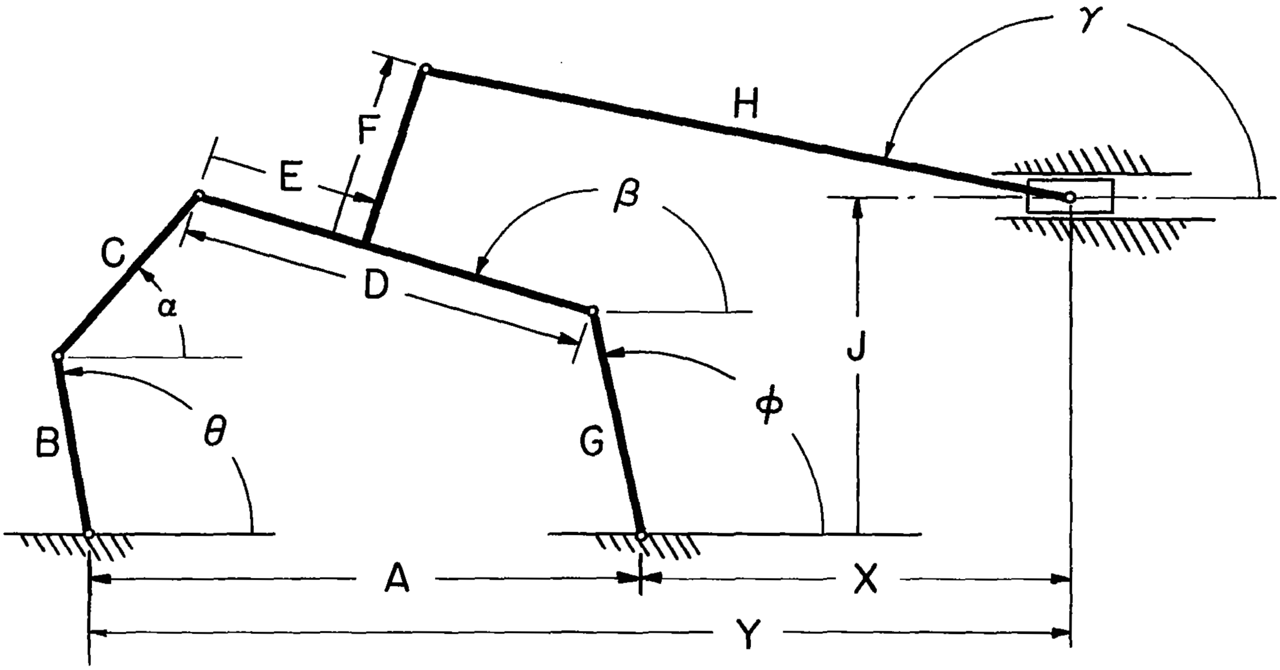


Figure 3-1

Figure 3-1 defines Mechanism #3. It is a multiple input mechanism in that two input angles must be prescribed and they in turn define one output linear quantity. The input angles θ and ϕ (the Greek letters theta and phi) are the angular positions of links B and G, respectively. The output variable is given as Y and describes the location of the slider.

In the graphs for this mechanism the two inputs have been related by the equation:

$$\phi = N \times \theta + PHI0 \quad . \tag{3-1}$$

This kind of relationship may be obtained physically by the use of gears in which N defines the ratio of the sizes of the gears. Note that N may be positive or negative--a positive value for N indicates that links B and G rotate in the same direction while a negative value for N indicates rotation in opposite directions. For $\theta = 0$, Eq. 3-1 yields ϕ = PHI0. In words, PHI0 is the initial phase angle between the two input angles θ and ϕ. Note that N and PHI0 as well as A, B, C, D, E, F, G, H, and J are required for defining the mechanism: numerical values of each are given as part of the heading for every graph.

Each graph shows Y versus θ as a solid line, the derivative of Y with respect to θ versus θ as a dashed line, and the second derivative of Y with respect to θ versus θ as a series of small triangles. Each curve begins with the maximum displacement Y. This has been accomplished by shifting the θ axis. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. The variable θ is presented in the units degrees. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement.

Scales have not been presented for the derivatives, but each graph heading includes both the maximum and minimum of the velocity and acceleration so that scales could be constructed if desired. The units for displacement, velocity, and acceleration will correspond to that chosen for the quantities A, B, C, et cetera. For example, if the link lengths are specified in inches, then the velocity, $dY/d\theta$, will be in inches per radian. A more conventional engineering unit for velocity may be obtained as:

$$\frac{dY}{dt} = \frac{dY}{d\theta} \times \frac{d\theta}{dt} \tag{3-2}$$

in which

$$\frac{d\theta}{dt} = \text{rpm} \times 2\pi \times \frac{1}{60}, \qquad \left(\frac{\text{rad}}{\text{sec}} = \frac{\text{rev}}{\text{min}} \times \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{\text{sec}}\right)$$

With this modification Eq. 3-2 may be written:

$$\frac{dY}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{dY}{d\theta}, \qquad \frac{\text{inches}}{\text{second}}. \tag{3-3}$$

In words, the velocity of the slider (inches/ second) is obtained by multiplying the angular speed of link B (rpm) by $\pi/30$ and then multiplying this product by $dY/d\theta$ (inches/ radian). Values for this latter term may be obtained from a graph (the dashed line) or from the heading of a graph (VEL. MAX or VEL. MIN) for Mechanism #3.

The acceleration of the slider may be written:

$$\frac{d^2Y}{dt^2} = \frac{d}{dt}\left[\frac{dY}{dt}\right]$$

$$= \frac{d}{dt}\left[\frac{dY}{d\theta} \times \frac{d\theta}{dt}\right]$$

$$= \frac{d}{d\theta}\left[\frac{dY}{d\theta}\right]\left[\frac{d\theta}{dt}\right]^2 + \left[\frac{dY}{d\theta}\right]\left[\frac{d^2\theta}{dt^2}\right]$$

$$= \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{d\theta}{dt}\right]^2 + \left[\frac{dY}{d\theta}\right]\left[\frac{d^2\theta}{dt^2}\right].$$

If the angular speed of link B remains constant (likewise for link G as the two are related by Eq. 3-1) then the angular acceleration of link B, $d^2\theta/dt^2$, is zero. The expression for the linear acceleration of the slider with link B turning with constant speed simplifies to:

$$\frac{d^2Y}{dt^2} = \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{d\theta}{dt}\right]^2$$

$$= \left[\frac{d^2Y}{d\theta^2}\right]\left[\frac{\pi}{30} \times \text{rpm}\right]^2, \quad \frac{\text{inches}}{\text{second}^2}. \tag{3-4}$$

The term $d^2Y/d\theta^2$ may be obtained from a graph (the series of small triangles), or from the heading of a graph (ACC. MAX or ACC. MIN), or from equations which follow.

Referring to Figure 3-1 the equations relating the output to the inputs for this mechanism may be derived. Looking at the basic five-bar mechanism, links A, B, C, D, and G, as projected onto a vertical line:

$$B \sin \theta + C \sin \alpha = G \sin (\pi - \phi) + D \sin (\pi - \beta).$$

This may be rewritten as

$$C \sin \alpha - D \sin \beta = G \sin \phi - B \sin \theta$$

$$= K \tag{3-5}$$

in which K is written for convenience in the manipulations which follow. Looking at projections onto a horizontal line, the five-bar portion of the mechanism gives

$$B \cos \theta + C \cos \alpha + D \cos (\pi - \beta) + G \cos (\pi - \phi) = A$$

which may be transposed to

$$C \cos \alpha - D \cos \beta = A - B \cos \theta + G \cos \phi$$

$$= L \tag{3-6}$$

in which L is employed for abbreviation. Eqs. 3-5 and 3-6 may be rewritten as

$$C \sin \alpha = K + D \sin \beta$$

$$C \cos \alpha = L + D \cos \beta$$

each of which may be squared and then added together forming

$$C^2 = K^2 + 2\,DK \sin \beta + D^2 + L^2 + 2\,DL \cos \beta.$$

Regrouping this equation

$$L \cos \beta + K \sin \beta = \frac{C^2 - D^2 - K^2 - L^2}{2D}$$

$$= M. \tag{3-7}$$

Note that with a given value for θ, which, via Eq. 3-1 determines a value for ϕ, and with the link lengths of the mechanism specified, K, L, and M may be calculated (using Eqs. 3-5 and 3-6). With K, L, and M determined, the only unknown in Eq. 7 is β which may be solved by different procedures. One procedure that has been employed satisfactorily is the Newton-Raphson method.

Knowing β, attention may be directed to that portion of the mechanism composed of links D, F, G, and H. The projection onto a vertical line reveals

$$J + H \sin (\pi - \gamma) = G \sin \phi + (D - E) \sin \beta - F \cos \beta$$

and this may be solved as

$$\sin \gamma = \frac{G \sin \phi + (D - E) \sin \beta - F \cos \beta - J}{H}$$

$$= P. \tag{3-8}$$

With Eq. 3-8, γ may be calculated. The projection onto a horizontal line will yield

$$F \sin \beta + H \cos (\pi - \gamma) = G \cos (\pi - \phi) + (D - E) \cos (\pi - \beta) + X.$$

The variable that defines the position of the slider is Y and, noting the Figure 3-1 of the mechanism, may be written

$$Y = A + X \tag{3-9}$$

in which

$$X = F \sin \beta - H \cos \gamma + G \cos \phi + (D - E) \cos \beta .$$

Linear Velocity

The velocity of the slider may be determined as the derivative of Eq. 3-9. Remembering that A is the center-to-center distance of the pivots of B and G and is a constant,

$$\frac{dY}{d\theta} = \frac{dX}{d\theta}$$

$$= F \cos \beta \frac{d\beta}{d\theta} + H \sin \gamma \frac{d\gamma}{d\theta} - G \sin \phi \frac{d\phi}{d\theta} - (D - E) \sin \beta \frac{d\beta}{d\theta} . \tag{3-10}$$

The derivatives of ϕ with respect to θ may be obtained from Eq. 3-1 as:

$$\frac{d\phi}{d\theta} = N$$

and

$$\frac{d^2\phi}{d\theta^2} = 0 . \tag{3-11}$$

Some manipulations are required to obtain the terms in Eq. 3-10. Taking the derivative of Eq. 3-7 with respect to θ results in:

$$-L \sin \beta \frac{d\beta}{d\theta} + \cos \beta \frac{dL}{d\theta} + K \cos \beta \frac{d\beta}{d\theta} + \sin \beta \frac{dK}{d\theta}$$

$$= \frac{-1}{2D} \left[2 K \frac{dK}{d\theta} + 2 L \frac{dL}{d\theta} \right] . \tag{3-12}$$

From Eqs. 3-5 and 3-6:

$$\frac{dK}{d\theta} = G \cos \phi \frac{d\phi}{d\theta} - B \cos \theta \tag{3-13}$$

$$\frac{dL}{d\theta} = B \sin \theta - G \sin \phi \frac{d\phi}{d\theta} \quad . \tag{3-14}$$

Rearranging Eq. 3-12 the following may be obtained:

$$\frac{d\beta}{d\theta} = \frac{\frac{dL}{d\theta}\left(-\cos \beta + \frac{L}{D}\right) - \frac{dK}{d\theta}\left(\sin \beta + \frac{K}{D}\right)}{K \cos \beta - L \sin \beta} \quad . \tag{3-15}$$

Differentiating Eq. 3-8 with respect to θ and dividing by cos γ yields:

$$\frac{d\gamma}{d\theta} = \frac{1}{H \cos \gamma} \left[G \cos \phi \frac{d\phi}{d\theta} + (D - E) \cos \beta \frac{d\beta}{d\theta} + F \sin \beta \frac{d\beta}{d\theta} \right] . \tag{3-16}$$

The Eqs. 3-11 through 3-16 may be substituted into Eq. 3-10 to determine the velocity dY/ dθ. This value may be substituted into Eq. 3-3 to obtain the linear velocity dY/ dt.

Linear Acceleration

The acceleration of the slider may be determined as the derivative of the velocity, as given by Eq. 3-10, with respect to θ.

$$\frac{d^2Y}{d\theta^2} = F \cos \beta \frac{d^2\beta}{d\theta^2} - F \sin \beta \left[\frac{d\beta}{d\theta}\right]^2 + H \sin \gamma \frac{d^2\gamma}{d\theta^2} + H \cos \gamma \left[\frac{d\gamma}{d\theta}\right]^2$$

$$- G \cos \phi \left[\frac{d\phi}{d\theta}\right]^2 - (D - E) \sin \beta \frac{d^2\beta}{d\theta^2} - (D - E) \cos \beta \left[\frac{d\beta}{d\theta}\right]^2 \quad . \tag{3-17}$$

If Eq. 3-15 is multiplied by the denominator of the right-hand side followed by taking the derivative of the resulting equation with respect to θ, and the terms collected, appropriately, the following equation may be obtained:

$$\frac{d^2\beta}{d\theta^2} = \left[\frac{1}{K\cos\beta - L\sin\beta}\right] \left\{ \left[\frac{d\beta}{d\theta}\right]^2 (L\cos\beta + K\sin\beta) - \frac{d^2L}{d\theta^2}\left[\cos\beta + \frac{L}{D}\right] + \frac{dL}{d\theta}\left[2\sin\beta\frac{d\beta}{d\theta} - \frac{1}{D}\frac{dL}{d\theta}\right] - \frac{d^2K}{d\theta^2}\left[\sin\beta + \frac{K}{D}\right] - \frac{dK}{d\theta}\left[2\cos\beta\frac{d\beta}{d\theta} + \frac{1}{D}\frac{dK}{d\theta}\right]\right\}. \tag{3-18}$$

The appropriate second derivatives may be determined from Eqs. 3-13 and 3-14.

$$\frac{d^2K}{d\theta^2} = -G\sin\phi\left[\frac{d\phi}{d\theta}\right]^2 + B\sin\theta \tag{3-19}$$

$$\frac{d^2L}{d\theta^2} = B\cos\theta - G\cos\phi\left[\frac{d\phi}{d\theta}\right]^2 . \tag{3-20}$$

Eq. 3-8 may be differentiated to:

$$\cos\gamma\frac{d\gamma}{d\theta} = \frac{dP}{d\theta} \tag{3-21}$$

and differentiating again to:

$$\cos\gamma\frac{d^2\gamma}{d\theta^2} - \sin\gamma\left[\frac{d\gamma}{d\theta}\right]^2 = \frac{d^2P}{d\theta^2} \tag{3-22}$$

in which

$$\frac{dP}{d\theta} = \frac{1}{H}\left[G\cos\phi\frac{d\phi}{d\theta} + (D-E)\cos\beta\frac{d\beta}{d\theta} + F\sin\beta\frac{d\beta}{d\theta}\right] \tag{3-23}$$

and

$$\frac{d^2P}{d\theta^2} = \frac{1}{H} \left\{ - G \sin\phi \left[\frac{d\phi}{d\theta}\right]^2 + (D - E) \cos\beta \frac{d^2\beta}{d\theta^2} - (D - E) \sin\beta \left[\frac{d\beta}{d\theta}\right]^2 \right.$$

$$\left. + F \sin\beta \frac{d^2\beta}{d\theta^2} + F \cos\beta \left[\frac{d\beta}{d\theta}\right]^2 \right\} . \qquad (3\text{-}24)$$

Eq. 3-22 may be rewritten as

$$\frac{d^2\gamma}{d\theta^2} = \frac{1}{\cos\gamma} \left\{ \sin\gamma \left[\frac{d\gamma}{d\theta}\right]^2 + \frac{d^2P}{d\theta^2} \right\} . \qquad (3\text{-}25)$$

With the equations that have been developed the acceleration, $d^2Y/d\theta^2$, as given by Eq. 3-17 may be evaluated. The linear acceleration of the slider may be determined using the above equations with Eq. 3-4.

The number of variations using this mechanism exceed the imagination. If N is an irrational number, the mechanism will not be cyclic. Even with N, an integer, the multiplicity of possible motion transformations is difficult to imagine. Getting a "feeling" for this mechanism is a real challenge.

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-1.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 3.31, VEL.MIN= -3.75, ACC.MAX= 8.29, ACC.MIN= -3.86,
PEAK TO PEAK DISPLACEMENT = 5.96

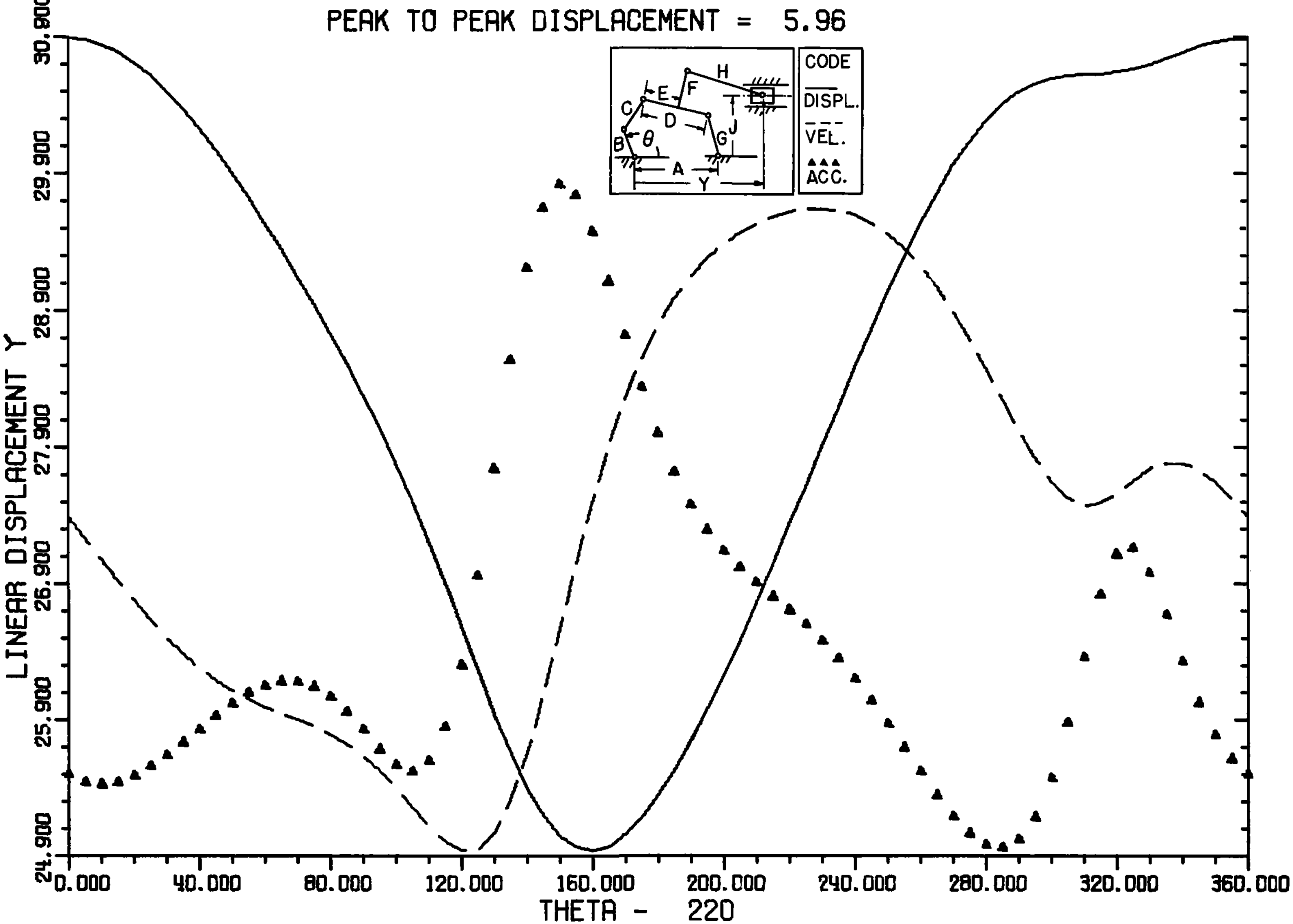
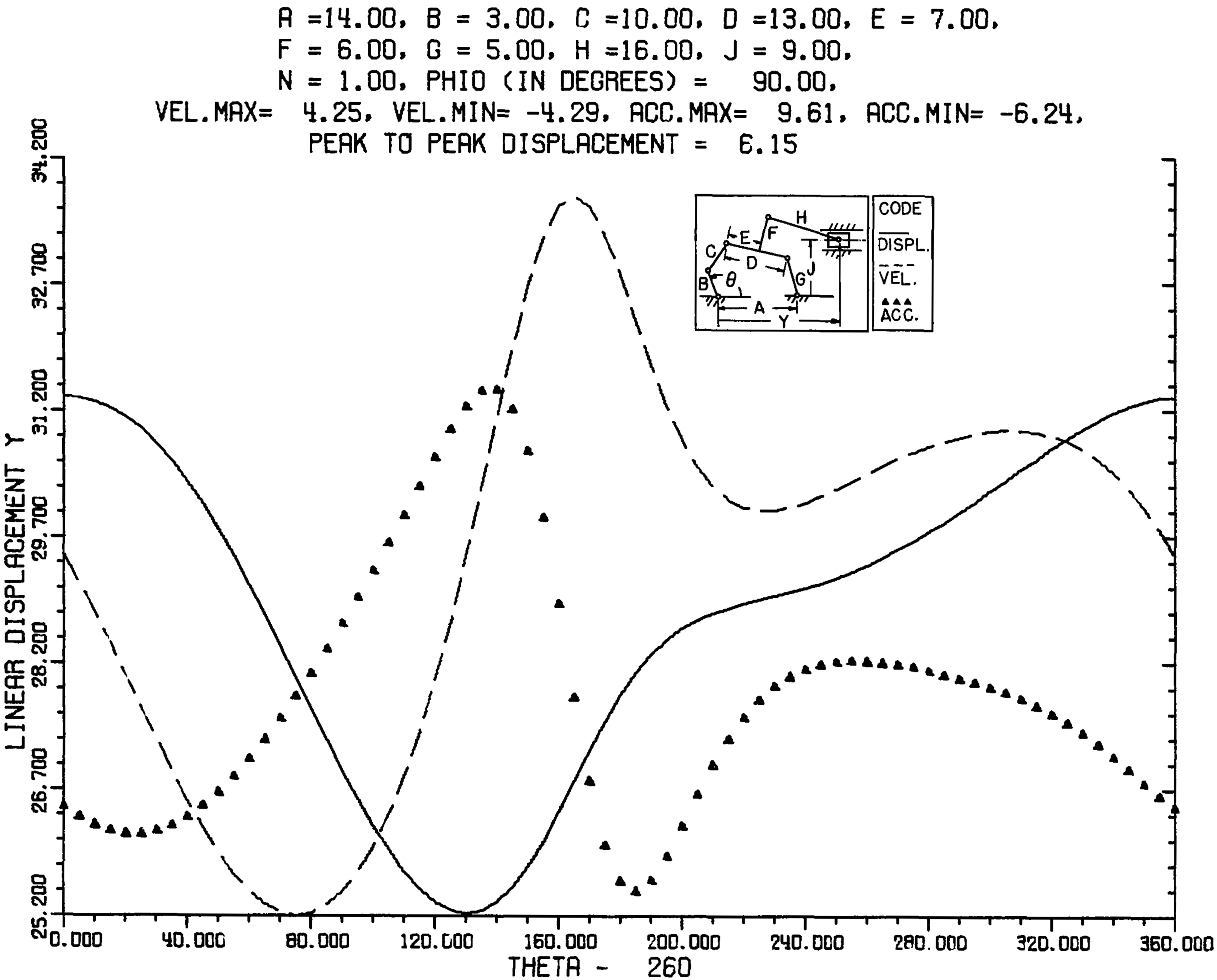
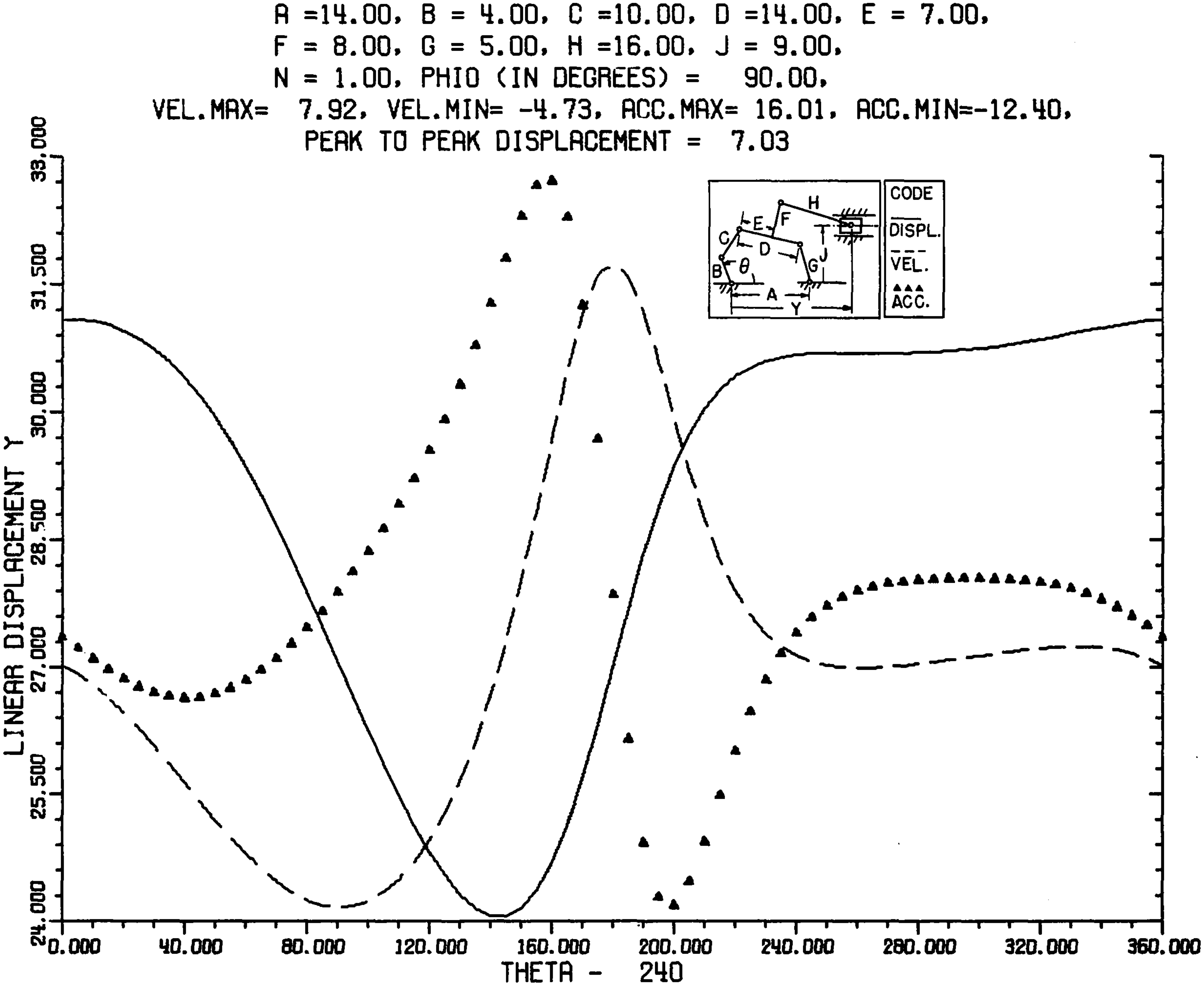


PLATE 3-1



A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 4.25, VEL.MIN= -4.29, ACC.MAX= 9.61, ACC.MIN= -6.24,
PEAK TO PEAK DISPLACEMENT = 6.15
LINEAR DISPLACEMENT Y
THETA - 260
CODE
DISPL.
VEL.
ACC.

PLATE 3-2



A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 8.00, G = 5.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 7.92, VEL.MIN= -4.73, ACC.MAX= 16.01, ACC.MIN=-12.40,
PEAK TO PEAK DISPLACEMENT = 7.03
LINEAR DISPLACEMENT Y
THETA - 240
CODE
DISPL.
VEL.
ACC.

PLATE 3-3

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 11.15, VEL.MIN= -2.63, ACC.MAX= 29.48, ACC.MIN=-23.11,
PEAK TO PEAK DISPLACEMENT = 7.04

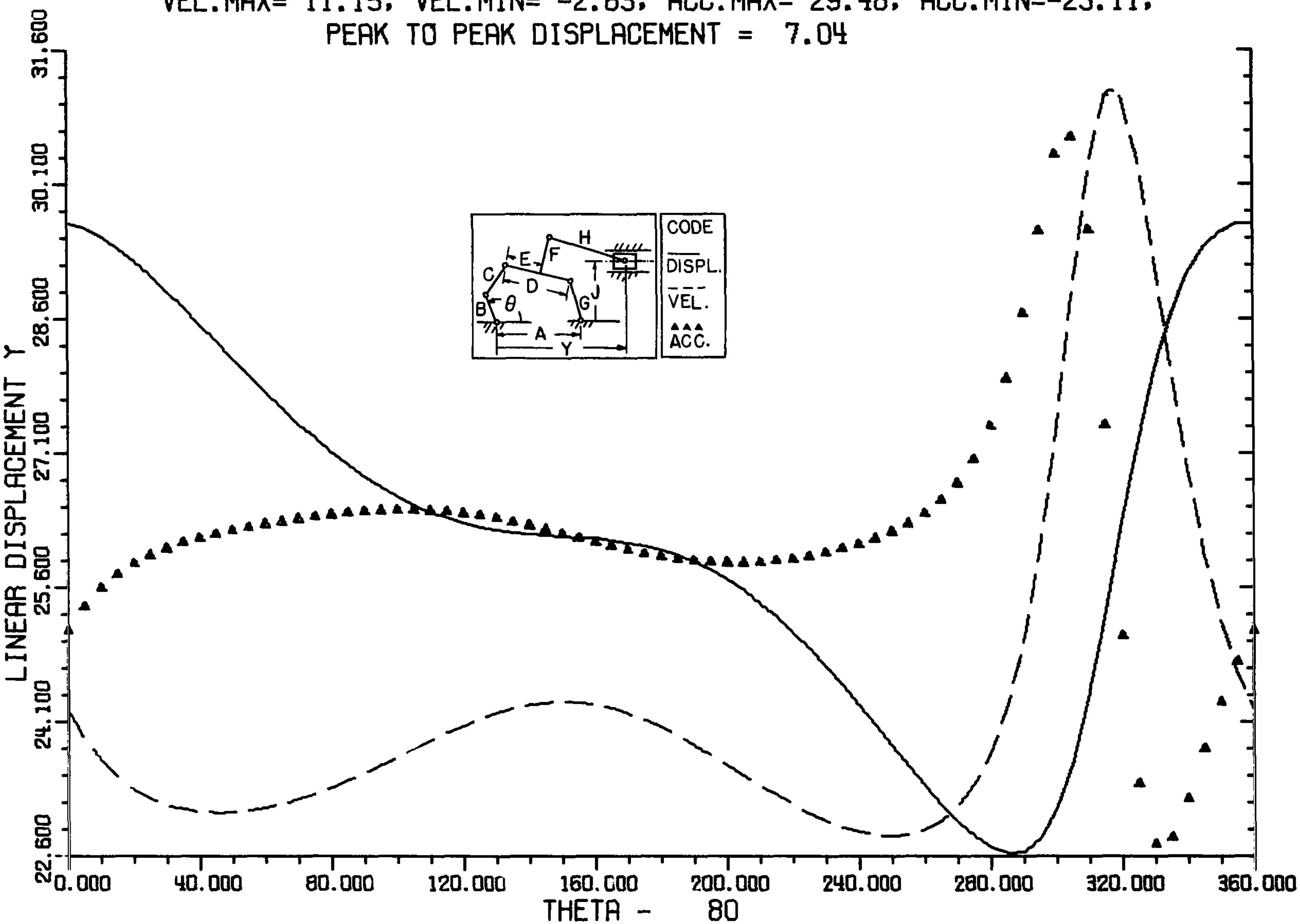


PLATE 3-4

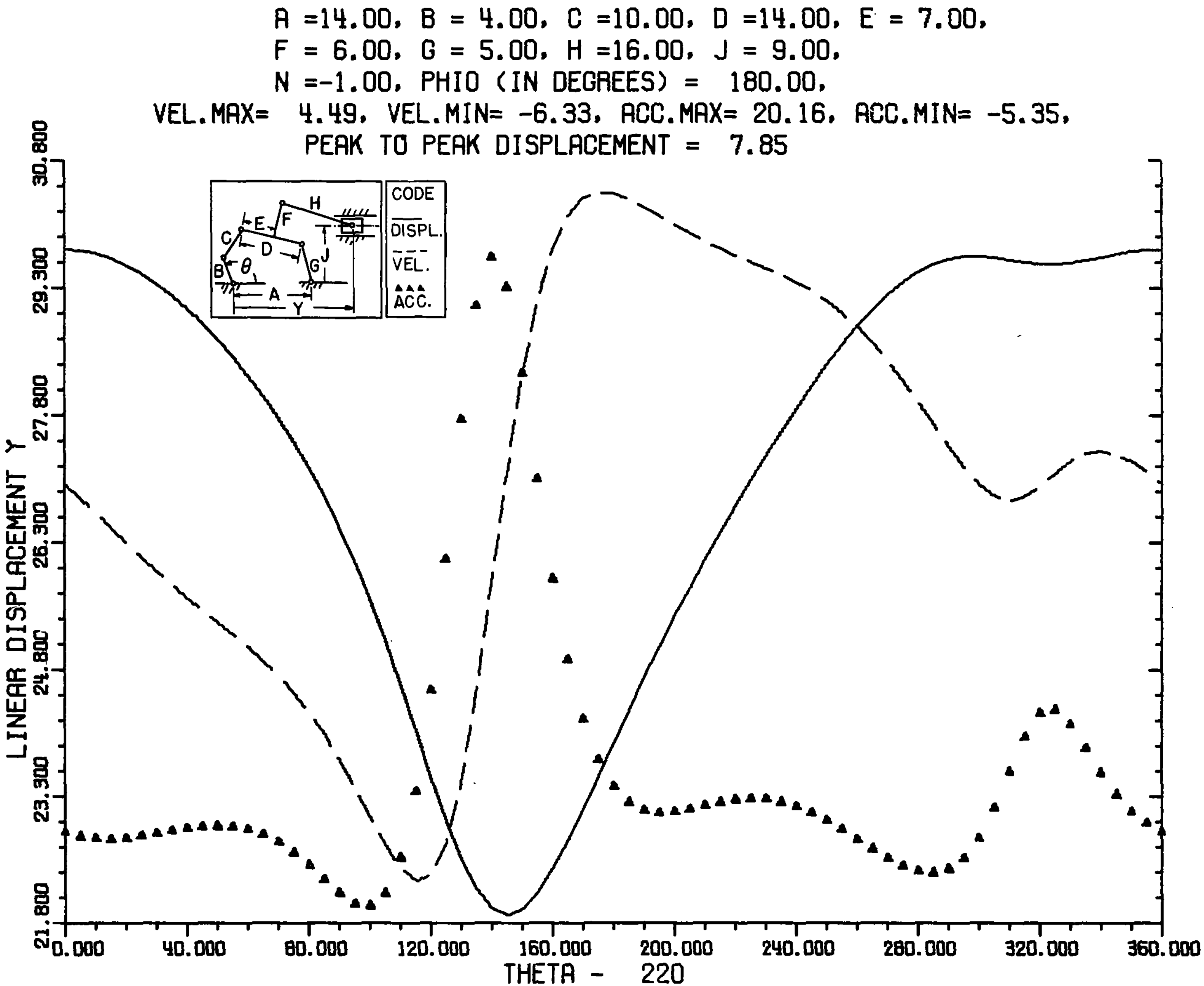

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-1.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 4.49, VEL.MIN= -6.33, ACC.MAX= 20.16, ACC.MIN= -5.35,
PEAK TO PEAK DISPLACEMENT = 7.85
CODE
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
Y
θ
LINEAR DISPLACEMENT Y
21.800
23.300
24.800
26.300
27.800
29.300
30.800
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 220

PLATE 3-5

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 3.99, VEL.MIN= -4.32, ACC.MAX= 3.27, ACC.MIN= -5.91,
PEAK TO PEAK DISPLACEMENT = 8.36

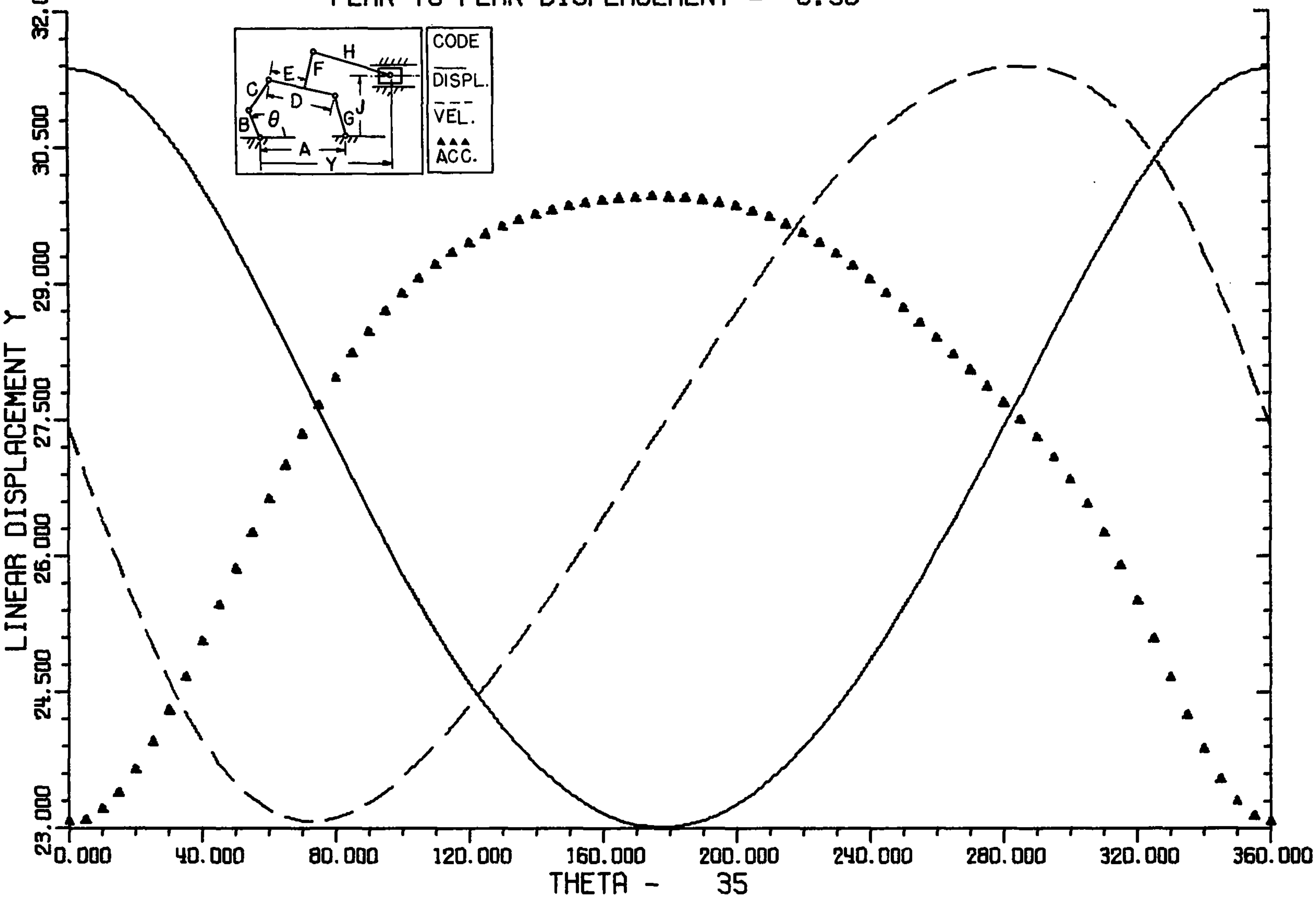


PLATE 3-6

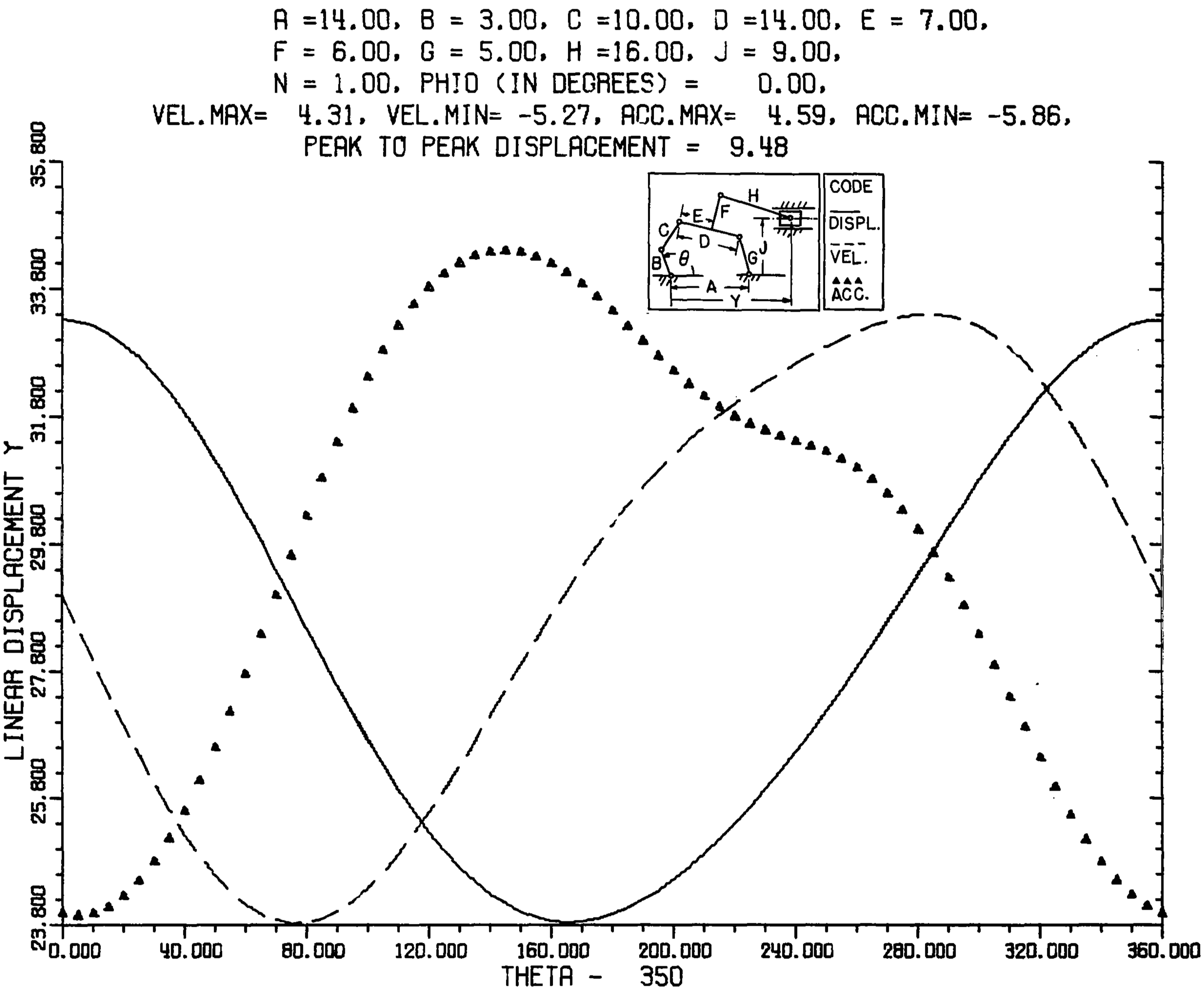
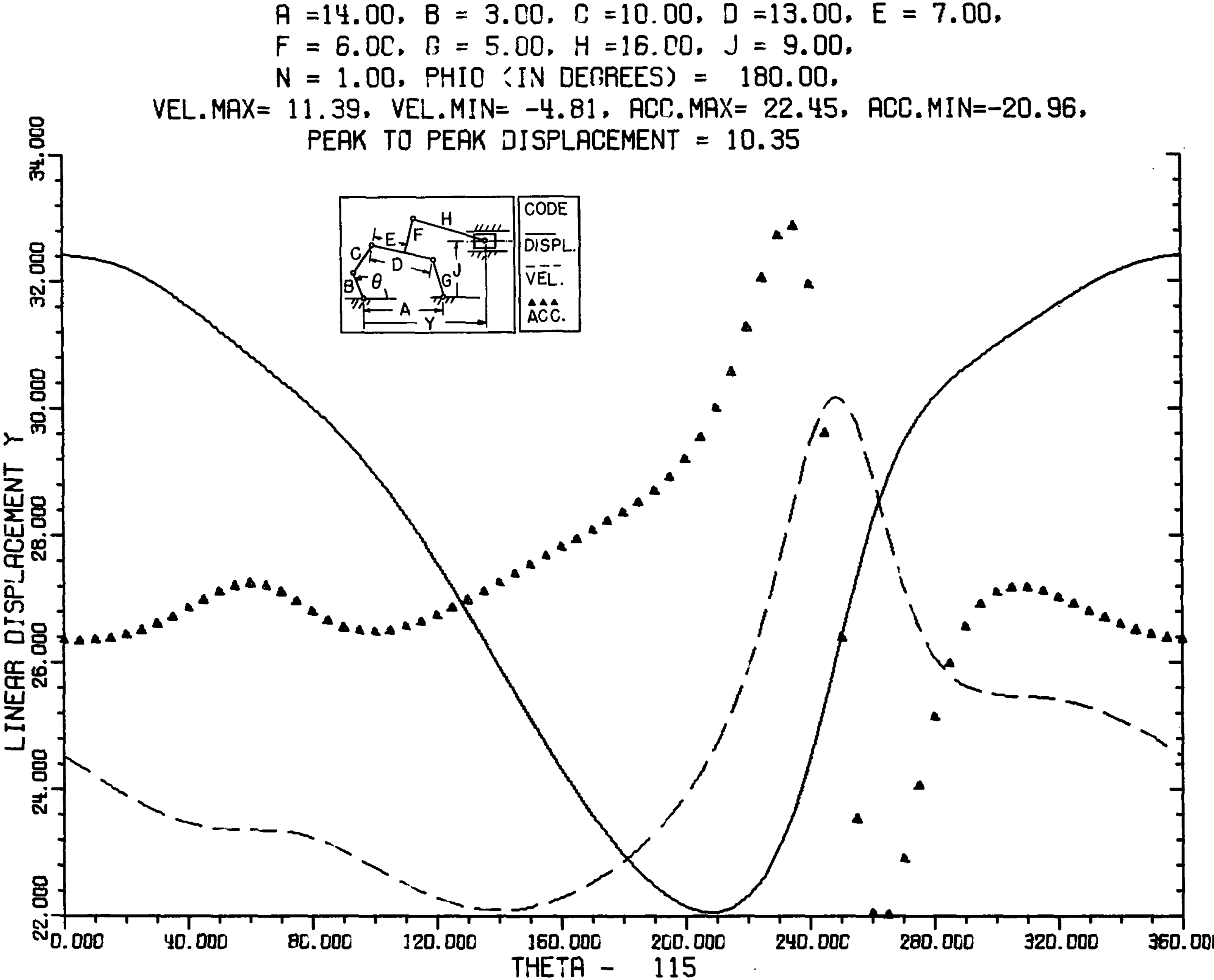

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 4.31, VEL.MIN= -5.27, ACC.MAX= 4.59, ACC.MIN= -5.86,
PEAK TO PEAK DISPLACEMENT = 9.48
CODE
DISPL.
VEL.
ACC.
LINEAR DISPLACEMENT Y
THETA - 350
23.800
25.800
27.800
29.800
31.800
33.800
35.800
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 3-7



A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 11.39, VEL.MIN= -4.81, ACC.MAX= 22.45, ACC.MIN=-20.96,
PEAK TO PEAK DISPLACEMENT = 10.35
CODE
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
Y
θ
LINEAR DISPLACEMENT Y
22.000
24.000
26.000
28.000
30.000
32.000
34.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 115

PLATE 3-8

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 7.45, VEL.MIN= -5.54, ACC.MAX= 10.22, ACC.MIN= -8.03,
PEAK TO PEAK DISPLACEMENT = 12.14

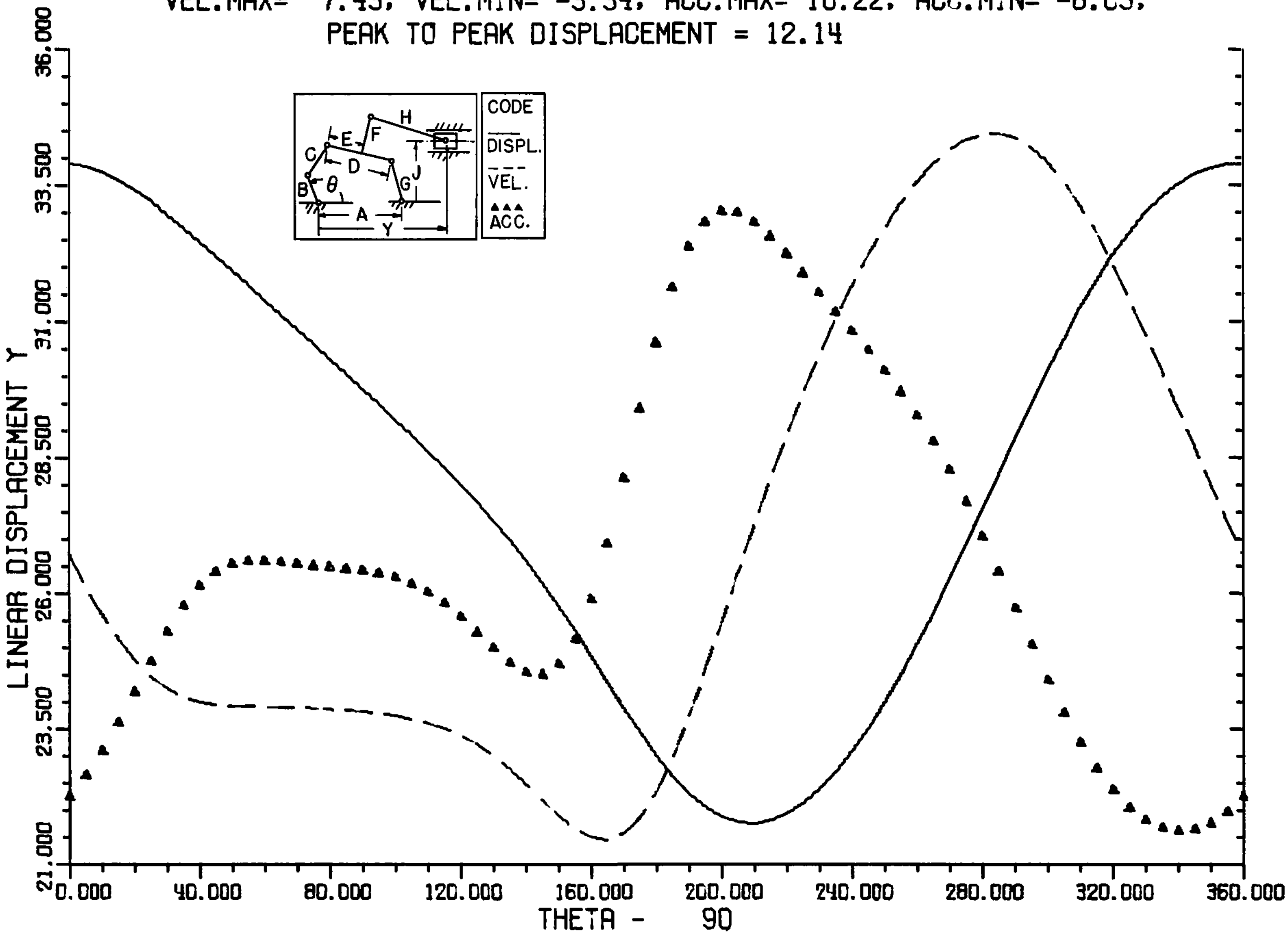


PLATE 3-9

A =14.00, B = 4.00, C =10.00, D =12.00, E = 5.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 8.44, VEL.MIN= -5.93, ACC.MAX= 10.93, ACC.MIN=-10.18,
PEAK TO PEAK DISPLACEMENT = 12.74

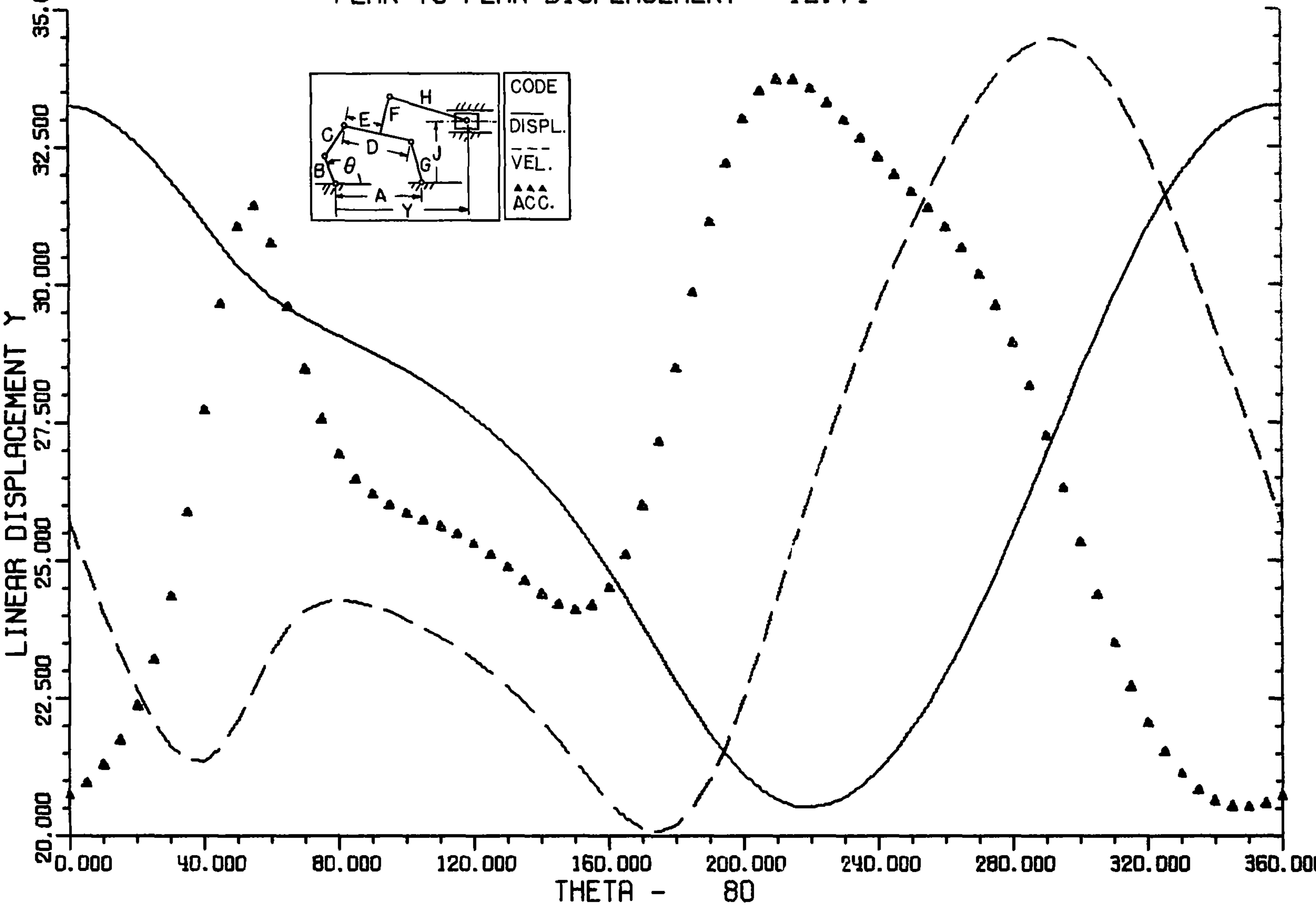


PLATE 3-10

A =16.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 7.00, H =16.00, J = 9.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 7.06, VEL.MIN= -7.56, ACC.MAX= 5.43, ACC.MIN= -9.60,
PEAK TO PEAK DISPLACEMENT = 13.41

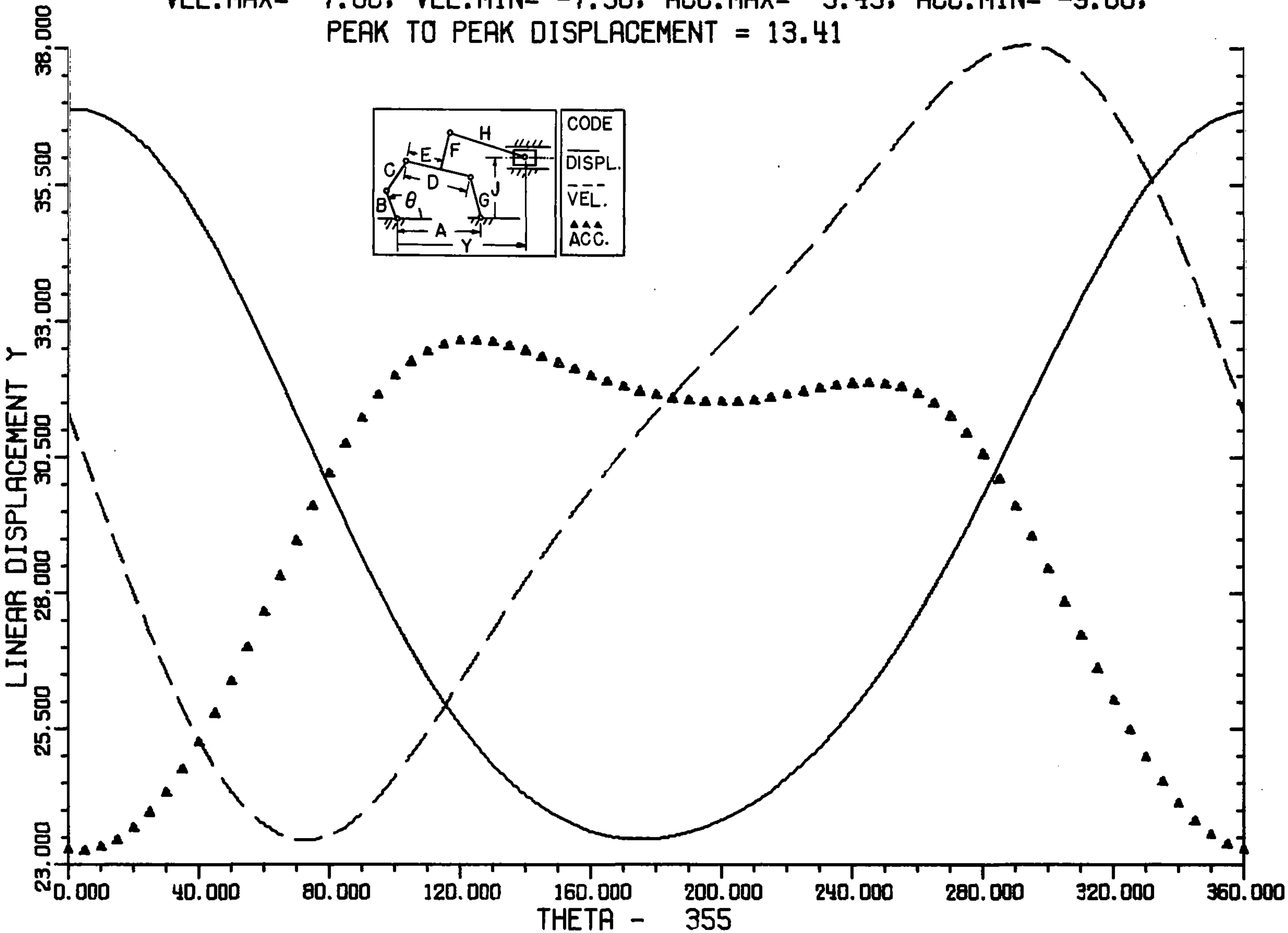


PLATE 3-11

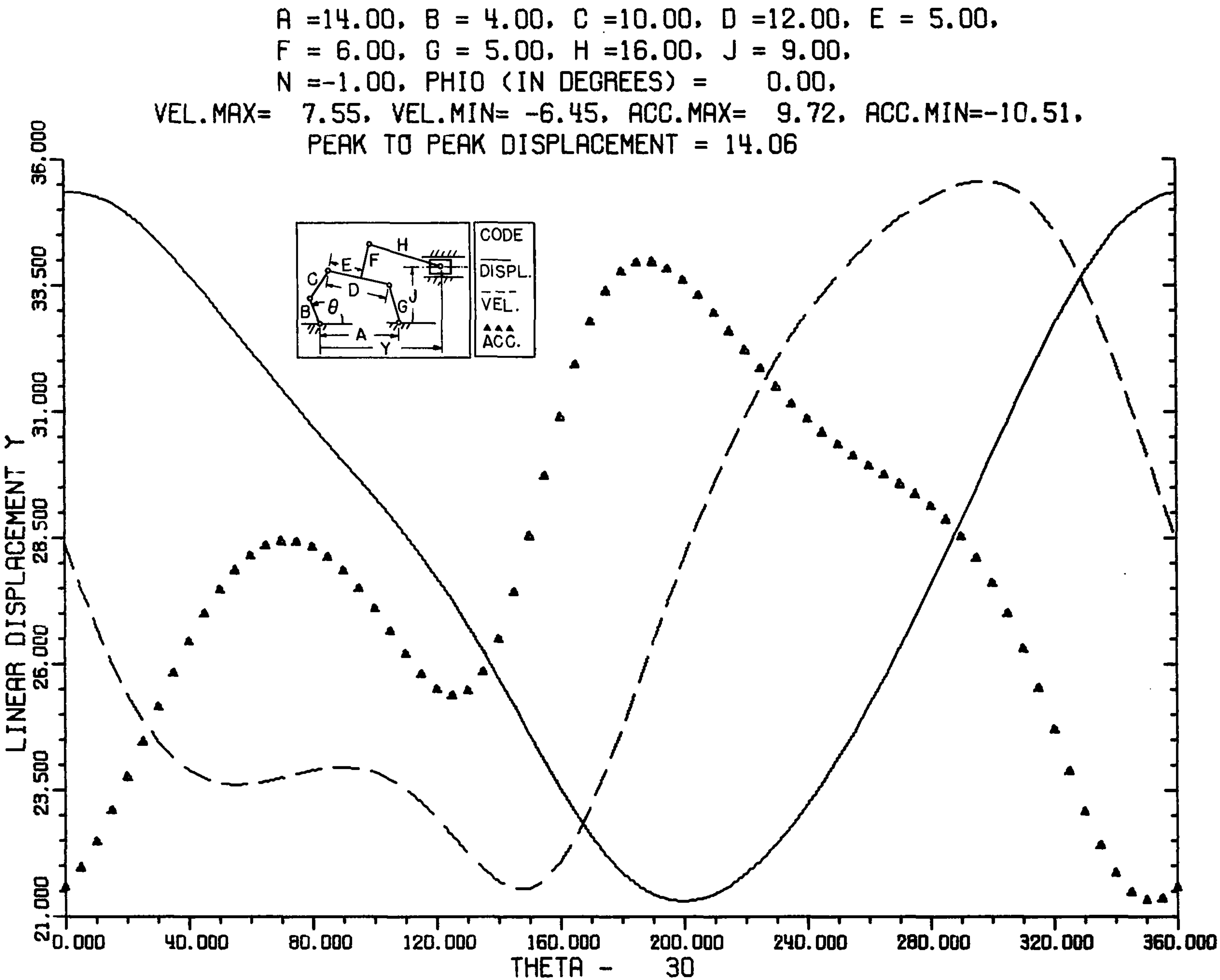

A =14.00, B = 4.00, C =10.00, D =12.00, E = 5.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 7.55, VEL.MIN= -6.45, ACC.MAX= 9.72, ACC.MIN=-10.51,
PEAK TO PEAK DISPLACEMENT = 14.06
LINEAR DISPLACEMENT Y
21.000
23.500
26.000
28.500
31.000
33.500
36.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 30
CODE
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
Y
θ

PLATE 3 - 12

A =14.00, B = 4.CO, C =10.00, D =14.00, E = 7.00,
F = 8.00, G = 5.00, H =16.00, J = 9.00,
N =-1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 9.84, VEL.MIN= -8.04, ACC.MAX= 12.79, ACC.MIN=-10.32,
PEAK TO PEAK DISPLACEMENT = 15.33

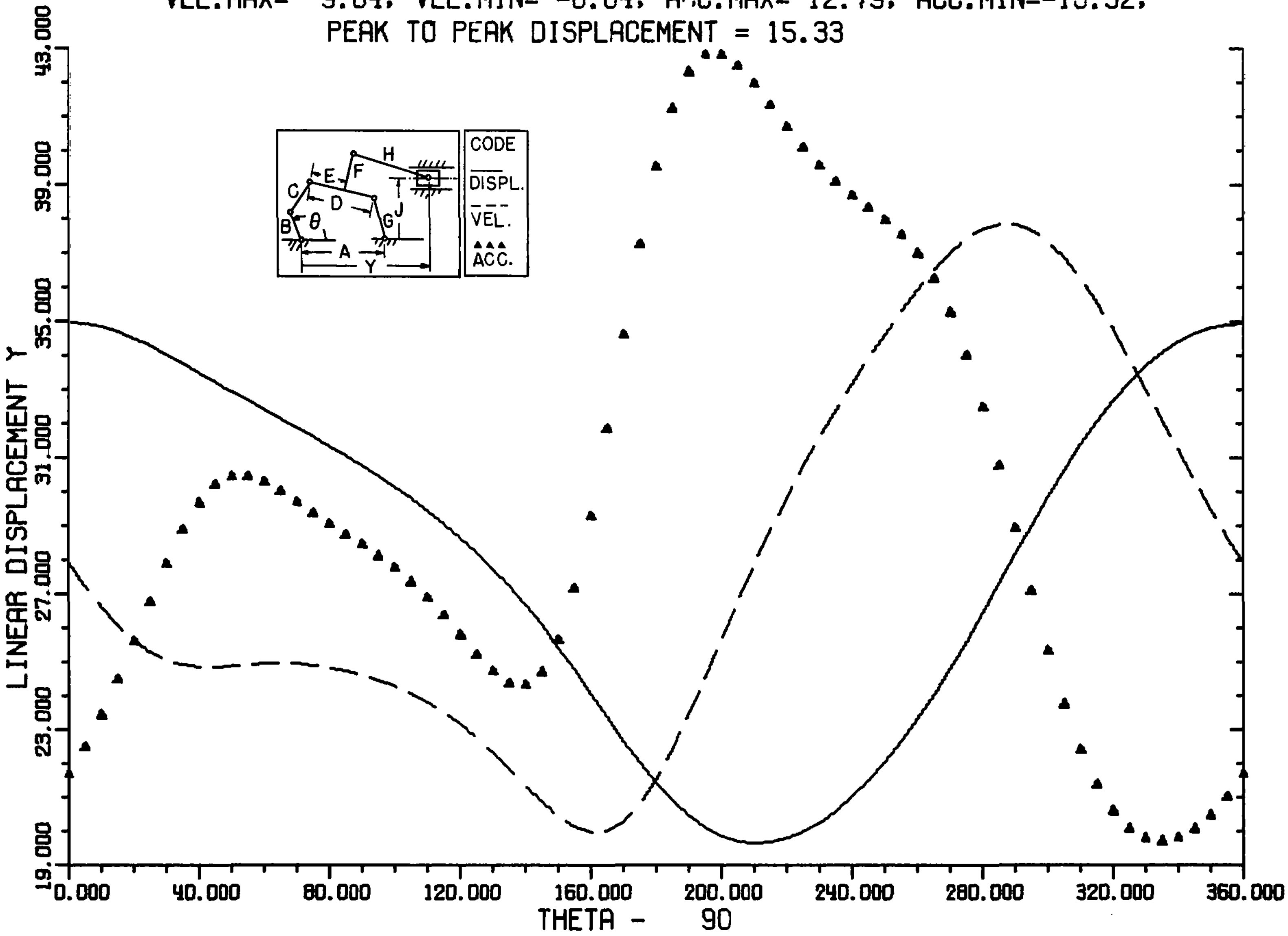


PLATE 3 - 13

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 2.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 13.18, VEL.MIN=-10.05, ACC.MAX= 49.45, ACC.MIN=-37.11,
PEAK TO PEAK DISPLACEMENT = 10.05

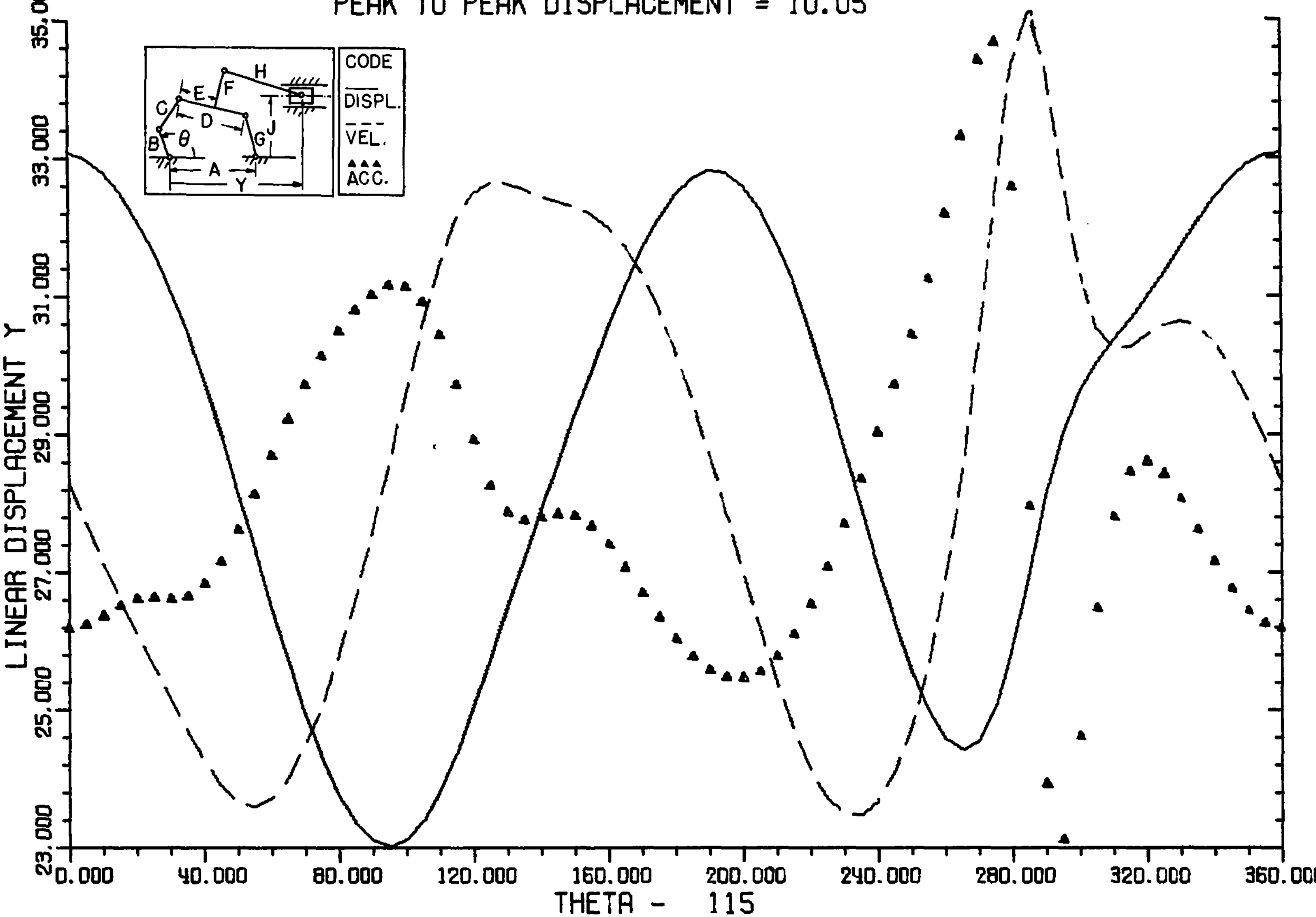


PLATE 3 - 14

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-2.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 11.01, VEL.MIN=-10.74, ACC.MAX= 41.09, ACC.MIN=-21.77,
PEAK TO PEAK DISPLACEMENT = 10.72

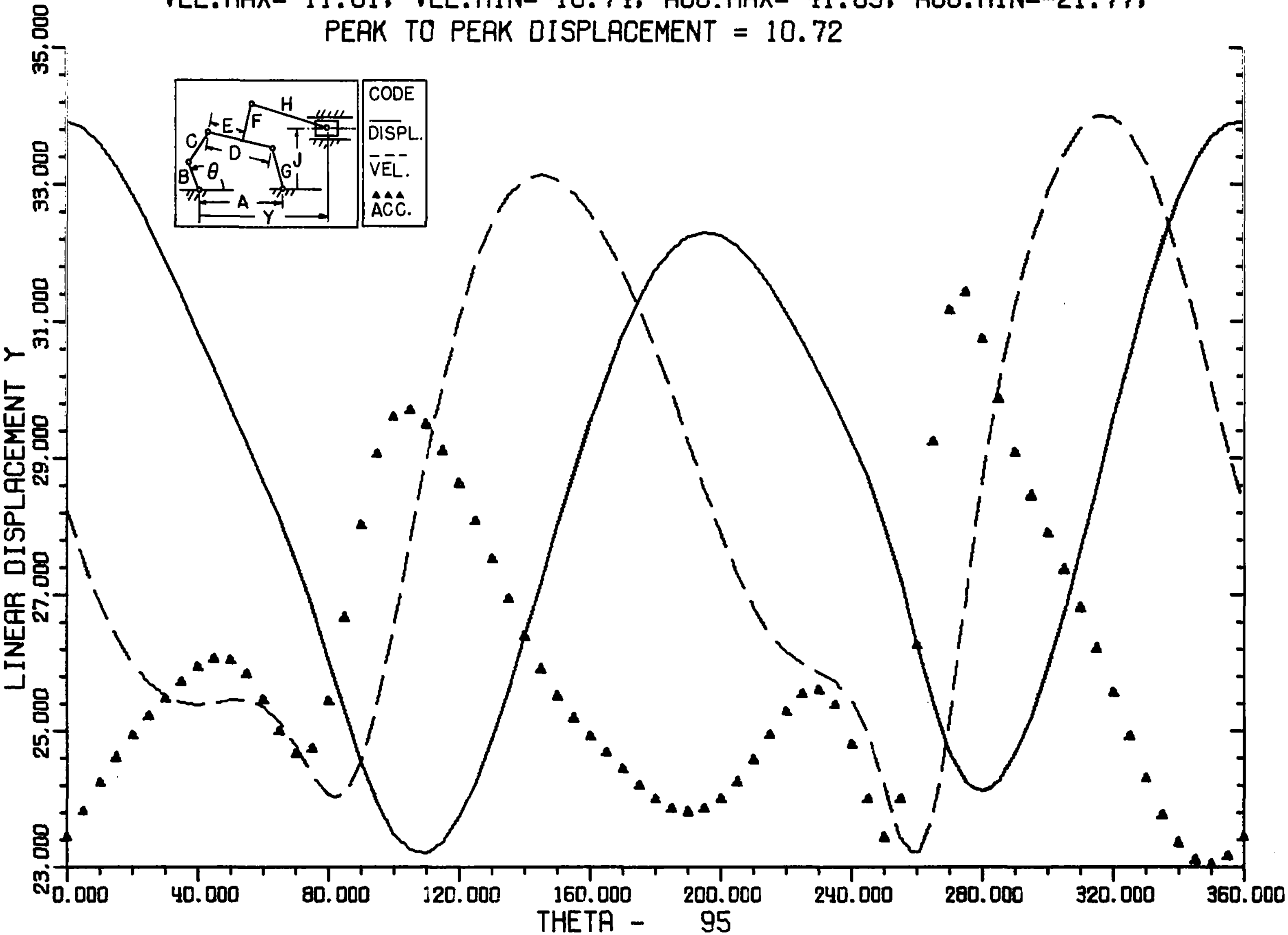


PLATE 3 - 15

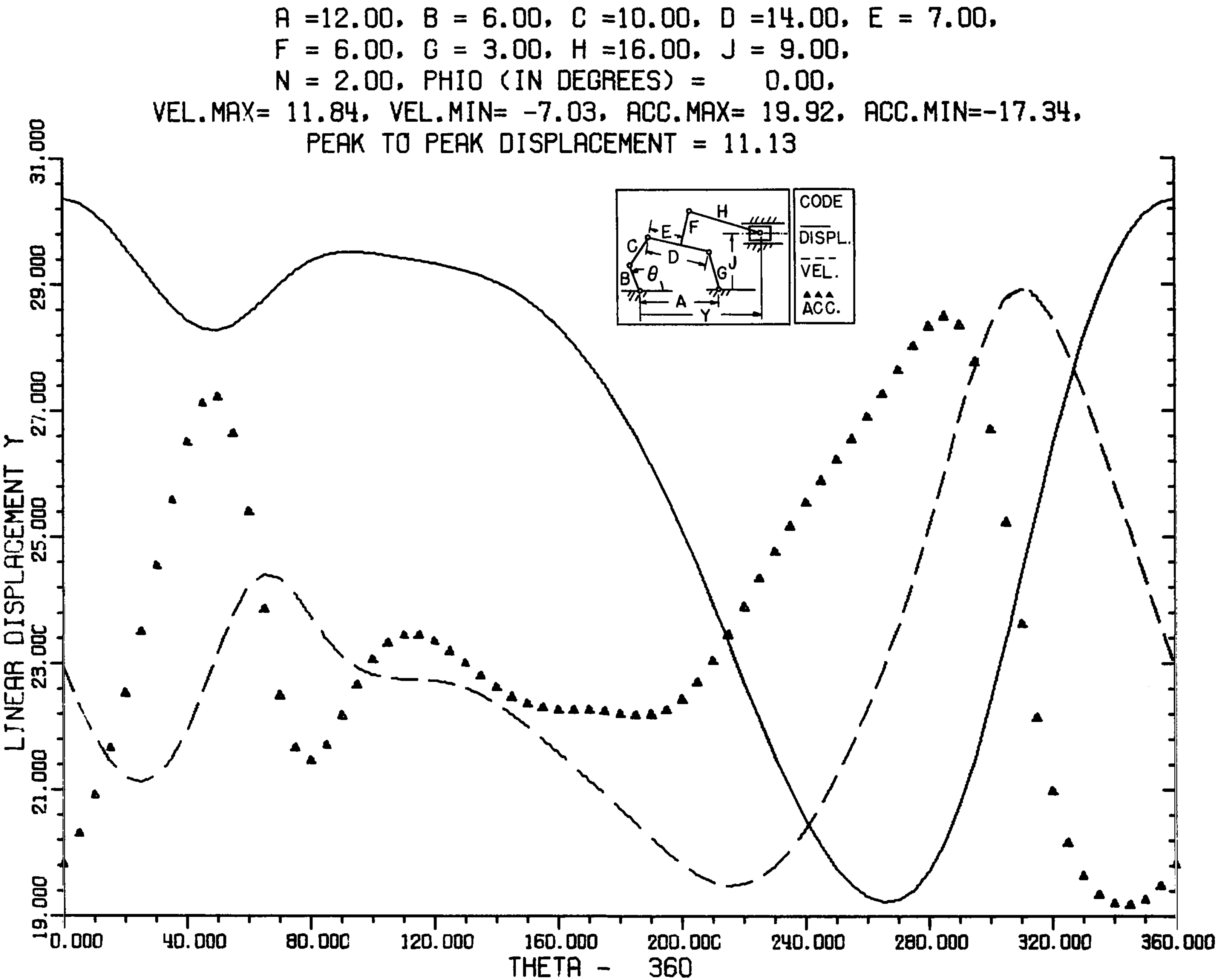

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 11.84, VEL.MIN= -7.03, ACC.MAX= 19.92, ACC.MIN=-17.34,
PEAK TO PEAK DISPLACEMENT = 11.13
LINEAR DISPLACEMENT Y
THETA - 360
CODE
DISPL.
VEL.
ACC.
A B C D E F G H J Y θ
19.000
21.000
23.000
25.000
27.000
29.000
31.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 3 - 16

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-2.00, PHIO (IN DEGREES) = 270.00,
VEL.MAX= 11.25, VEL.MIN= -9.61, ACC.MAX= 30.60, ACC.MIN=-19.55,
PEAK TO PEAK DISPLACEMENT = 11.59

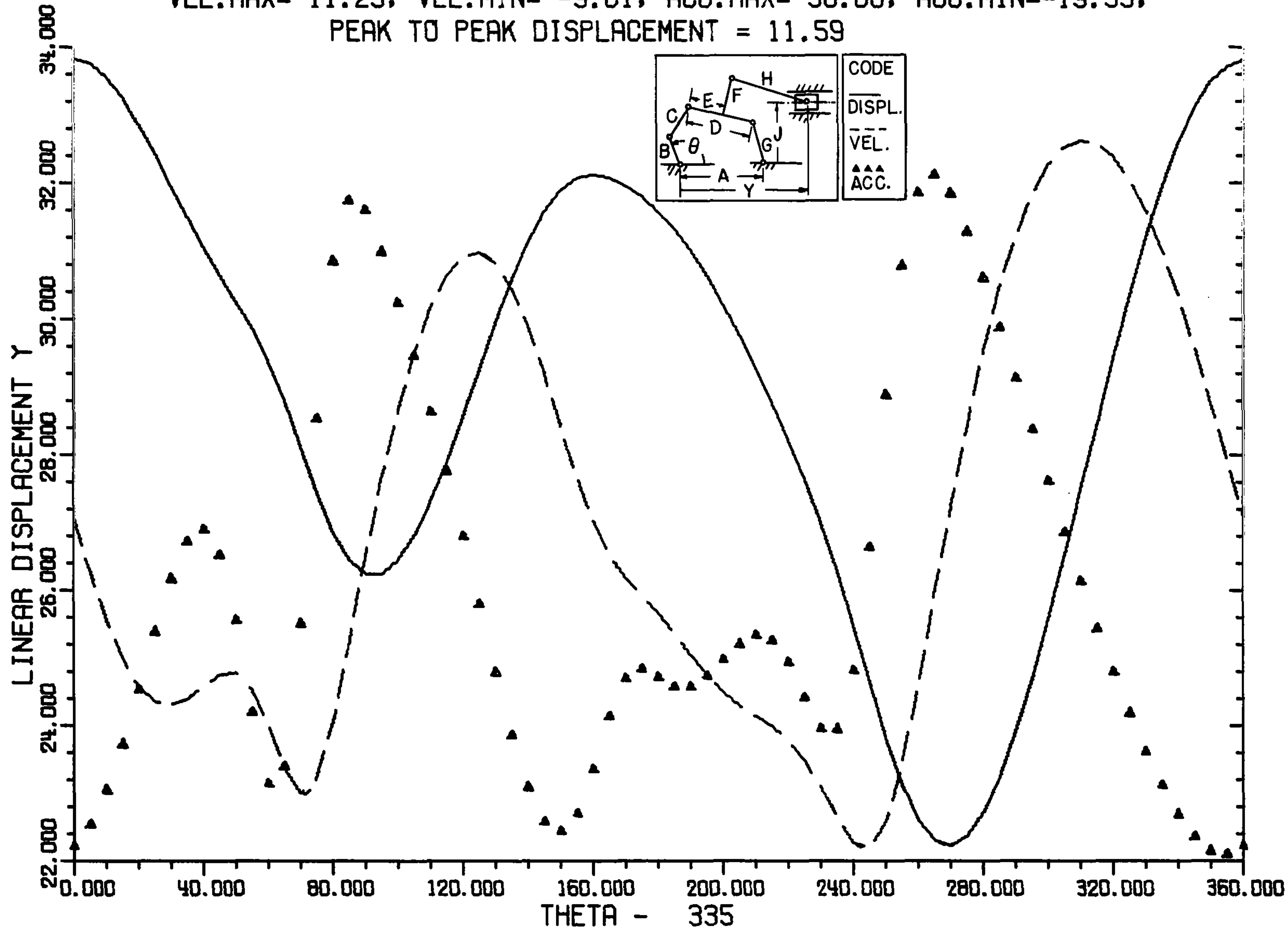


PLATE 3 - 17

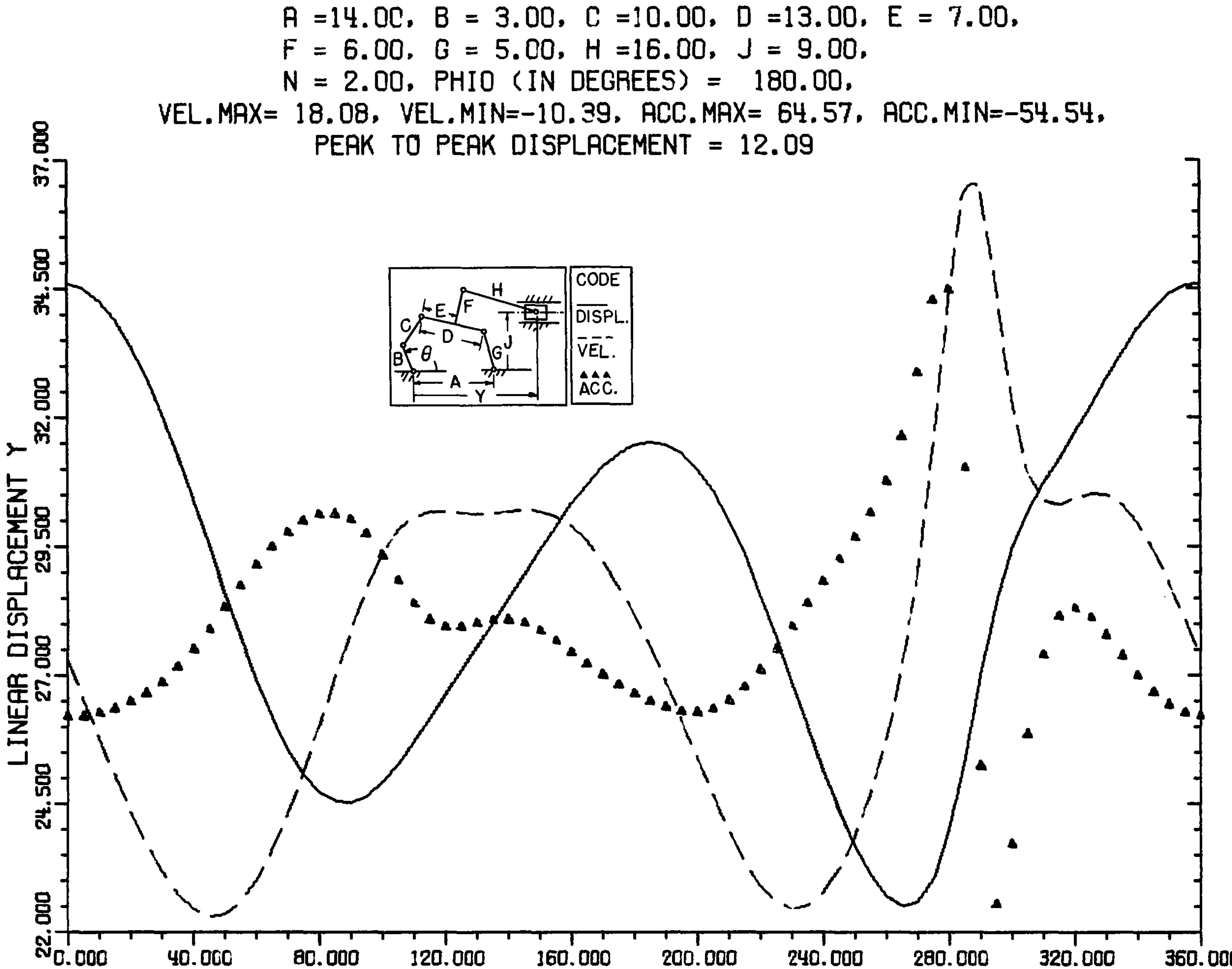

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 2.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 18.08, VEL.MIN=-10.39, ACC.MAX= 64.57, ACC.MIN=-54.54,
PEAK TO PEAK DISPLACEMENT = 12.09
LINEAR DISPLACEMENT Y
THETA -
CODE
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
Y
θ
22.000
24.500
27.000
29.500
32.000
34.500
37.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 3-18

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N =-2.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 12.55, VEL.MIN=-16.87, ACC.MAX= 76.73, ACC.MIN=-33.26,
PEAK TO PEAK DISPLACEMENT = 12.46

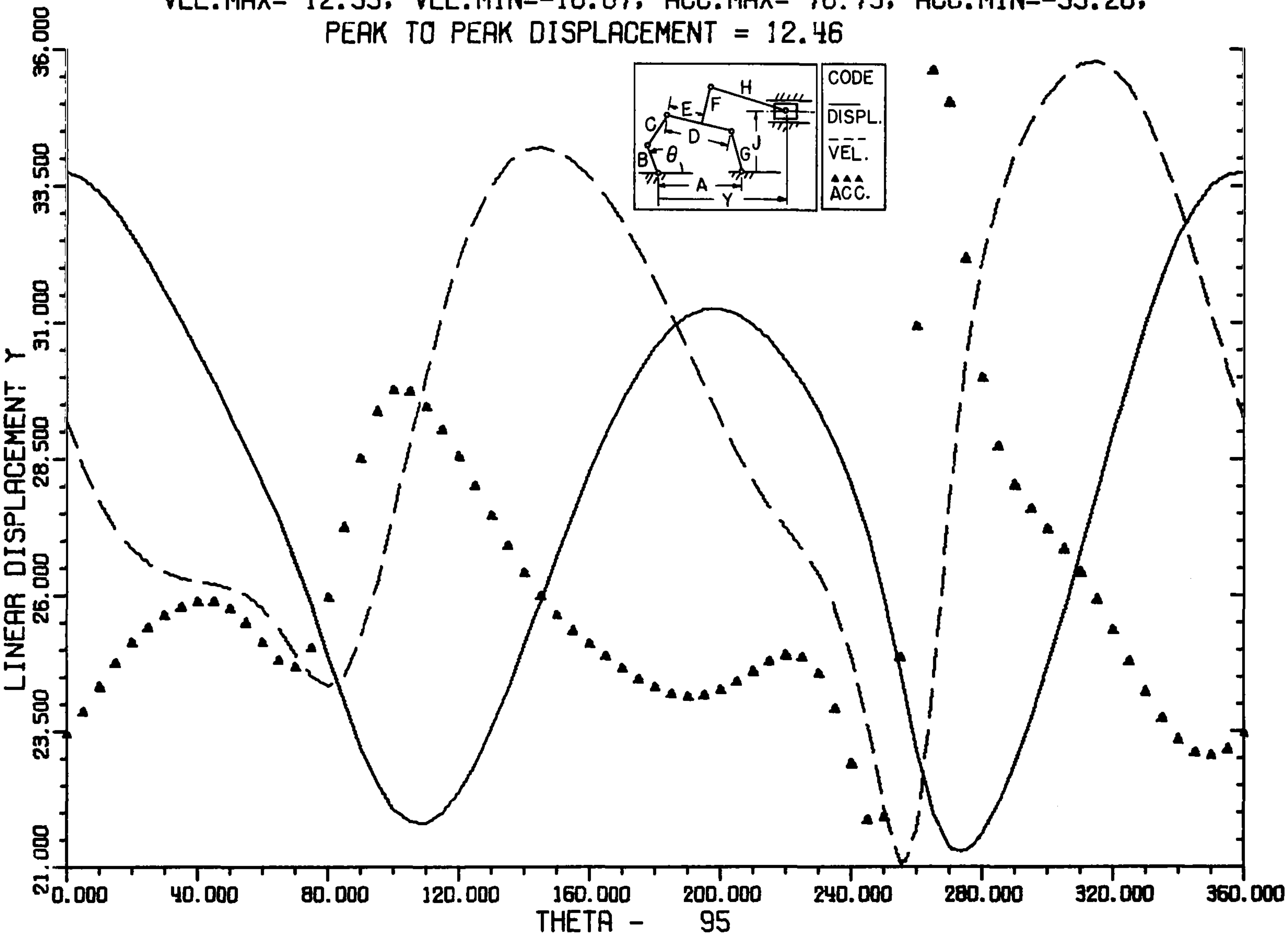


PLATE 3 - 19

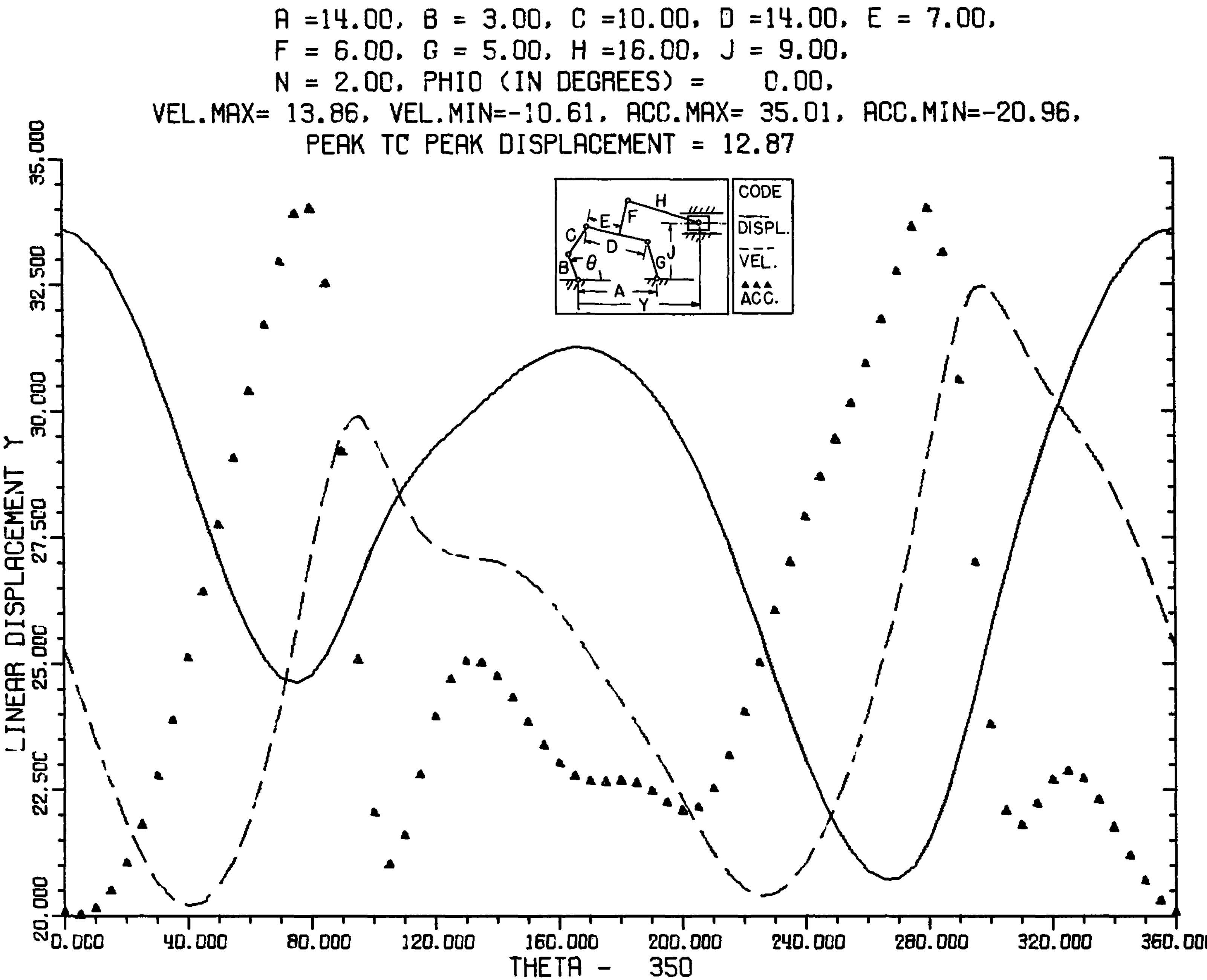

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 13.86, VEL.MIN=-10.61, ACC.MAX= 35.01, ACC.MIN=-20.96,
PEAK TO PEAK DISPLACEMENT = 12.87
CODE
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
Y
θ
LINEAR DISPLACEMENT Y
20.000
22.500
25.000
27.500
30.000
32.500
35.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 350

PLATE 3-20

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 2.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 32.20, VEL.MIN=-11.17, ACC.MAX=154.04, ACC.MIN=108.44,
PEAK TO PEAK DISPLACEMENT = 14.88

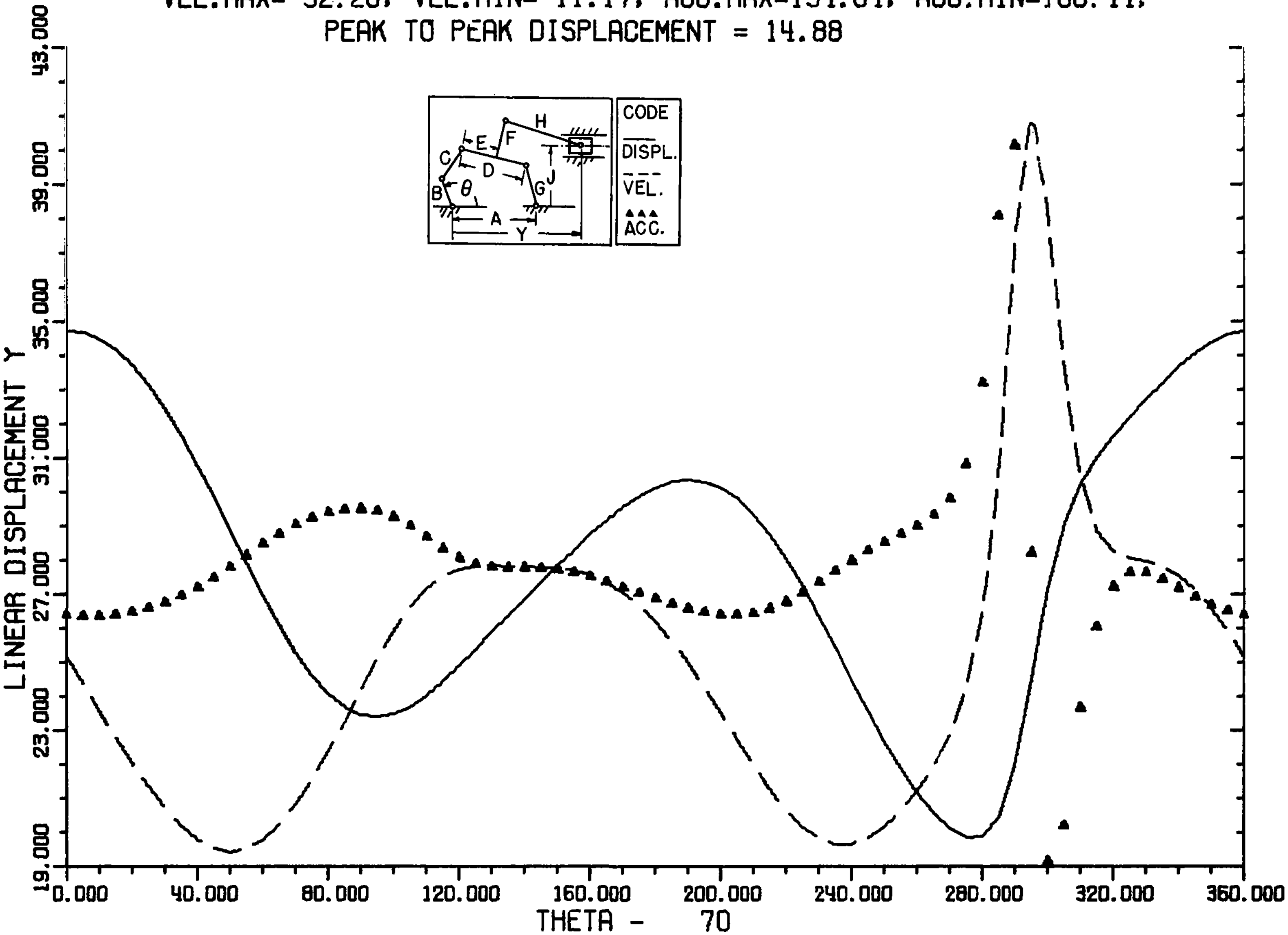


PLATE 3-21

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 3.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 24.90, VEL.MIN=-15.77, ACC.MAX=137.08, ACC.MIN=-87.34,
PEAK TO PEAK DISPLACEMENT = 12.55

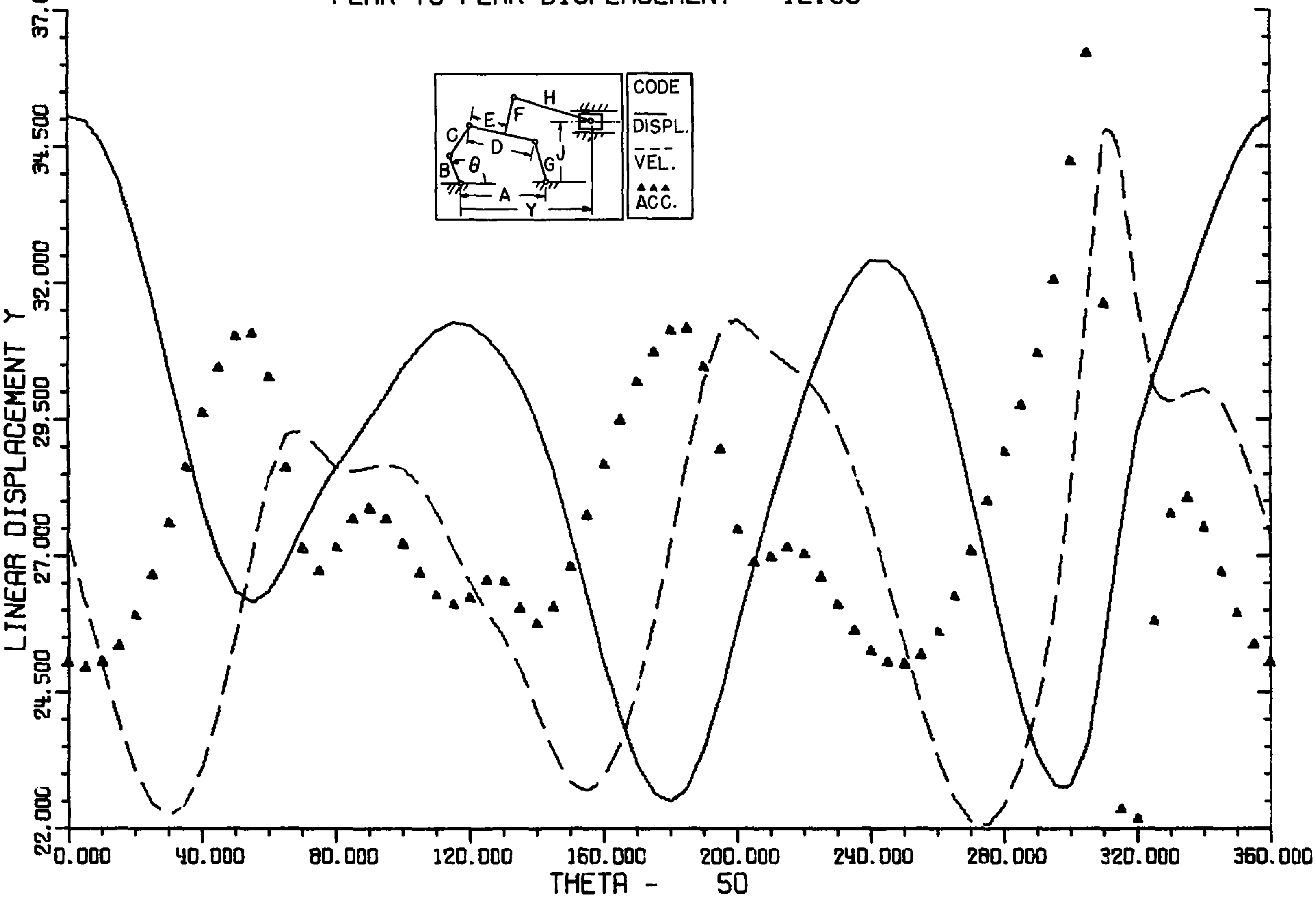


PLATE 3 - 22

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 3.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 24.94, VEL.MIN=-16.68, ACC.MAX=105.43, ACC.MIN=-62.96,
PEAK TO PEAK DISPLACEMENT = 13.42

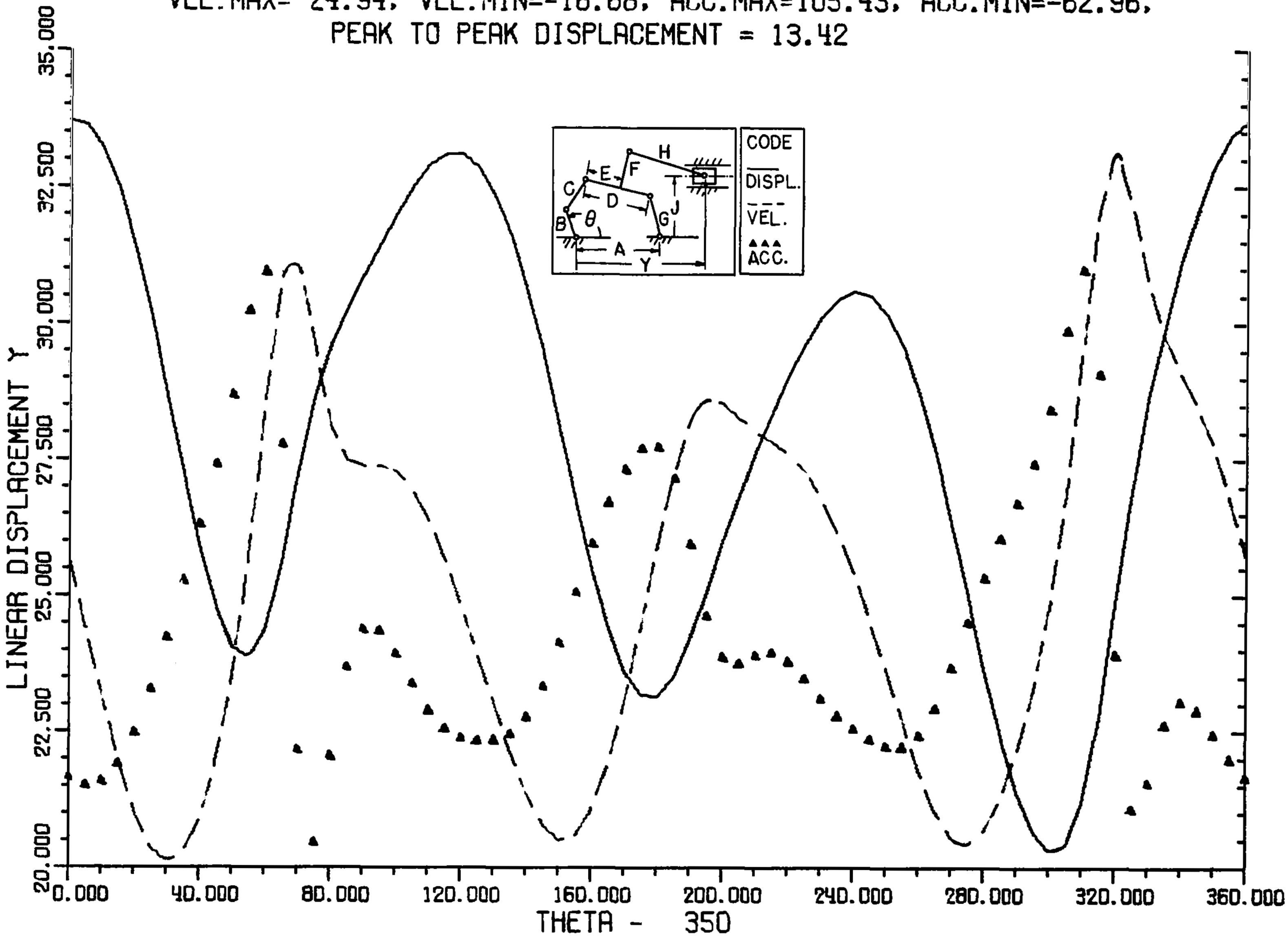


PLATE 3-23

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 3.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 27.65, VEL.MIN=-16.84, ACC.MAX=111.62, ACC.MIN=-71.42,
PEAK TO PEAK DISPLACEMENT = 14.40

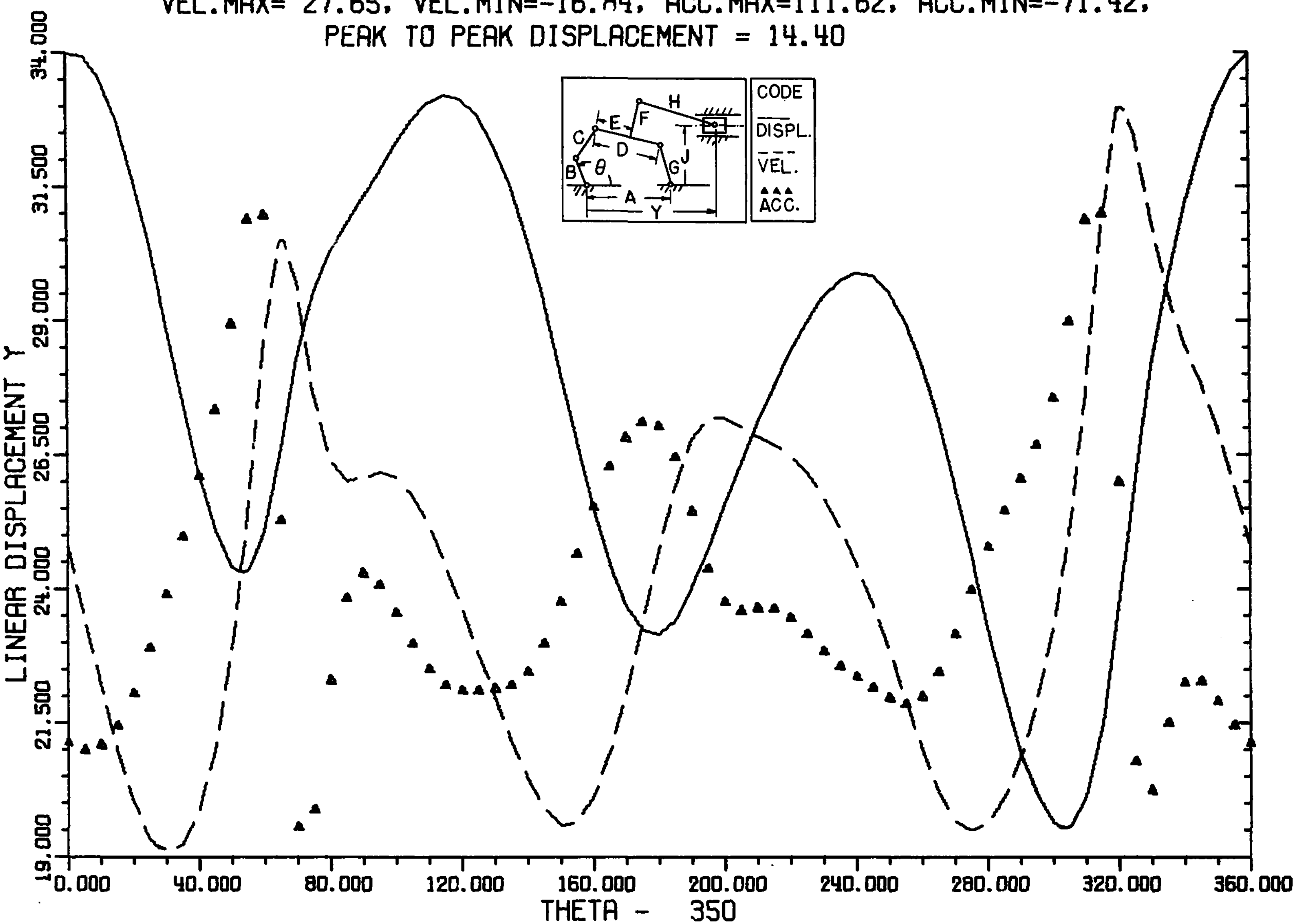


PLATE 3-24

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00,
N = 3.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 42.97, VEL.MIN=-16.88, ACC.MAX=299.87, ACC.MIN=192.00,
PEAK TO PEAK DISPLACEMENT = 15.23

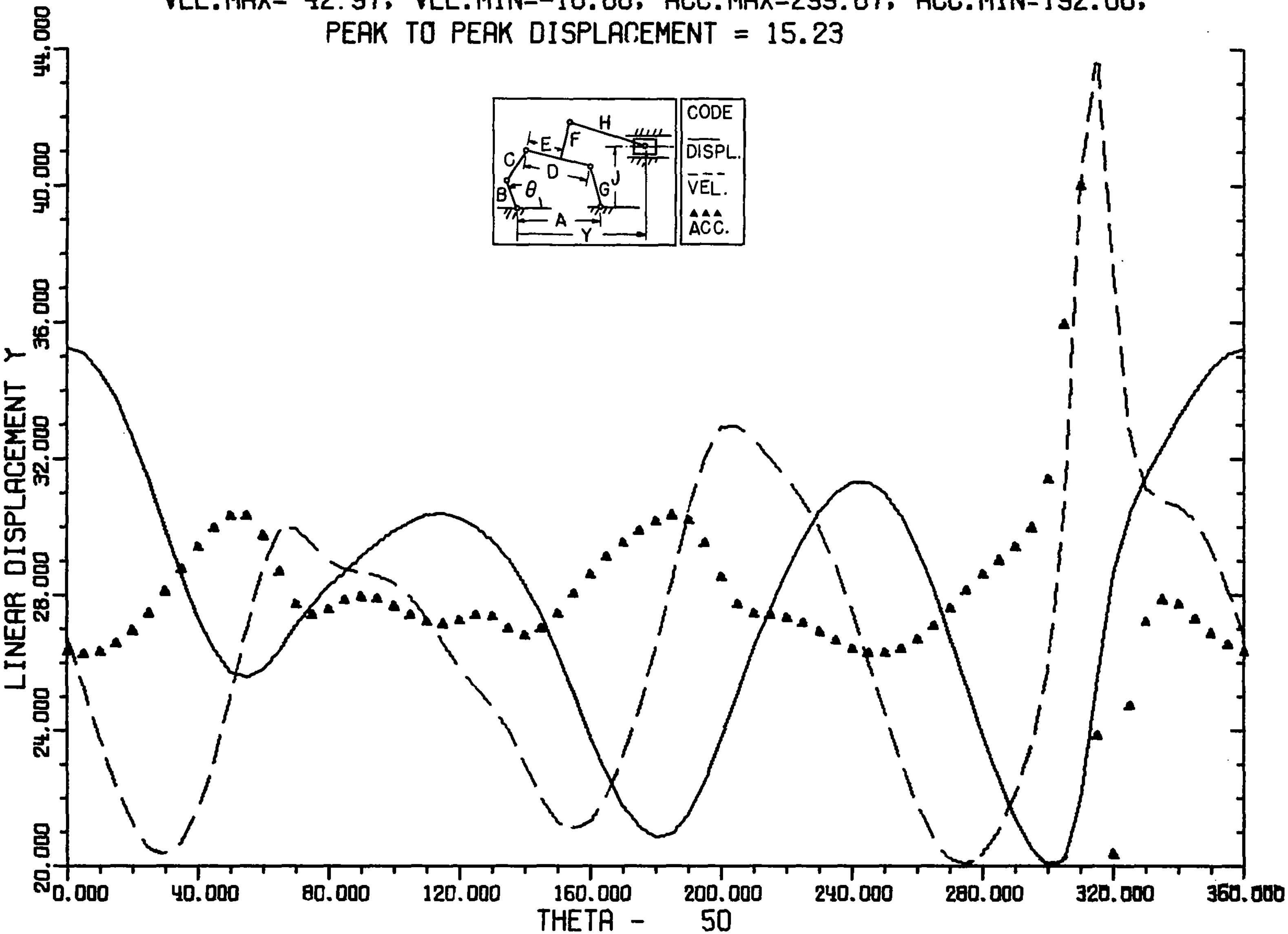


PLATE 3-25

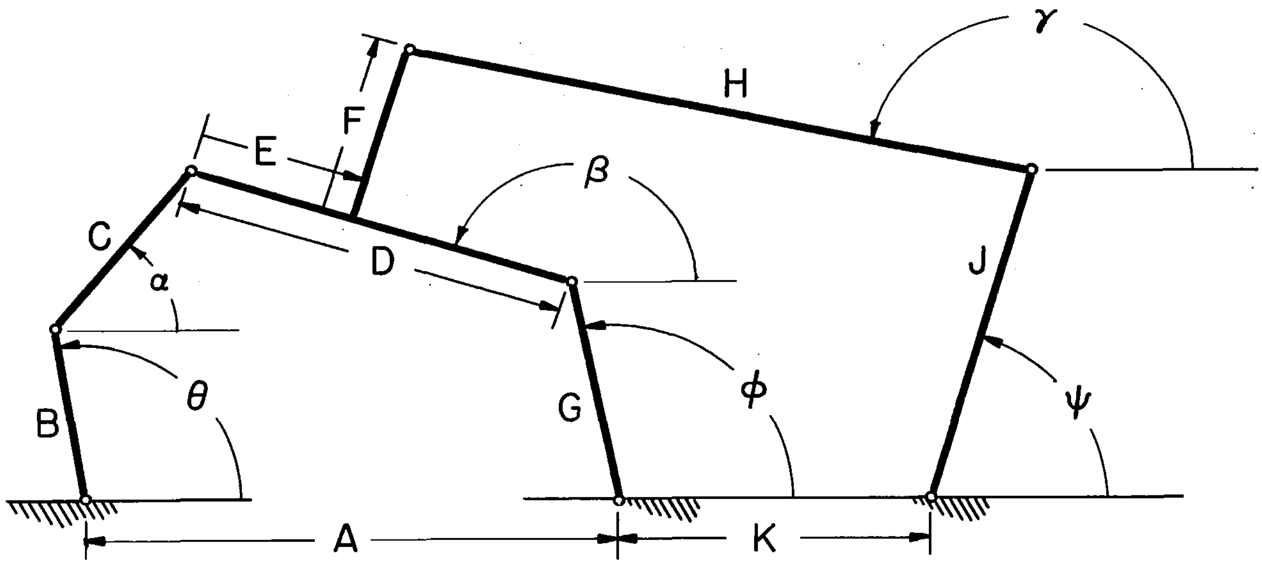


Figure 4-1

Figure 4-1 defines Mechanism #4. It is a multiple input mechanism in that two input angles must be prescribed; they in turn define one angular quantity, the output. The input angles θ and ϕ (theta and phi) identify the angular positions of links B and G, respectively. The output variable psi, ψ defines the angular position of link J.

In the graphs for this mechanism the two input angles have been related by the equation:

$$\phi = N \times \theta + PHI0 \qquad (4\text{-}1)$$

This kind of relationship may be obtained physically by the use of suitable gearing. In this equation N may be positive or negative--a positive value for N indicates that the links B and G rotate in the same direction while a negative value for N indicates rotation in opposite direction. In Eq. 4-1 if

$\theta = 0$ then ϕ = PHI0. Or in words, PHI0 is the initial phase angle between the two input angles θ and ϕ. The complete specification for the mechanism requires values for N and PHI0 as well as A, B, C, et cetera. Values for these variables are given as part of the heading for every graph.

Each graph shows ψ with respect to θ (PSI vs. THETA) as a solid line, the derivative of ψ with respect to θ versus θ as a dashed line, and the second derivative of ψ with respect to θ versus θ as a series of small triangles. Each curve begins with the maximum angular displacement of ψ. This has been accomplished by shifting the θ axis. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. The variables θ and ψ are presented in the units degrees. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement.

Scales have not been presented for the derivatives but each graph heading includes the maximum and minimum for both the angular velocity, $d\psi/d\theta$, and the angular acceleration, $d^2\psi/d\theta^2$. From these data scales may be established if so desired. As the input and output variables are angles the overall size of the mechanism is unimportant. The link lengths A, B, C, and so forth are presented as integers, though this need not be, and so long as all of the link lengths are changed by the same proportion the angular relationships for the mechanism will remain unchanged.

For the velocity $d\psi/d\theta$ the units are radians per radian and for the acceleration the units are radians per radian squared. A more applicable engineering unit for the angular velocity may be deduced:

$$\frac{d\psi}{dt} = \frac{d\psi}{d\theta} \times \frac{d\theta}{dt}$$

$$= \frac{d\psi}{d\theta} \times \text{rpm} \times 2\pi \times \frac{1}{60}$$

$$\left(\frac{\text{rad}}{\text{sec}} = \frac{\text{rad}}{\text{rad}} \times \frac{\text{rev}}{\text{min}} \times \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{\text{sec}}\right).$$

This equation may be rewritten as

$$\frac{d\psi}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{d\psi}{d\theta}, \quad \frac{\text{radians}}{\text{second}}. \tag{4-2}$$

In words, Eq. 4-2 states, that the angular velocity of link J (radians/ second) equals the product of $\pi/30$ times the angular speed (revolutions per minutes) of link B and $d\psi/d\theta$ (radians per radian). The value for $d\psi/d\theta$ may be obtained from the graph as the dashed line or the extreme values may be obtained from the heading of the graph as VEL. MAX or VEL. MIN.

The angular acceleration of link J may be derived as:

$$\frac{d^2\psi}{dt^2} = \frac{d^2\psi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2 + \frac{d^2\theta}{dt^2}\,\frac{d\psi}{d\theta}. \tag{4-3}$$

For the special case of the angular velocity of link B turning with constant speed, $d^2\theta/dt^2 = 0$ and, similar to the derivation of Eq. 4-2, the expression for the angular acceleration, Eq. 4-3, may be simplified to:

$$\frac{d^2\psi}{dt^2} = \left[\frac{\pi}{30} \times \text{rpm}\right]^2 \frac{d^2\psi}{d\theta^2}$$

$$\frac{\text{radians}}{\text{second}^2} = \frac{\text{radians}^2}{\text{second}^2} \times \frac{\text{radians}}{\text{radian}^2}. \tag{4-4}$$

Values for $d^2\psi/d\theta^2$ may be determined from the series of small triangles for each graph or the extreme values may be read from the graph title as ACC. MAX or ACC. MIN.

Referring to Figure 4-1 for this mechanism the equations relating the output to the inputs may be derived. Looking at the basic five-bar mechanism as projected onto a vertical line:

$$B \sin\theta + C \sin\alpha = G \sin(\pi - \phi) + D \sin(\pi - \beta).$$

This may be rewritten as

$$C \sin\alpha - D \sin\beta = G \sin\phi - B \sin\theta$$

$$= Q \tag{4-5}$$

in which Q is written for convenience for the manipulations which follow. Looking at projections onto a horizontal line the five-bar portion of the mechanism yields

$$B \cos\theta + C \cos\alpha + D \cos(\pi - \beta) + G \cos(\pi - \phi) = A$$

which may be transposed to

$$C \cos\alpha - D \cos\beta = A - B \cos\theta + G \cos\phi$$

$$= L \tag{4-6}$$

in which L is an abbreviation. Eqs. 4-5 and 4-6 may be rewritten as

$$C \sin\alpha = Q + D \sin\beta$$

$$C \cos\alpha = L + D \cos\beta$$

each of which may be squared and then the resulting equations summed to form

$$C^2 = Q^2 + 2\,DQ \sin\beta + D^2 + L^2 + 2\,DL \cos\beta\,.$$

Regrouping, this equation becomes

$$L \cos\beta + Q \sin\beta = \left[C^2 - D^2 - Q^2 - L^2\right] / (2D)$$

$$= M. \tag{4-7}$$

Note that with a given value for θ (which implies a value for ϕ via Eq. 4-1) and for specified values for the link lengths, Q, L, and M may be calculated using Eqs. 4-5 and 4-6. This in turn means that the only unknown in Eq. 4-7 is β. The value for β may be determined by different procedures; the Newton-Raphson method has been quite satisfactory.

Knowing β, the remaining portion of the mechanism may be considered. Looking again at projections onto a vertical line will reveal:

$$J \sin \psi + H \sin (\pi - \gamma) = G \sin \phi + (D - E) \sin \beta - F \cos \beta$$

which may be rewritten as:

$$H \sin \gamma = V - J \sin \psi \tag{4-8}$$

in which

$$V = G \sin \phi + (D - E) \sin \beta - F \cos \beta . \tag{4-9}$$

Similarly, projection onto a horizontal line will produce:

$$H \cos \gamma = U - J \cos \psi \tag{4-10}$$

in which

$$U = G \cos \phi + (D - E) \cos \beta + F \sin \beta - K . \tag{4-11}$$

Squaring Eqs. 4-8 and 4-10 and summing the resulting equations will eliminate γ and yield an equation in the single unknown ψ:

$$H^2 = U^2 + V^2 + J^2 - 2\,JV \sin \psi - 2\,JU \cos \psi . \tag{4-12}$$

Let

$$W = (U^2 + V^2 + J^2 - H^2)/\ (2J) \quad ,$$

then Eq. 4-12 may be written

$$U \cos \psi + V \sin \psi = W. \tag{4-13}$$

From this equation may be solved ψ. The Newton-Raphson method has been quite satisfactory for calculating ψ from Eq. 4-13. The graphs with the solid lined curves have been obtained using Eq. 4-13. They depict ψ as a function of θ for the particular mechanism which is defined by the link lengths given in the graph title. Both ψ and θ are presented in the units of degrees and θ has been shifted (by the amount indicated by the abscissa title, in degrees) so that the graph begins with the maximum value for ψ.

Angular Velocity

The angular velocity of the output link, J, may be determined by taking the derivative of Eq. 4-13 with respect to θ. This will yield:

$$-U \sin \psi \frac{d\psi}{d\theta} + \cos \psi \frac{dU}{d\theta} + V \cos \psi \frac{d\psi}{d\theta} + \sin \psi \frac{dV}{d\theta} = \frac{U}{J}\frac{dU}{d\theta} + \frac{V}{J}\frac{dV}{d\theta}.$$

From this may be solved

$$\frac{d\psi}{d\theta} = \frac{\left\{\left[\frac{U}{J} - \cos \psi\right] \frac{dU}{d\theta} + \left[\frac{V}{J} - \sin \psi\right] \frac{dV}{d\theta}\right\}}{V \cos \psi - U \sin \psi}. \tag{4-14}$$

In this equation $dU/d\theta$ and $dV/d\theta$ are not known but may be determined. By differentiating Eq. 4-11 with respect to θ the following results:

$$\frac{dU}{d\theta} = - G \sin \phi \frac{d\phi}{d\theta} - (D - E) \sin \beta \frac{d\beta}{d\theta} + F \cos \beta \frac{d\beta}{d\theta}. \tag{4-15}$$

An expression for $d\beta/d\theta$ may be obtained by differentiating Eq. 4-7 with respect to θ as:

$$\frac{d\beta}{d\theta} = \frac{\frac{dL}{d\theta}\left[\frac{L}{D} - \cos\beta\right] - \frac{dQ}{d\theta}\left[\sin\beta + \frac{Q}{D}\right]}{Q\cos\beta - L\sin\beta} . \qquad (4\text{-}16)$$

From Eqs. 4-5 and 4-6, by differentiation with respect to θ, one may obtain:

$$\frac{dQ}{d\theta} = G\cos\phi\,\frac{d\phi}{d\theta} - B\cos\theta$$

$$\frac{dL}{d\theta} = B\sin\theta - G\sin\phi\,\frac{d\phi}{d\theta} . \qquad (4\text{-}17)$$

And from Eq. 4-1:

$$\frac{d\phi}{d\theta} = N \quad \text{and} \quad \frac{d^2\phi}{d\theta^2} = 0 . \qquad (4\text{-}18)$$

Differentiating Eq. 4-9 with respect to θ yields:

$$\frac{dV}{d\theta} = G\cos\phi\,\frac{d\phi}{d\theta} + (D - E)\cos\beta\,\frac{d\beta}{d\theta} + F\sin\beta\,\frac{d\beta}{d\theta} . \qquad (4\text{-}19)$$

With Eq. 4-14 and those following it, $d\psi/d\theta$ may be evaluated. Values have been plotted as the dashed lines on the graphs which follow. One may think of $d\psi/d\theta$ as being in the units of radians per radian or just as correctly in the units of degrees per degree. As $d\psi/d\theta$ involves a ratio of identical units, the choice of units is immaterial. With Eq. 4-2 and those equations just cited, the angular velocity of link J, $d\psi/dt$, may be obtained in the form of the more conventional engineering units.

Angular Acceleration

The angular acceleration of the output link J may be obtained by the differentiation of Eq. 4-14 to produce:

$$\frac{d^2\psi}{d\theta^2} = \left\{ \frac{d\psi}{d\theta} \left[(U \cos\psi + V \sin\psi) \frac{d\psi}{d\theta} + \sin\psi \frac{d\psi}{d\theta} \right.\right.$$

$$\left. - \cos\psi \frac{dV}{d\theta} \right] + \frac{d^2U}{d\theta^2} \left[\frac{U}{J} - \cos\psi \right] + \frac{dU}{d\theta} \left[\frac{1}{J} \frac{dU}{d\theta} + \sin\psi \frac{d\psi}{d\theta} \right]$$

$$\left. + \frac{d^2V}{d\theta^2} \left[\frac{V}{J} - \sin\psi \right] + \frac{dV}{d\theta} \left[\frac{1}{J} \frac{dV}{d\theta} - \cos\psi \frac{d\psi}{d\theta} \right] \right\} \Big/ (V \cos\psi - U \sin\psi). \tag{4-20}$$

From Eq. 4-15 may be derived:

$$\frac{d^2U}{d\theta^2} = - G \cos\phi \left[\frac{d\phi}{d\theta} \right]^2 - (D - E) \sin\beta \frac{d^2\beta}{d\theta^2} - (D - E) \cos\beta \left[\frac{d\beta}{d\theta} \right]^2$$

$$+ F \cos\beta \frac{d^2\beta}{d\theta^2} - F \sin\beta \left[\frac{d\beta}{d\theta} \right]^2 . \tag{4-21}$$

And from Eq. 4-19 may be derived:

$$\frac{d^2V}{d\theta^2} = - G \sin\phi \left[\frac{d\phi}{d\theta} \right]^2 + (D - E) \cos\beta \frac{d^2\beta}{d\theta^2} - (D - E) \sin\beta \left[\frac{d\beta}{d\theta} \right]^2$$

$$+ F \sin\beta \frac{d^2\beta}{d\theta^2} + F \cos\beta \left[\frac{d\beta}{d\theta} \right]^2 . \tag{4-22}$$

Eq. 4-16 may be differentiated with respect to θ to afford:

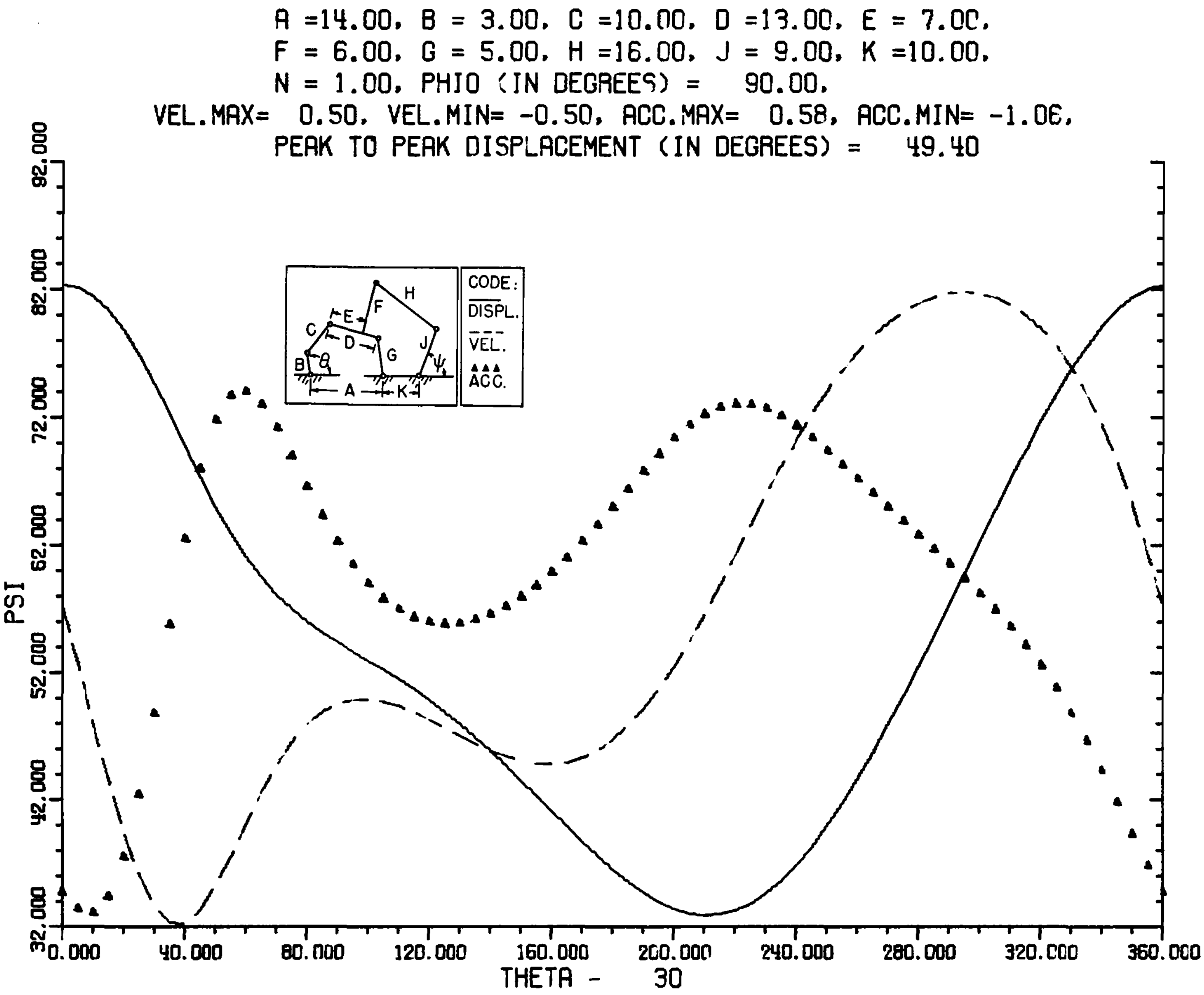
$$\frac{d^2\beta}{d\theta^2} = \left\{ \left[\frac{d\beta}{d\theta}\right]^2 (L \cos\beta + Q \sin\beta) - \frac{d^2L}{d\theta^2}\left[\cos\beta + \frac{L}{D}\right] + \frac{dL}{d\theta}\left[2 \sin\beta \frac{d\beta}{d\theta} - \frac{1}{D}\frac{dL}{d\theta}\right] - \frac{d^2Q}{d\theta^2}\left[\sin\beta + \frac{Q}{D}\right] - \frac{dQ}{d\theta}\left[2 \cos\beta \frac{d\beta}{d\theta} + \frac{1}{D}\frac{dQ}{d\theta}\right]\right\} \Big/ (Q \cos\beta - L \sin\beta) \,. \tag{4-23}$$

From Eq. 4-17 may be determined:

$$\frac{d^2Q}{d\theta^2} = - G \sin\phi \left[\frac{d\phi}{d\theta}\right]^2 + B \sin\theta$$

$$\frac{d^2L}{d\theta^2} = B \cos\theta - G \cos\phi \left[\frac{d\phi}{d\theta}\right]^2 .$$

The expression for $d^2\psi/d\theta^2$, as given by Eq. 4-20 and the several equations following it, has been presented for several mechanisms as the series of small triangles on the graphs for this Mechanism #4. This expression with Eq. 4-4 affords the means for calculating the angular acceleration of the output link J, $d^2\psi/dt^2$, in the usual engineering set of units.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F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 0.50, VEL.MIN= -0.50, ACC.MAX= 0.58, ACC.MIN= -1.06,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 49.40
CODE:
DISPL.
VEL.
ACC.
PSI
THETA - 30

PLATE 4-1

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-1.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 0.47, VEL.MIN= -0.35, ACC.MAX= 0.71, ACC.MIN= -0.94,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 50.34

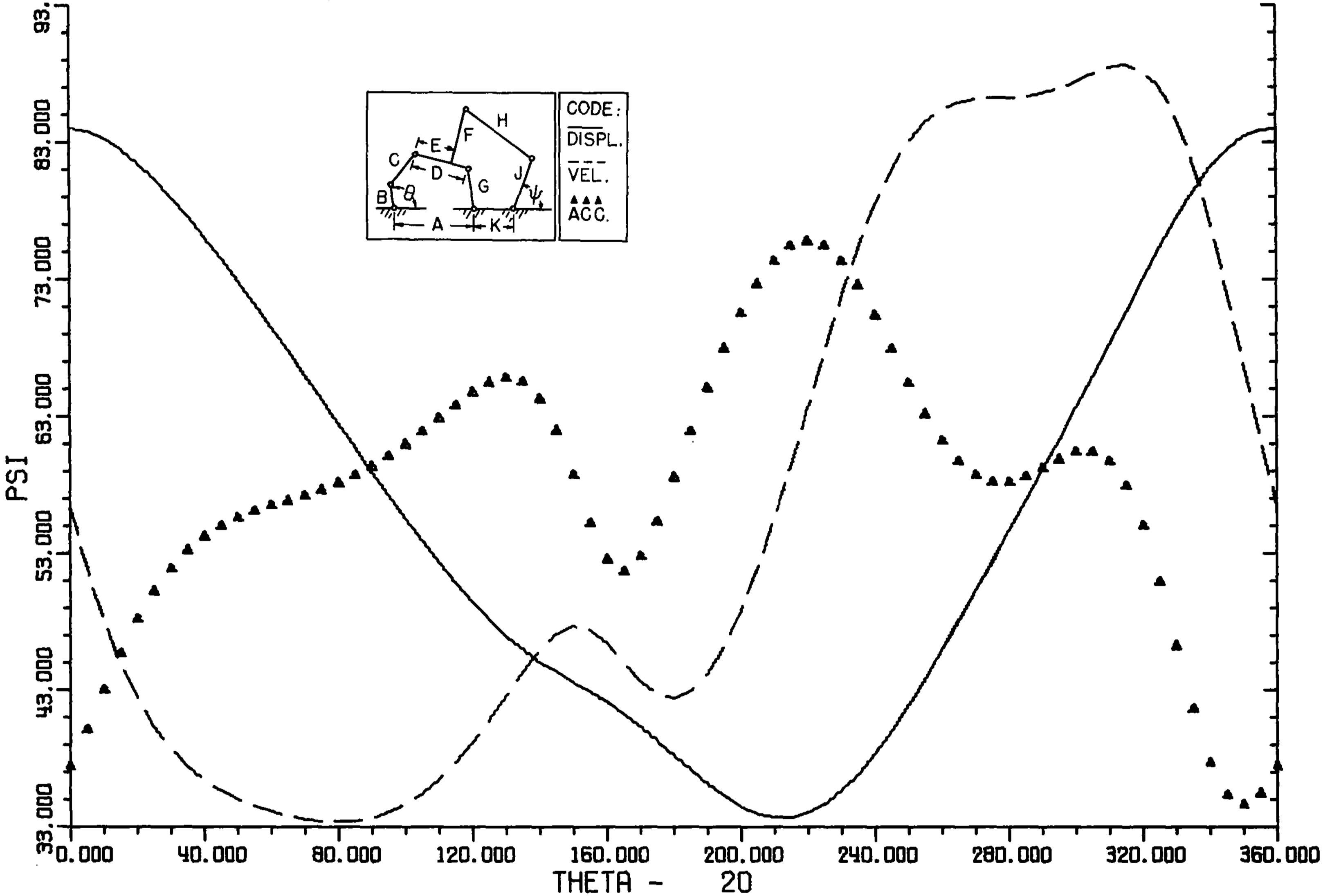


PLATE 4-2

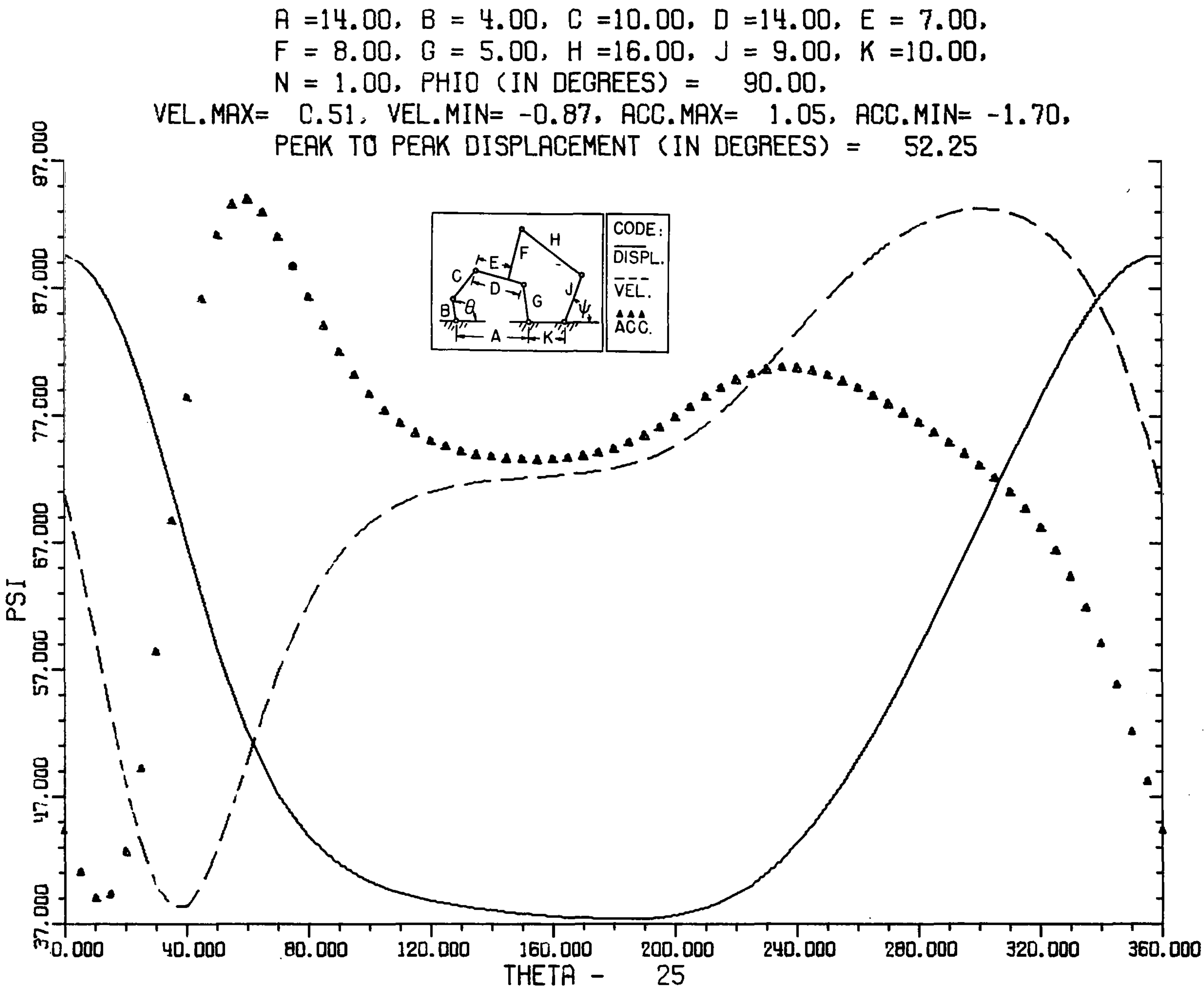

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 8.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 0.51, VEL.MIN= -0.87, ACC.MAX= 1.05, ACC.MIN= -1.70,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 52.25
CODE:
DISPL.
VEL.
ACC.
PSI
THETA - 25
37.000
47.000
57.000
67.000
77.000
87.000
97.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 4-3

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00, K = 8.00,
N = 1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 0.33, VEL.MIN= -1.34, ACC.MAX= 2.11, ACC.MIN= -3.30,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 56.49

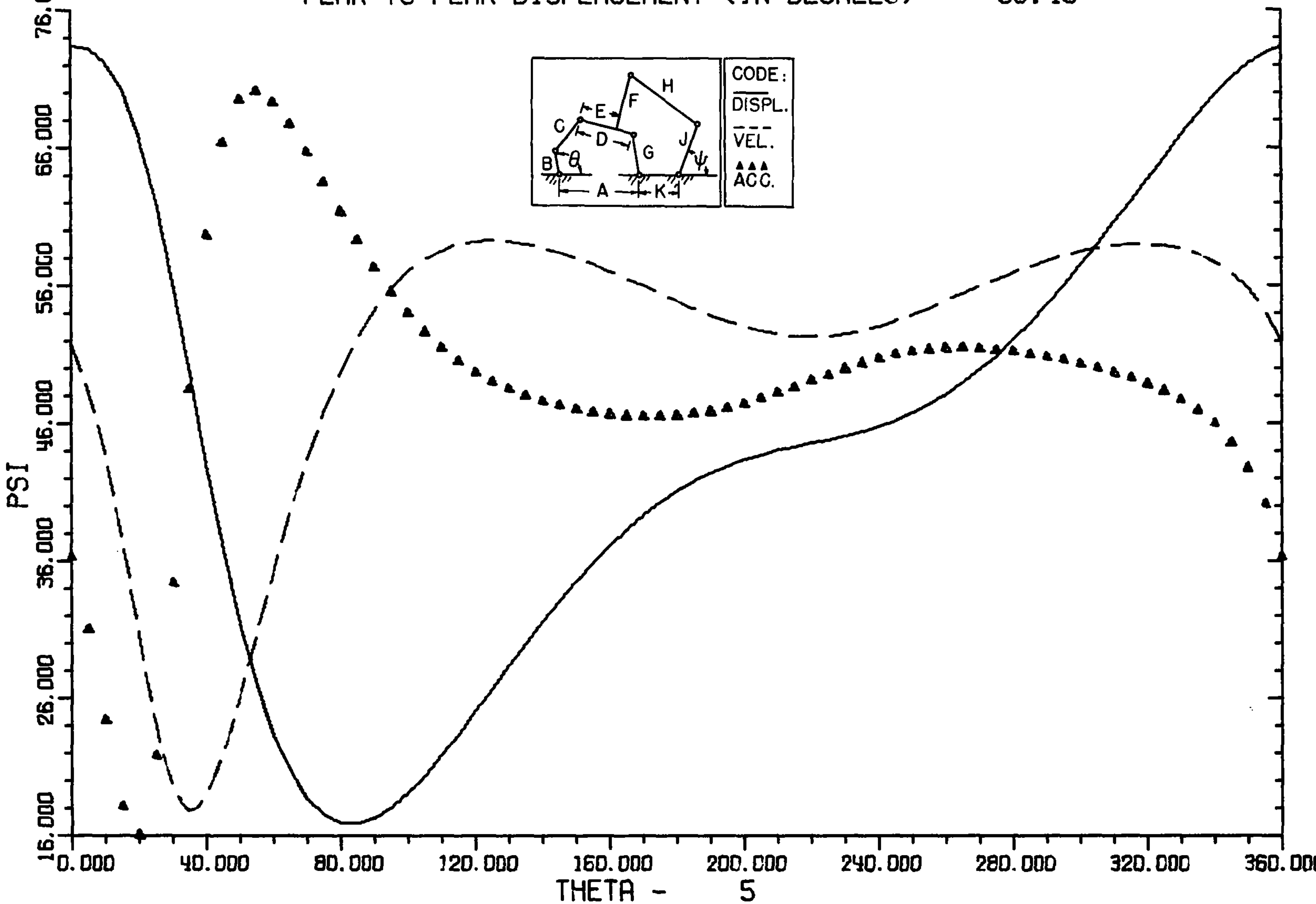


PLATE 4-4

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00, K = 8.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.48, VEL.MIN= -0.58, ACC.MAX= 0.65, ACC.MIN= -0.42,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 58.56

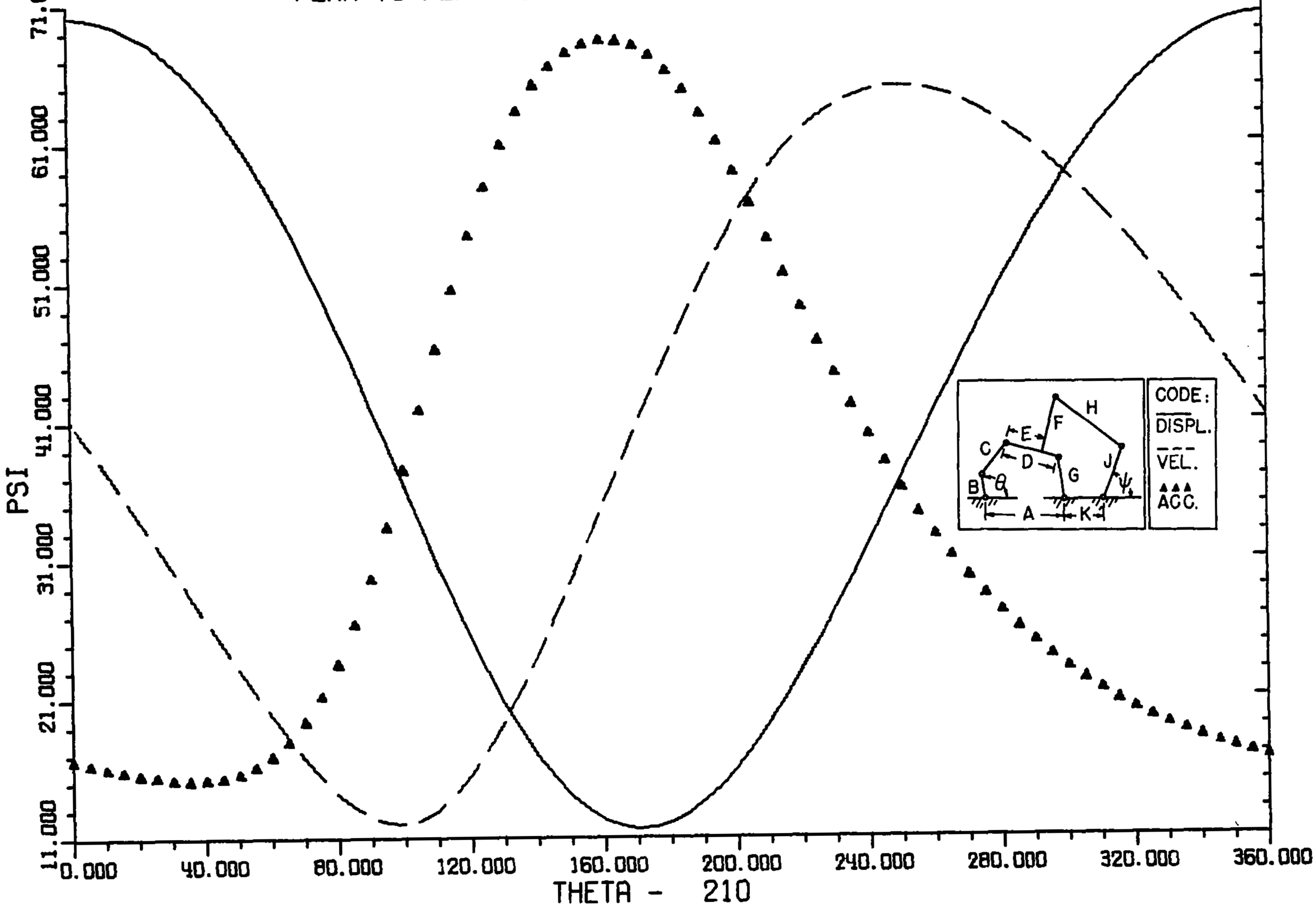


PLATE 4-5

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 8.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-1.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 0.85, VEL.MIN= -0.50, ACC.MAX= 0.75, ACC.MIN= -2.54,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 62.09

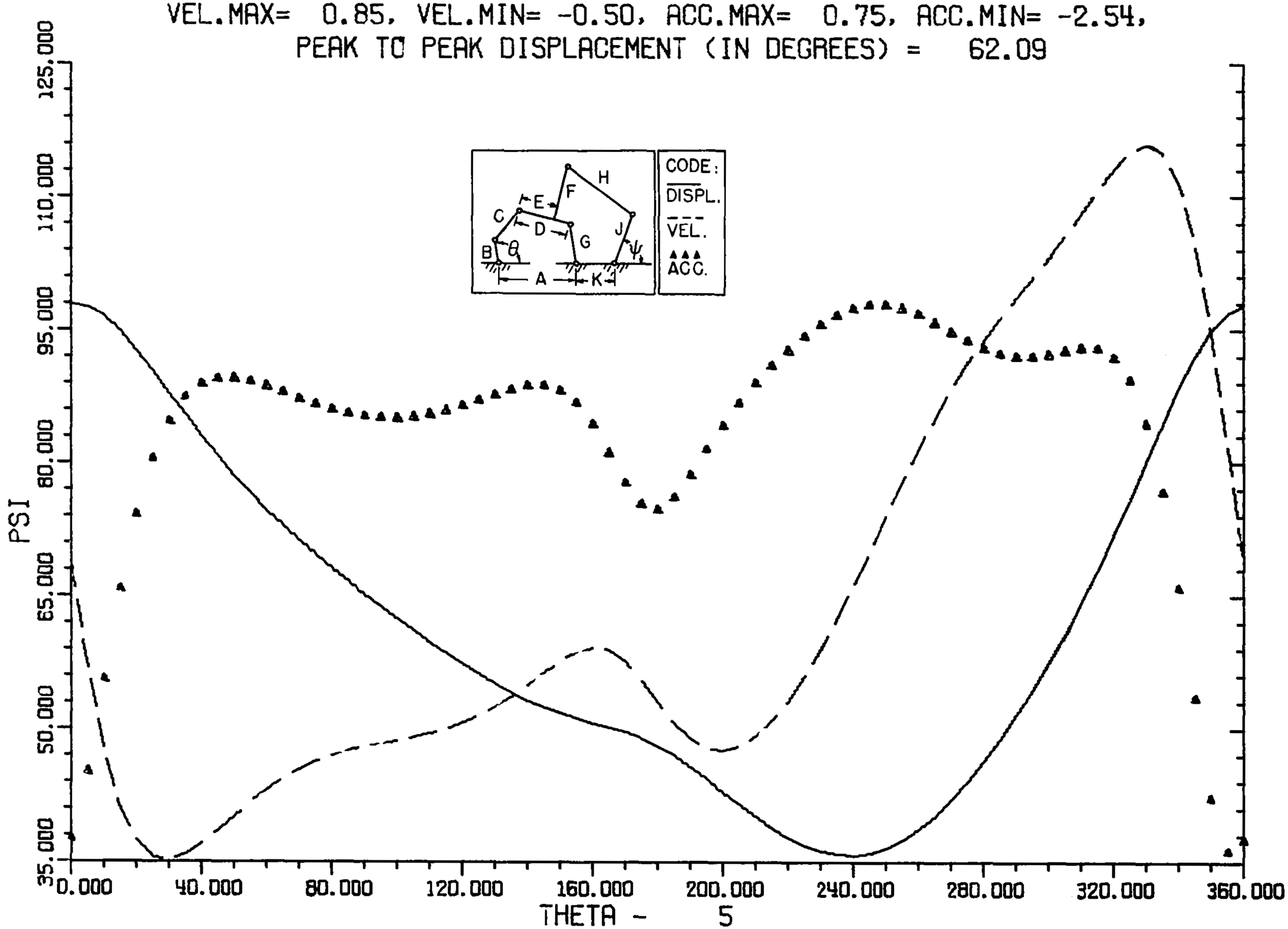


PLATE 4-6

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =12.00,
N =-1.00, PHIO (IN DEGREES) = 270.00,
VEL.MAX= 0.55, VEL.MIN= -0.51, ACC.MAX= 1.21, ACC.MIN= -0.73,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 68.03

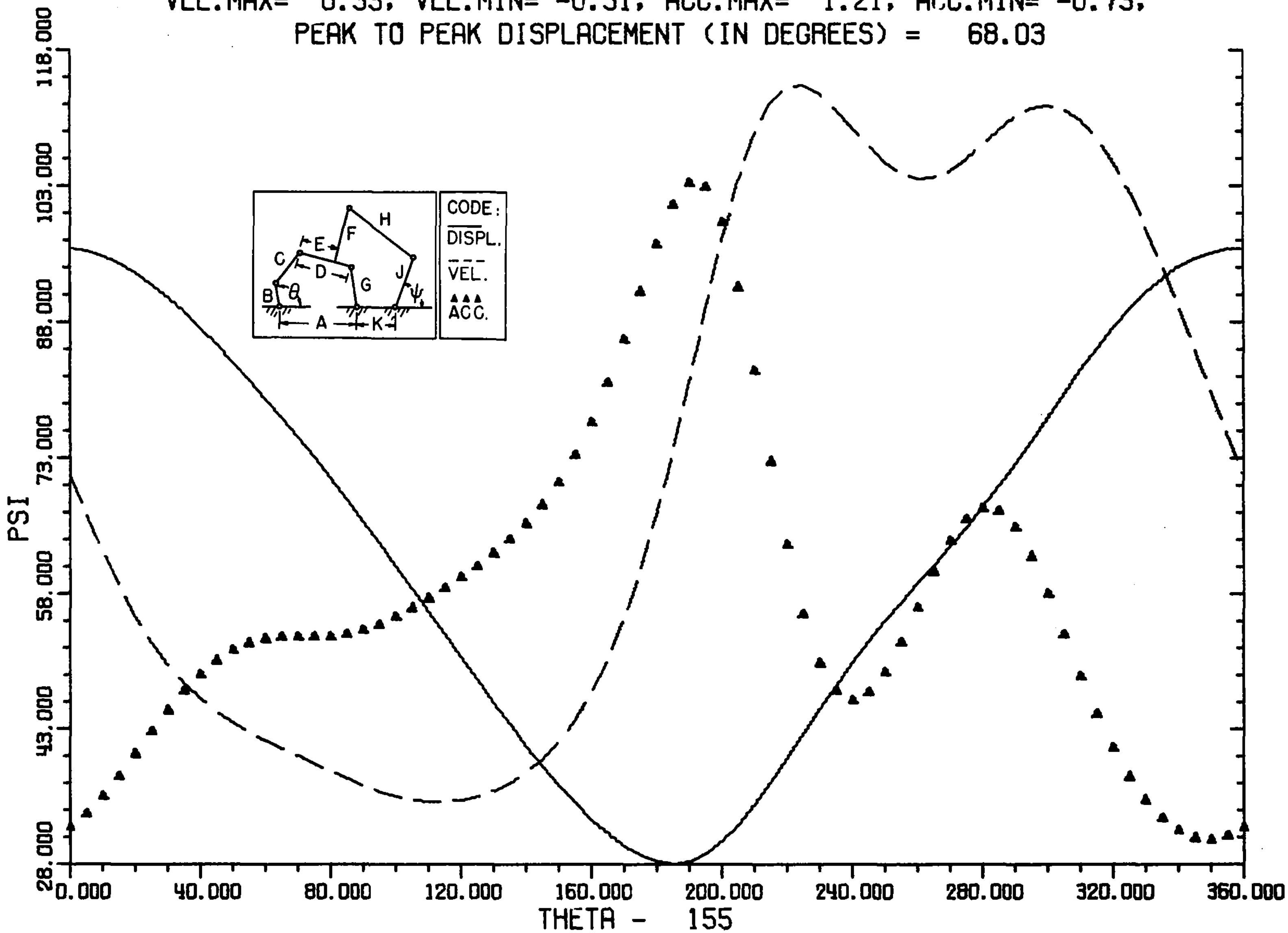


PLATE 4-7

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 1.00, PHIO (IN DEGREES) = 0.CO,
VEL.MAX= 0.59, VEL.MIN= -0.78, ACC.MAX= 1.42, ACC.MIN= -0.50,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 74.89

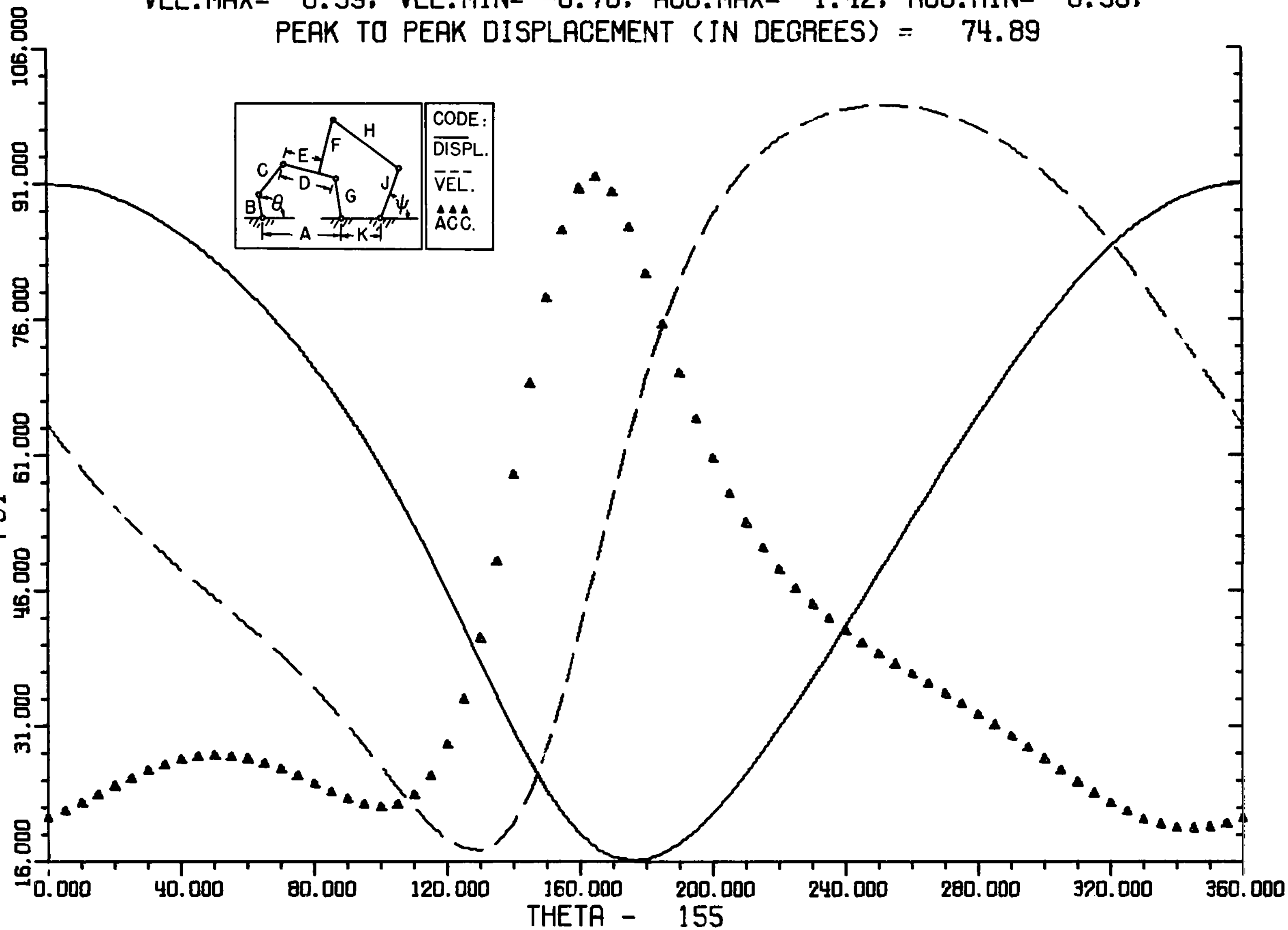


PLATE 4-8

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.62, VEL.MIN= -0.97, ACC.MAX= 2.13, ACC.MIN= -0.67,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 81.35

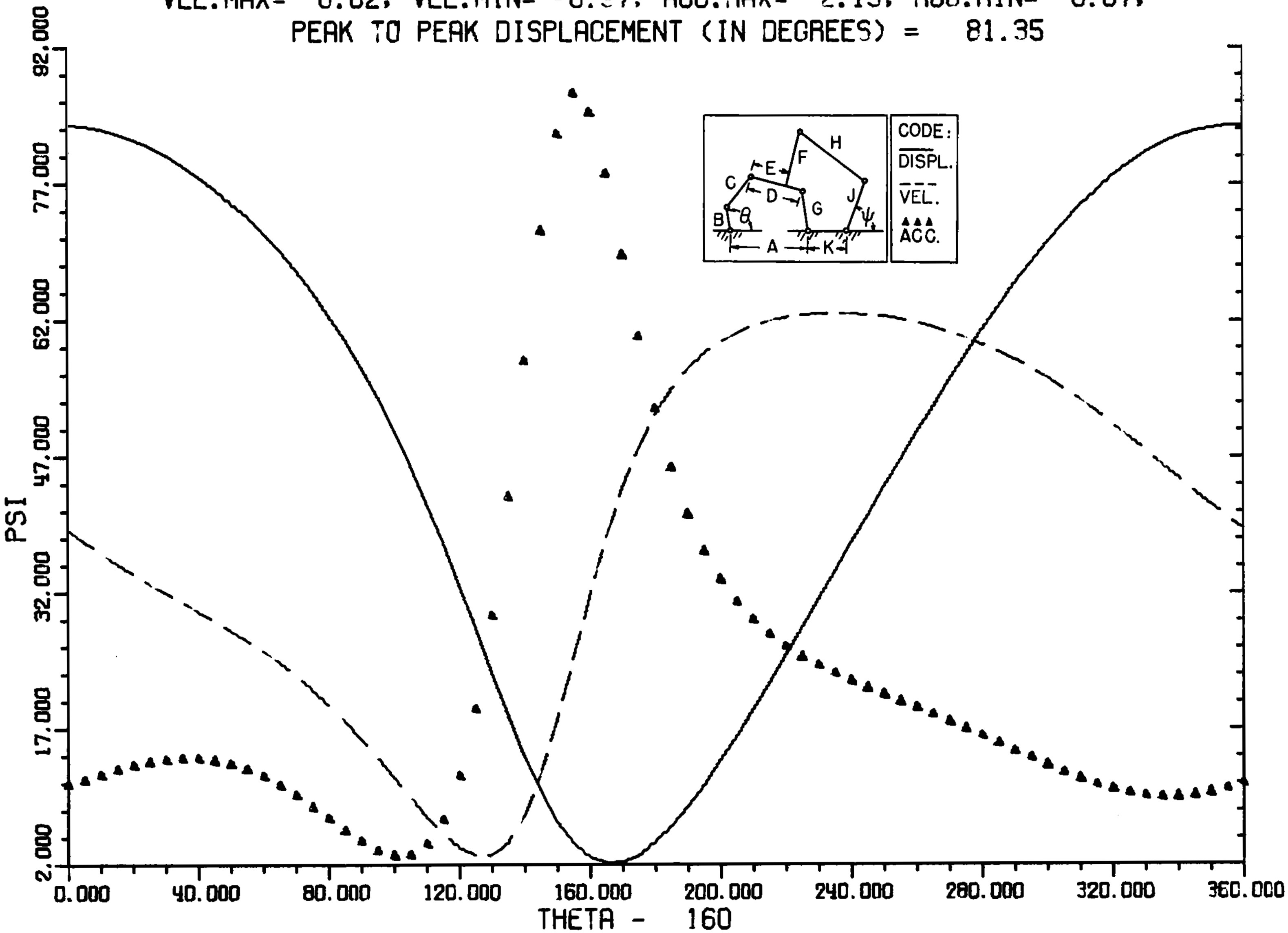


PLATE 4-9

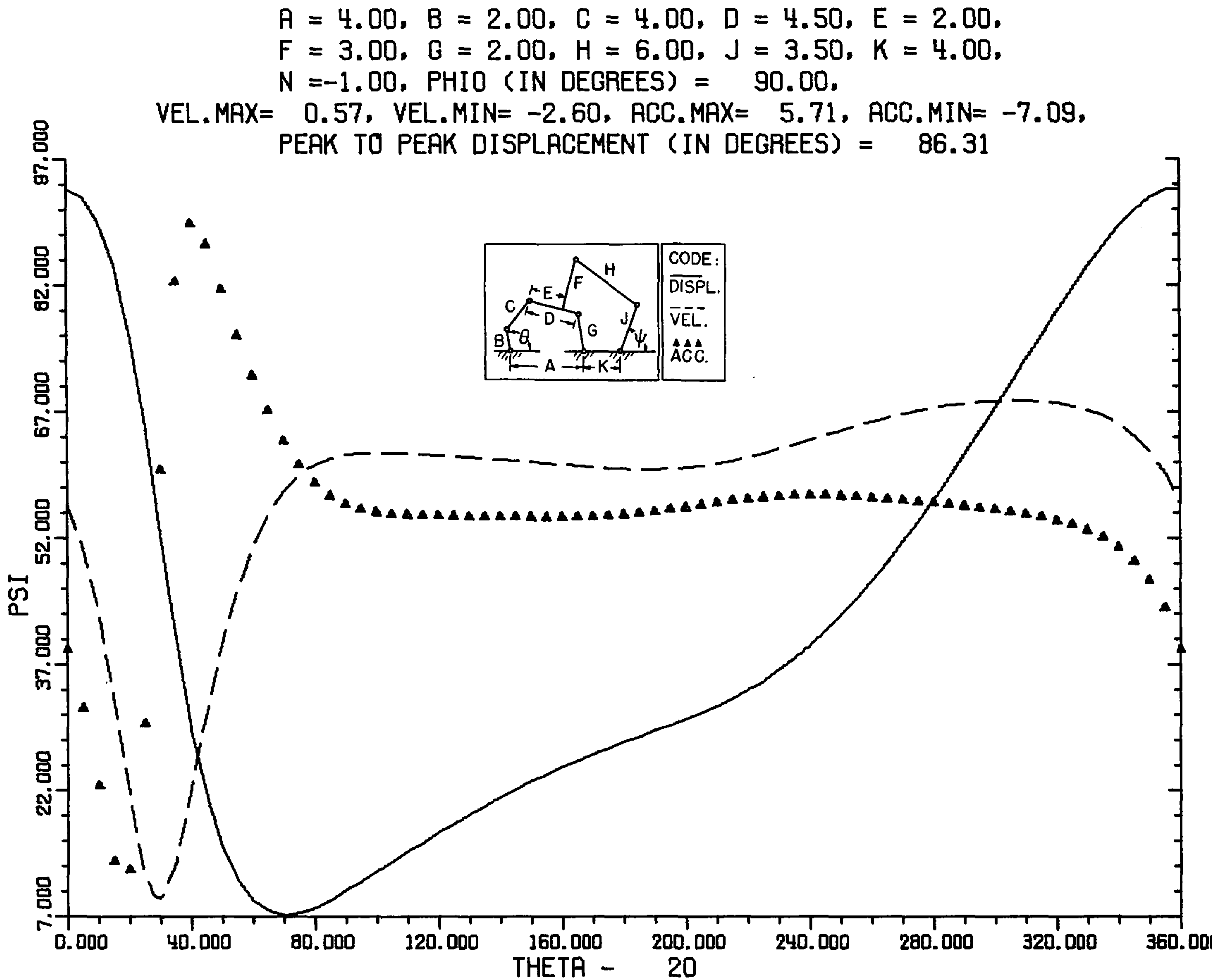

A = 4.00, B = 2.00, C = 4.00, D = 4.50, E = 2.00,
F = 3.00, G = 2.00, H = 6.00, J = 3.50, K = 4.00,
N =-1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 0.57, VEL.MIN= -2.60, ACC.MAX= 5.71, ACC.MIN= -7.09,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 86.31
PSI
97.000
82.000
67.000
52.000
37.000
22.000
7.000
THETA - 20
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
K
θ
ψ

PLATE 4-10

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-1.00, PHIO (IN DEGREES) = 270.00,
VEL.MAX= 0.95, VEL.MIN= -0.62, ACC.MAX= 3.52, ACC.MIN= -0.78,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 92.71

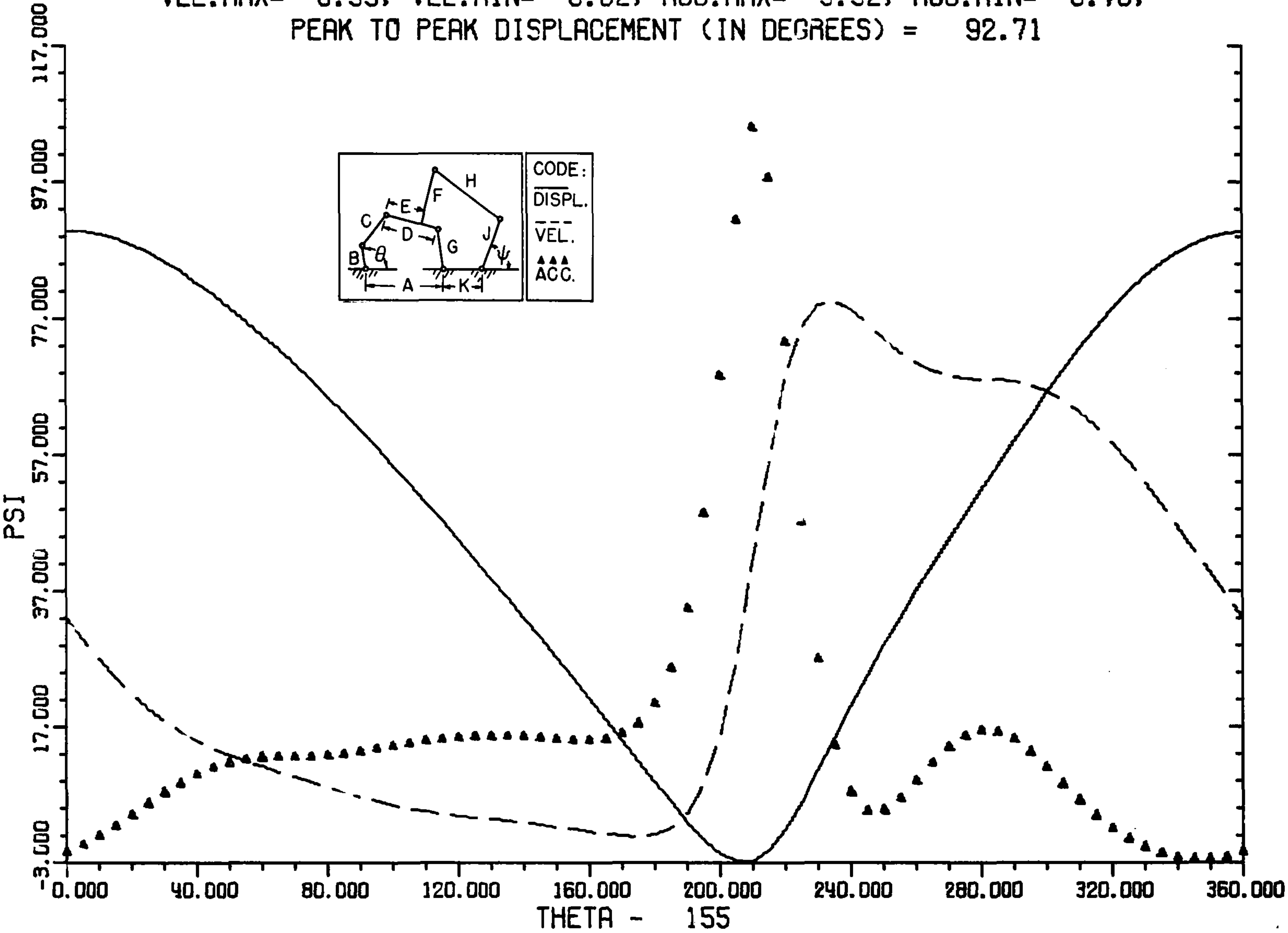


PLATE 4-11

A =12.00, B =10.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K = 8.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.76, VEL.MIN= -1.03, ACC.MAX= 1.17, ACC.MIN= -0.74,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 93.61

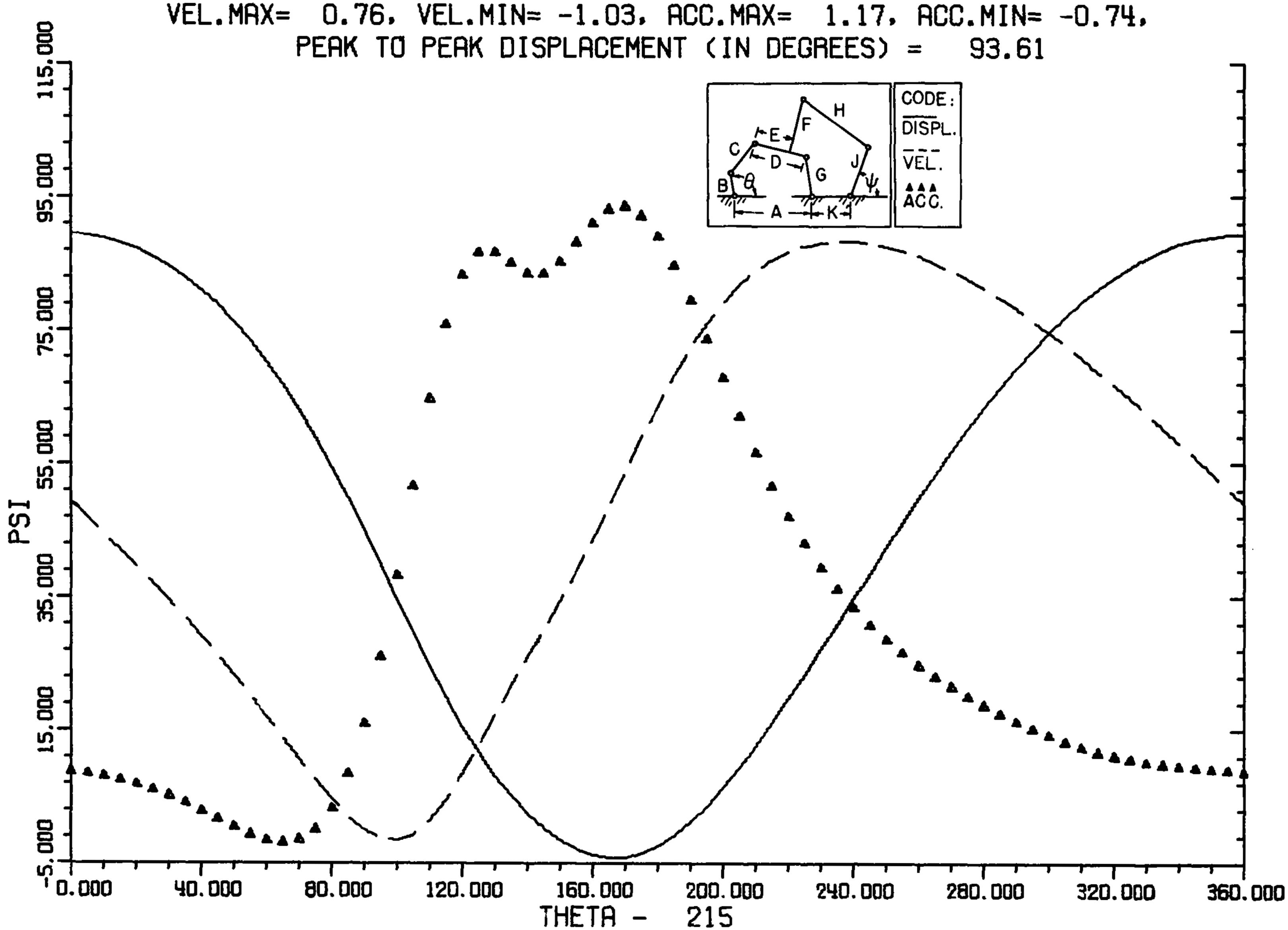


PLATE 4-12

A =16.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 7.00, H =16.00, J = 9.00, K =12.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.78, VEL.MIN= -1.12, ACC.MAX= 2.28, ACC.MIN= -0.86,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 99.48

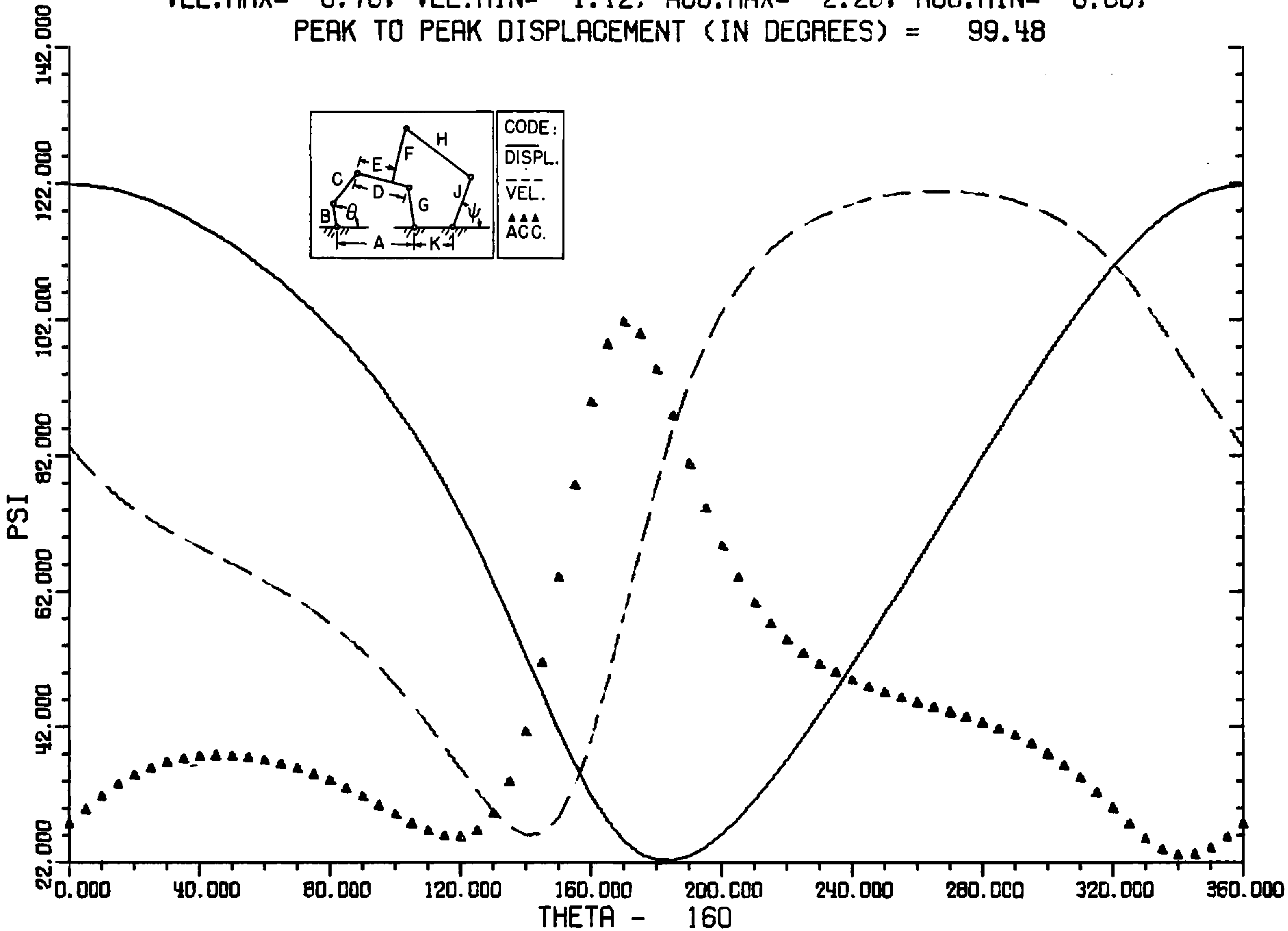


PLATE 4-13

A =12.00, B = 7.00, C =10.00, D =15.00, E = 7.00,
F = 8.00, G = 7.00, H =16.00, J = 9.00, K =12.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.79, VEL.MIN= -1.12, ACC.MAX= 2.18, ACC.MIN= -0.95,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 102.01

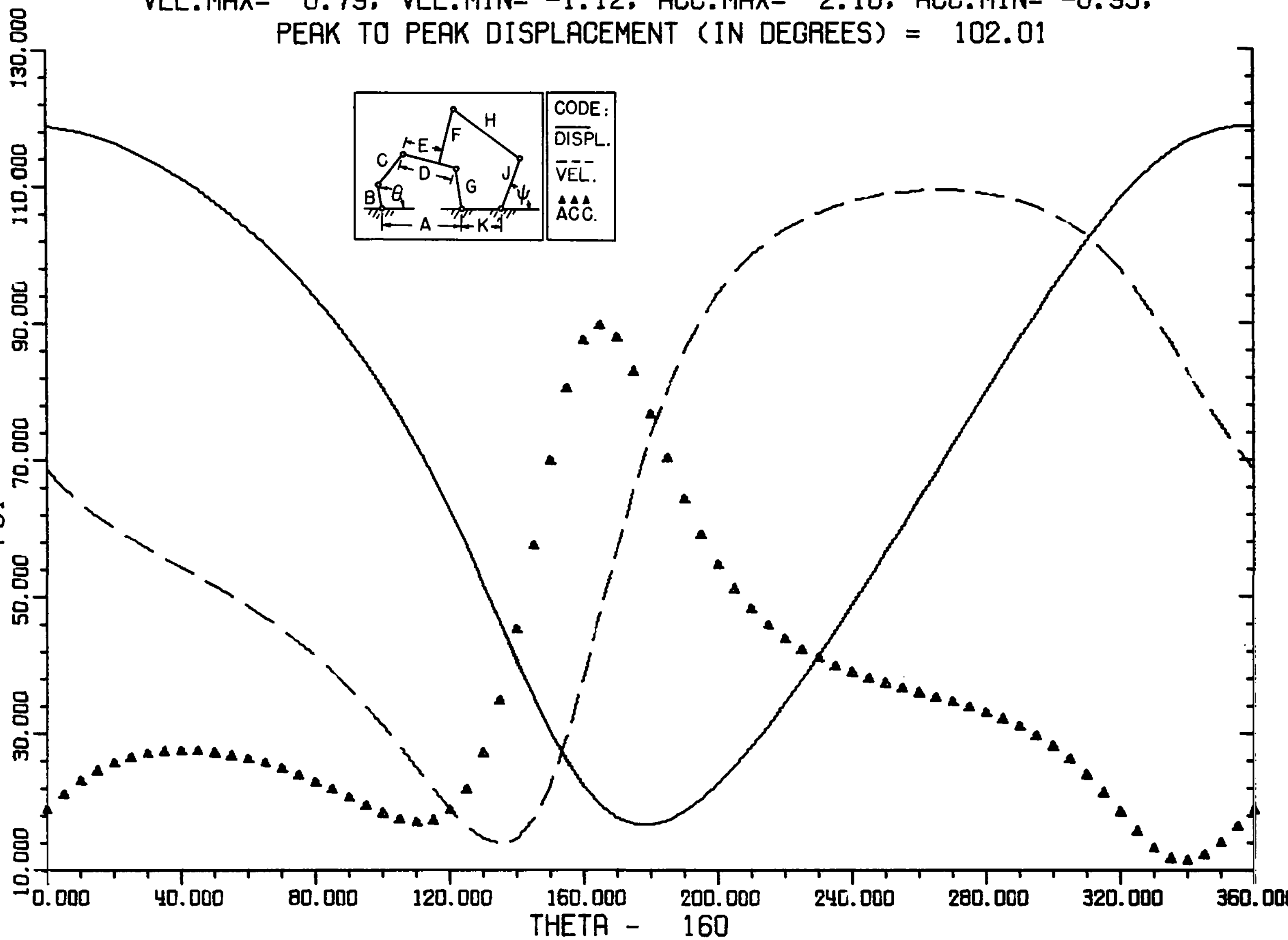


PLATE 4-14

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 1.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 0.62, VEL.MIN= -1.35, ACC.MAX= 3.28, ACC.MIN= -2.44,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 105.44

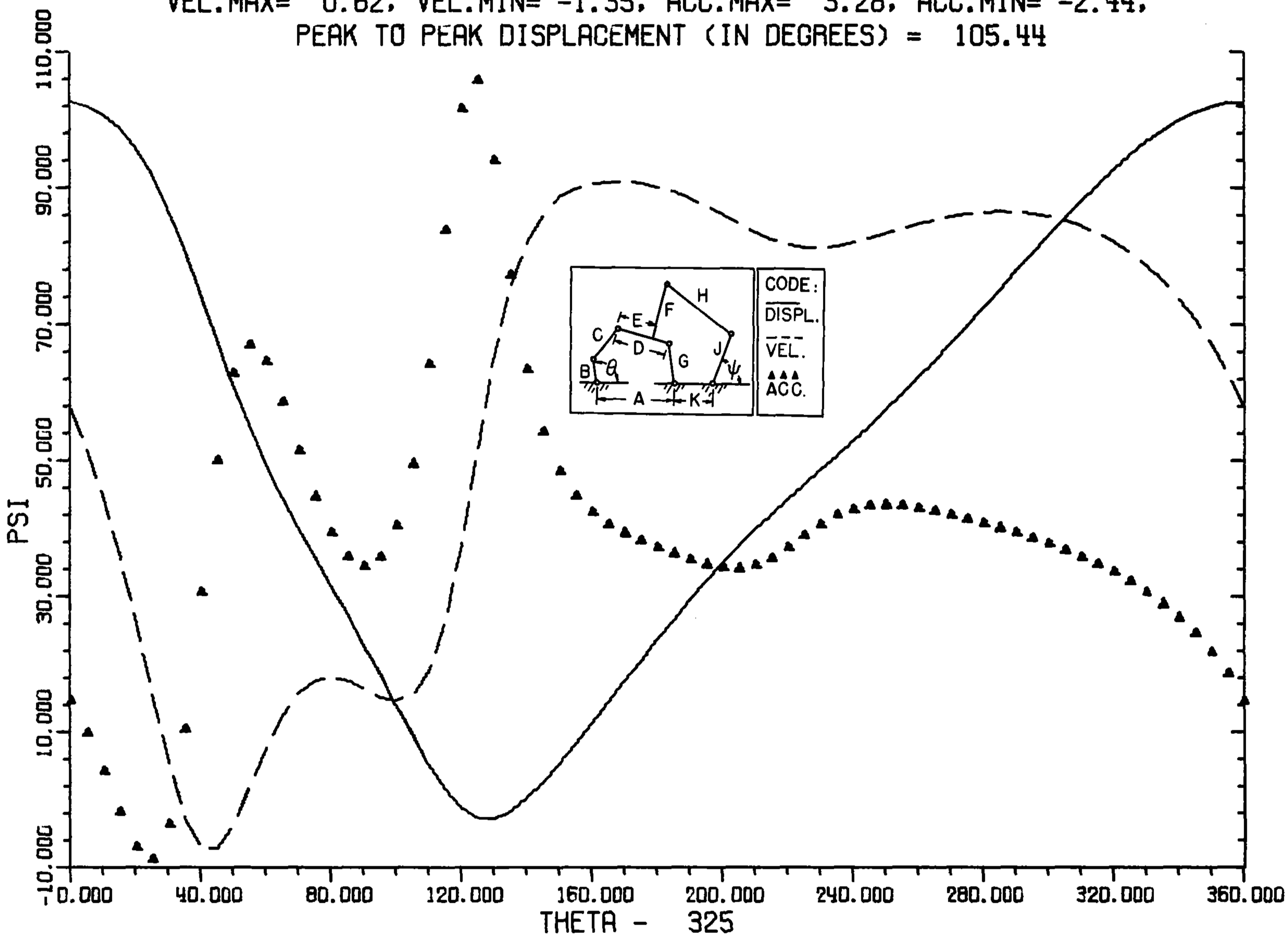


PLATE 4-15

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 8.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-1.00, PHIO (IN DEGREES) = 90.00,
VEL.MAX= 0.89, VEL.MIN= -1.00, ACC.MAX= 0.95, ACC.MIN= -1.74,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 109.30

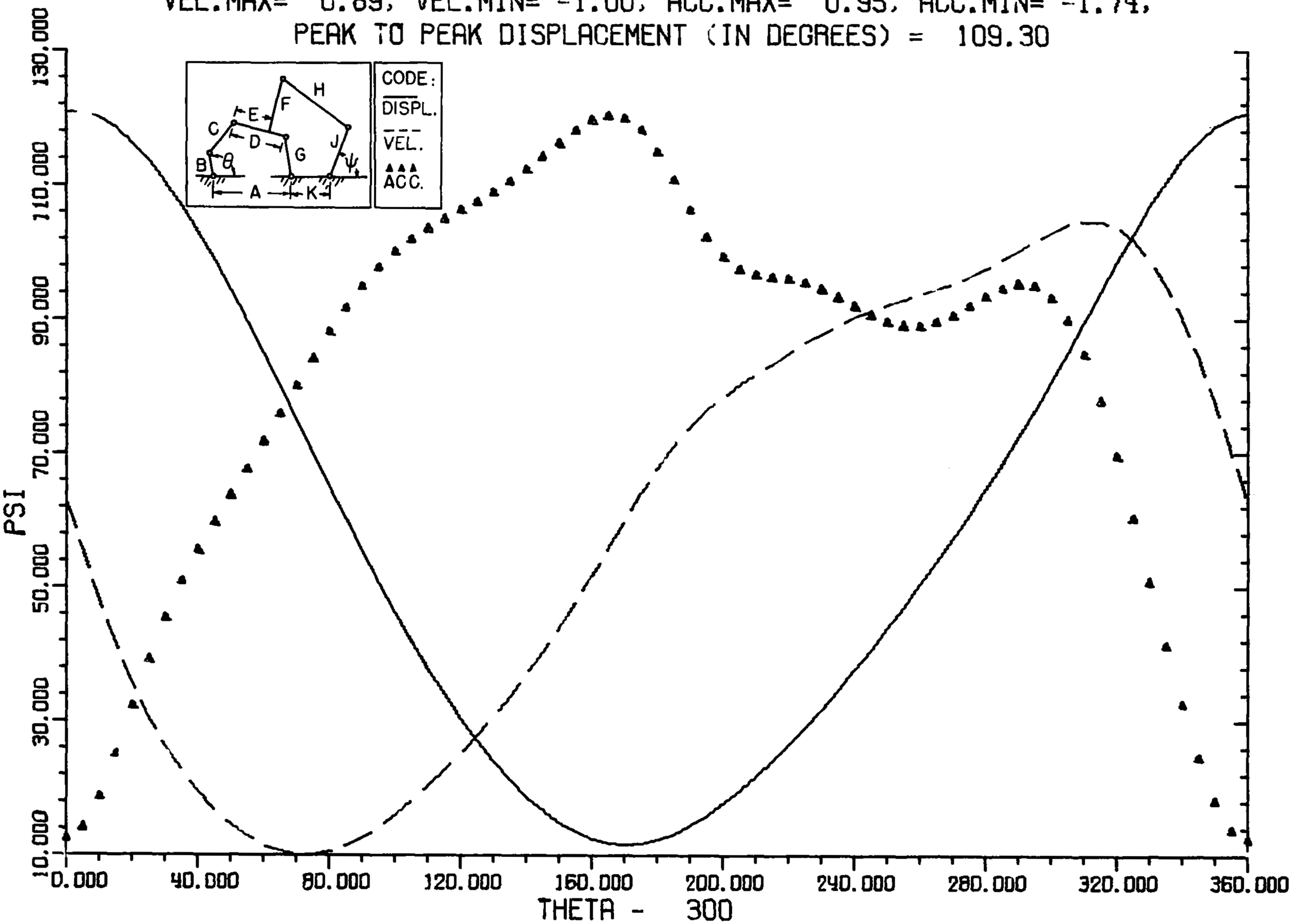


PLATE 4-16

A =12.00, B = 7.00, C =10.00, D =12.00, E = 5.00,
F = 8.00, G = 7.00, H =16.00, J = 9.00, K =12.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.80, VEL.MIN= -1.66, ACC.MAX= 4.66, ACC.MIN= -1.44,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 113.50

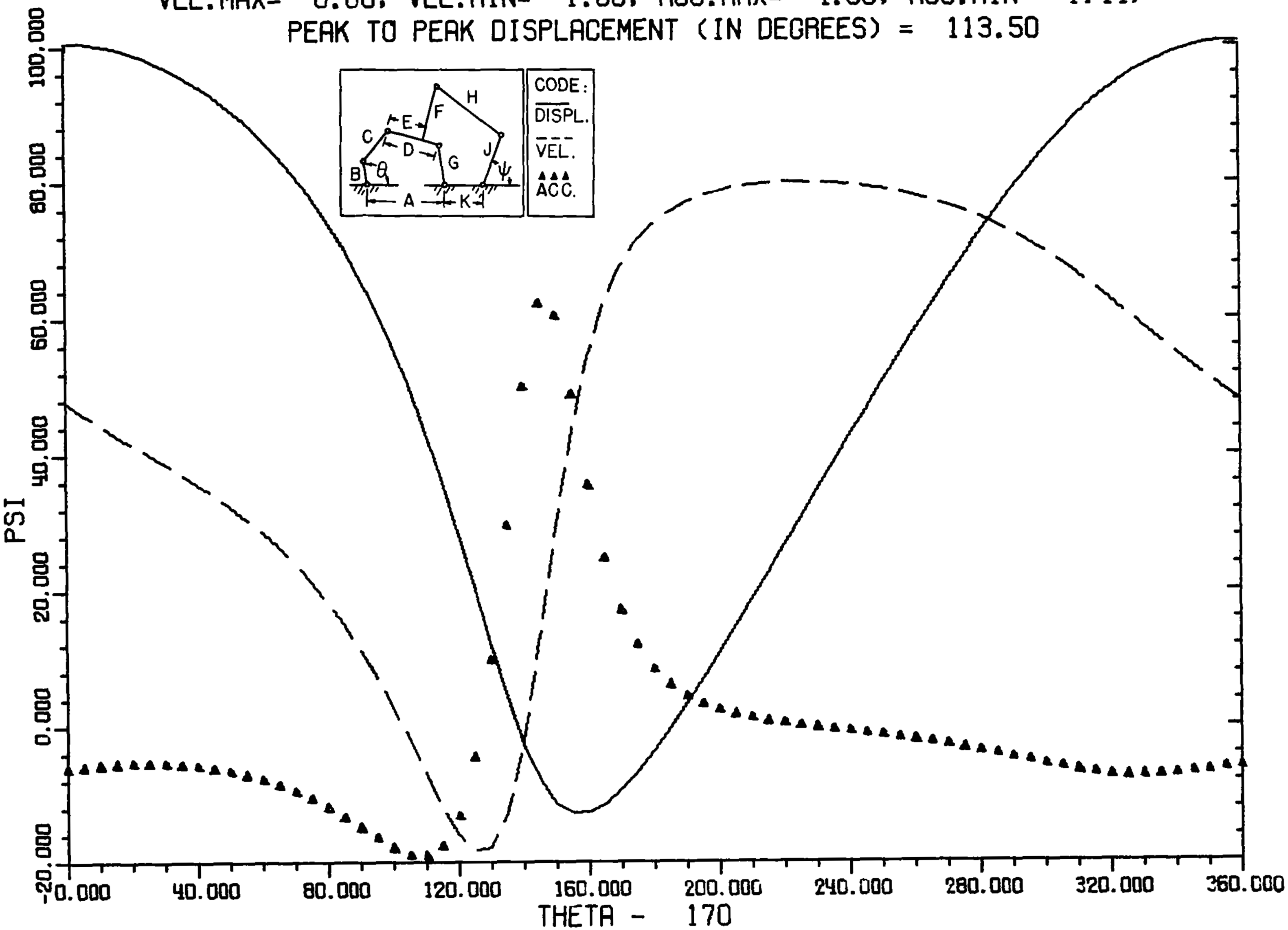


PLATE 4-17

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-1.00, PHIC (IN DEGREES) = 0.00,
VEL.MAX= 1.40, VEL.MIN= -0.92, ACC.MAX= 5.88, ACC.MIN= -1.10,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 127.99

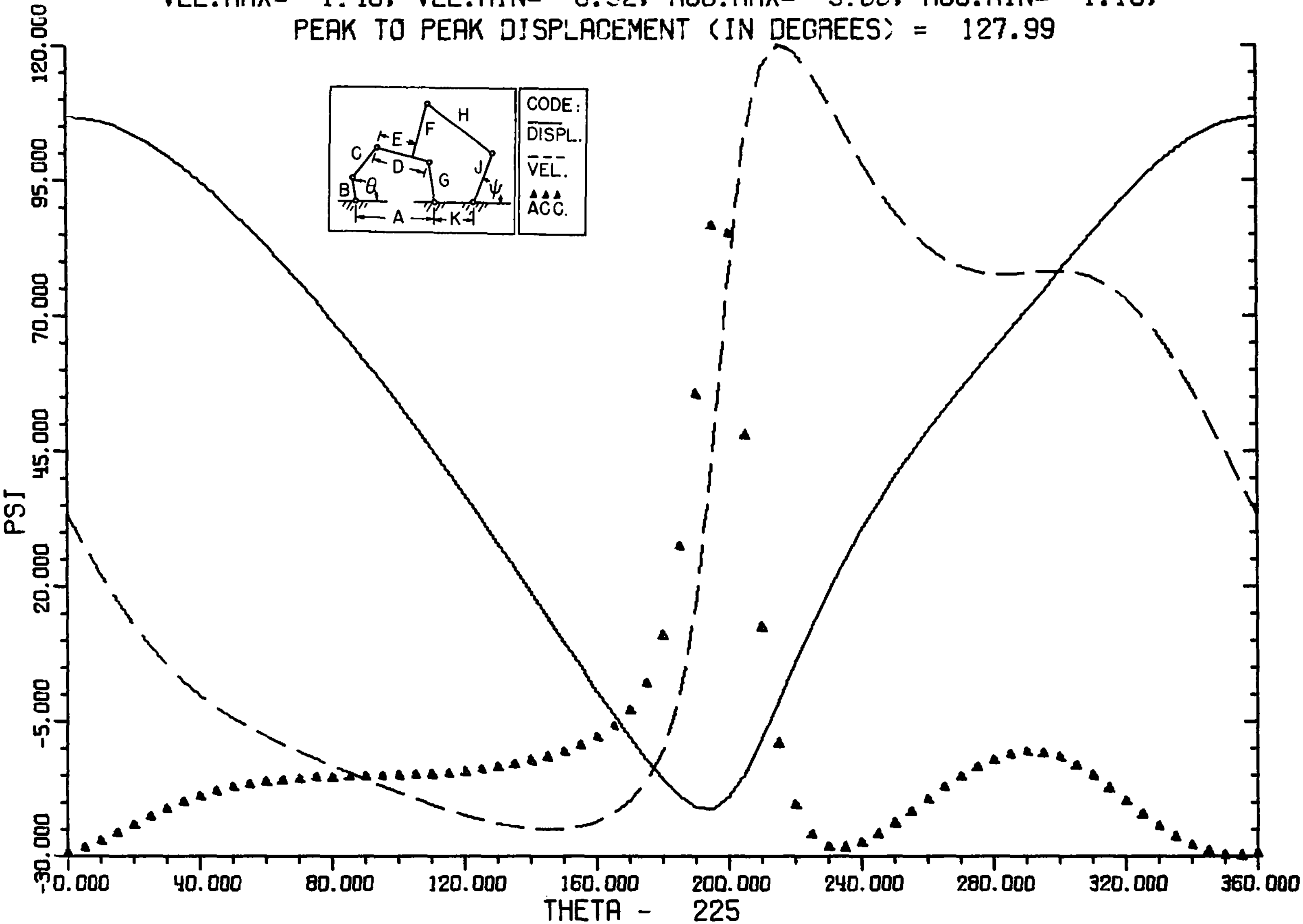


PLATE 4-18

A =12.00, B = 9.00, C = 8.00, D =12.00, E = 5.00,
F = 8.00, G = 7.00, H =16.00, J = 9.00, K = 8.00,
N = 1.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.75, VEL.MIN= -2.43, ACC.MAX= 12.56, ACC.MIN= -2.56,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 129.00

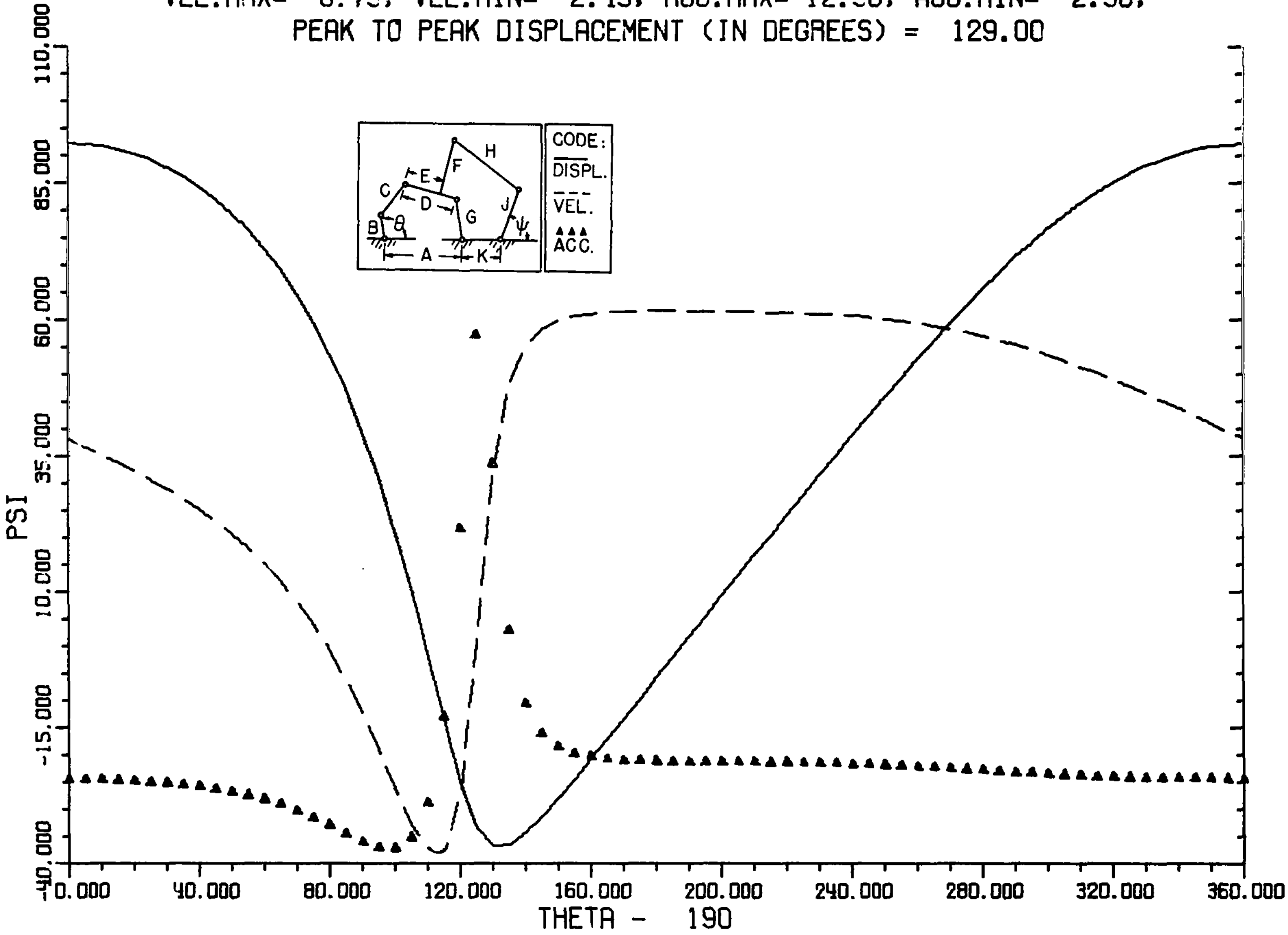


PLATE 4-19

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00, K =12.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.79, VEL.MIN= -1.35, ACC.MAX= 2.68, ACC.MIN= -1.98,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 74.92

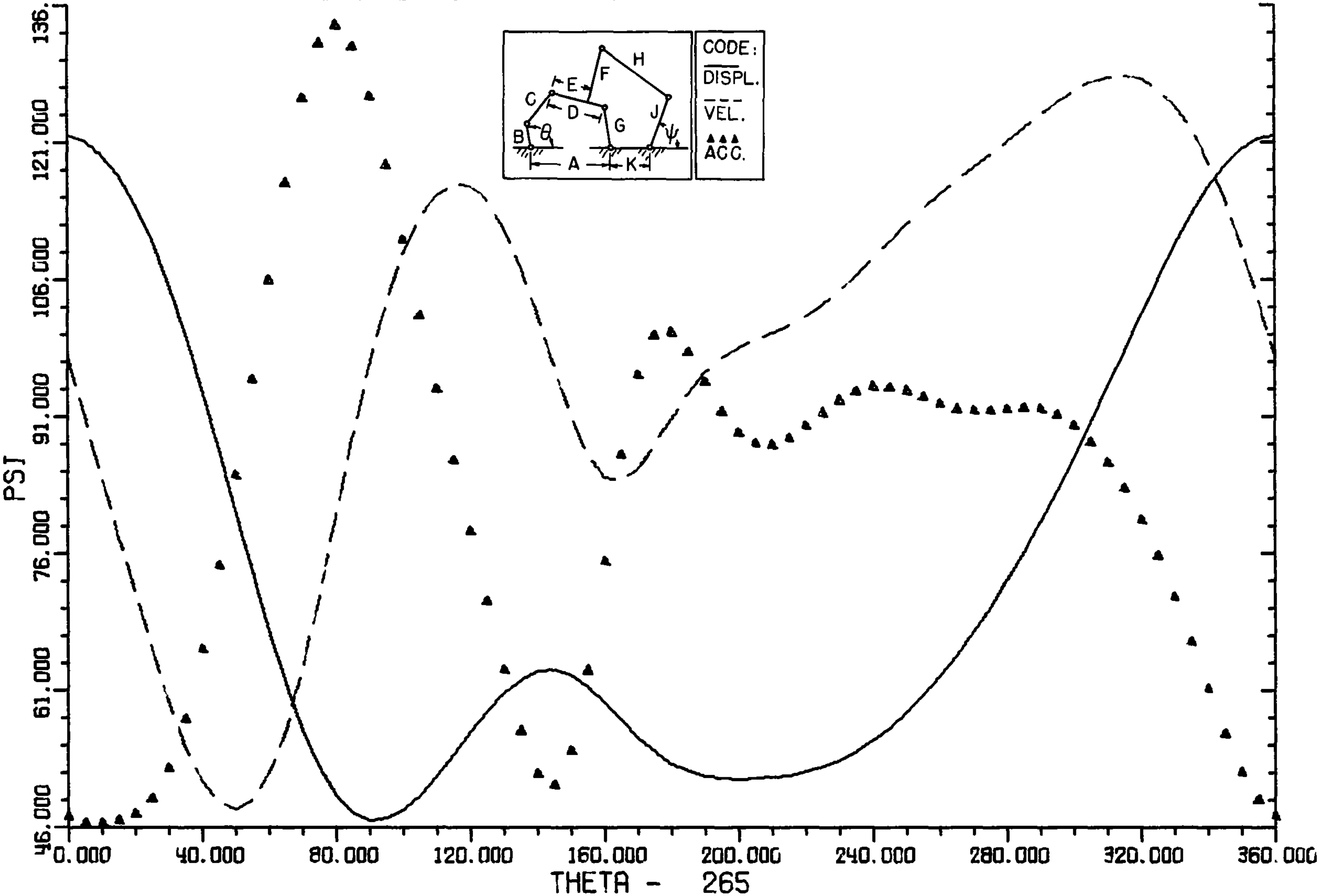


PLATE 4-20

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00, K =10.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.78, VEL.MIN= -1.49, ACC.MAX= 3.33, ACC.MIN= -2.11,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 77.42

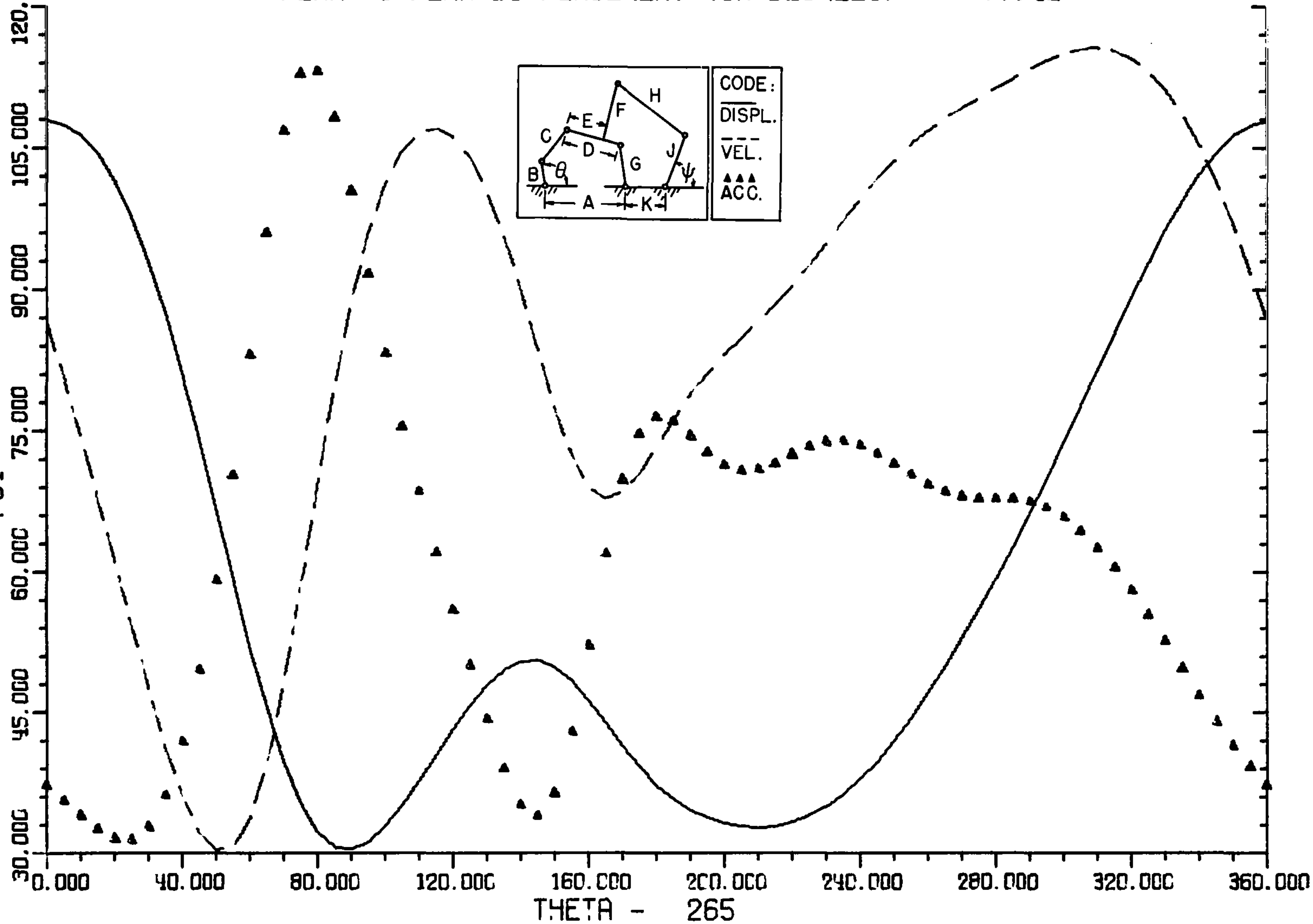


PLATE 4-21

A =12.00, B = 6.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 3.00, H =16.00, J = 9.00, K = 8.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 0.82, VEL.MIN= -1.74, ACC.MAX= 4.56, ACC.MIN= -2.44,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 86.12

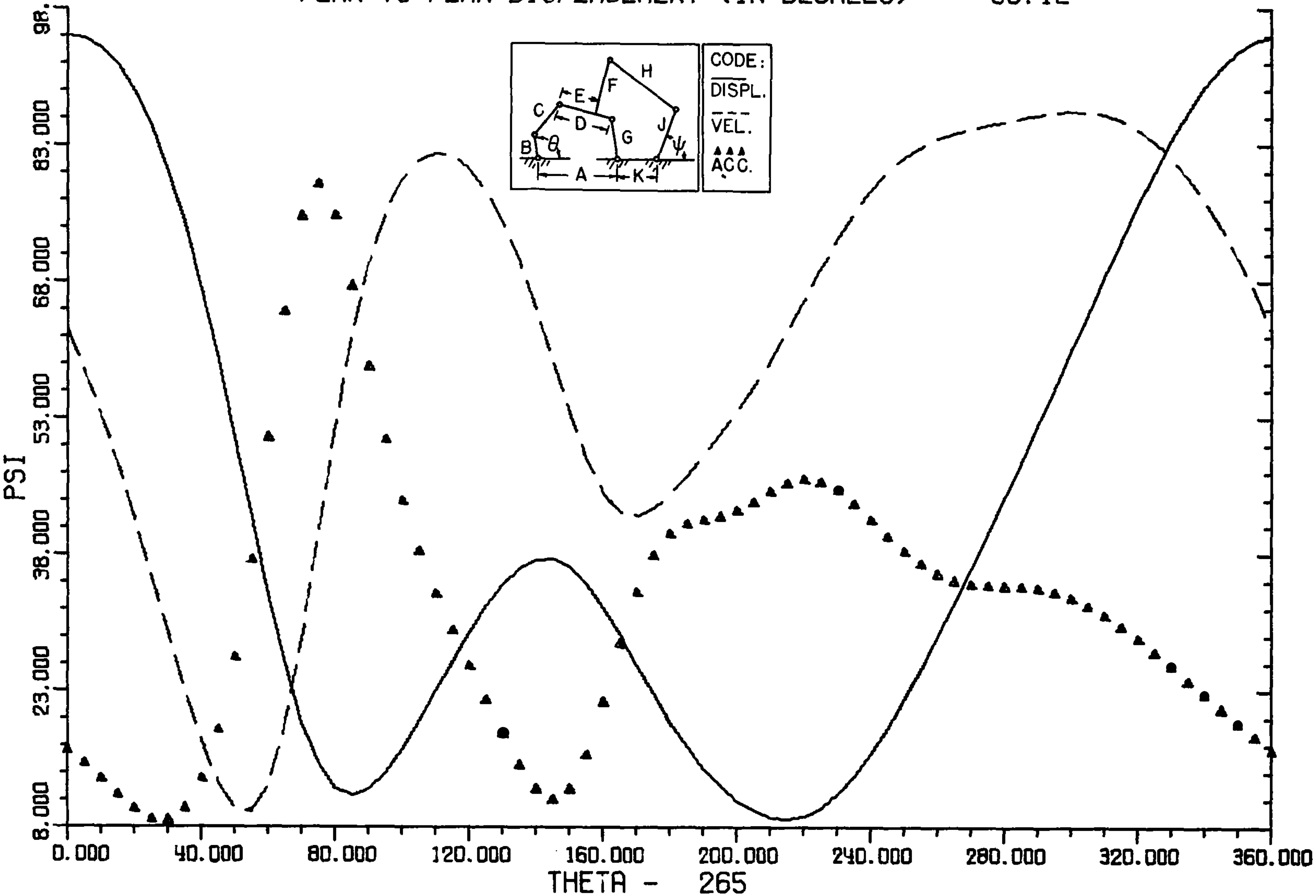


PLATE 4-22

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-2.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 1.54, VEL.MIN= -1.24, ACC.MAX= 5.85, ACC.MIN= -4.56,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 91.78

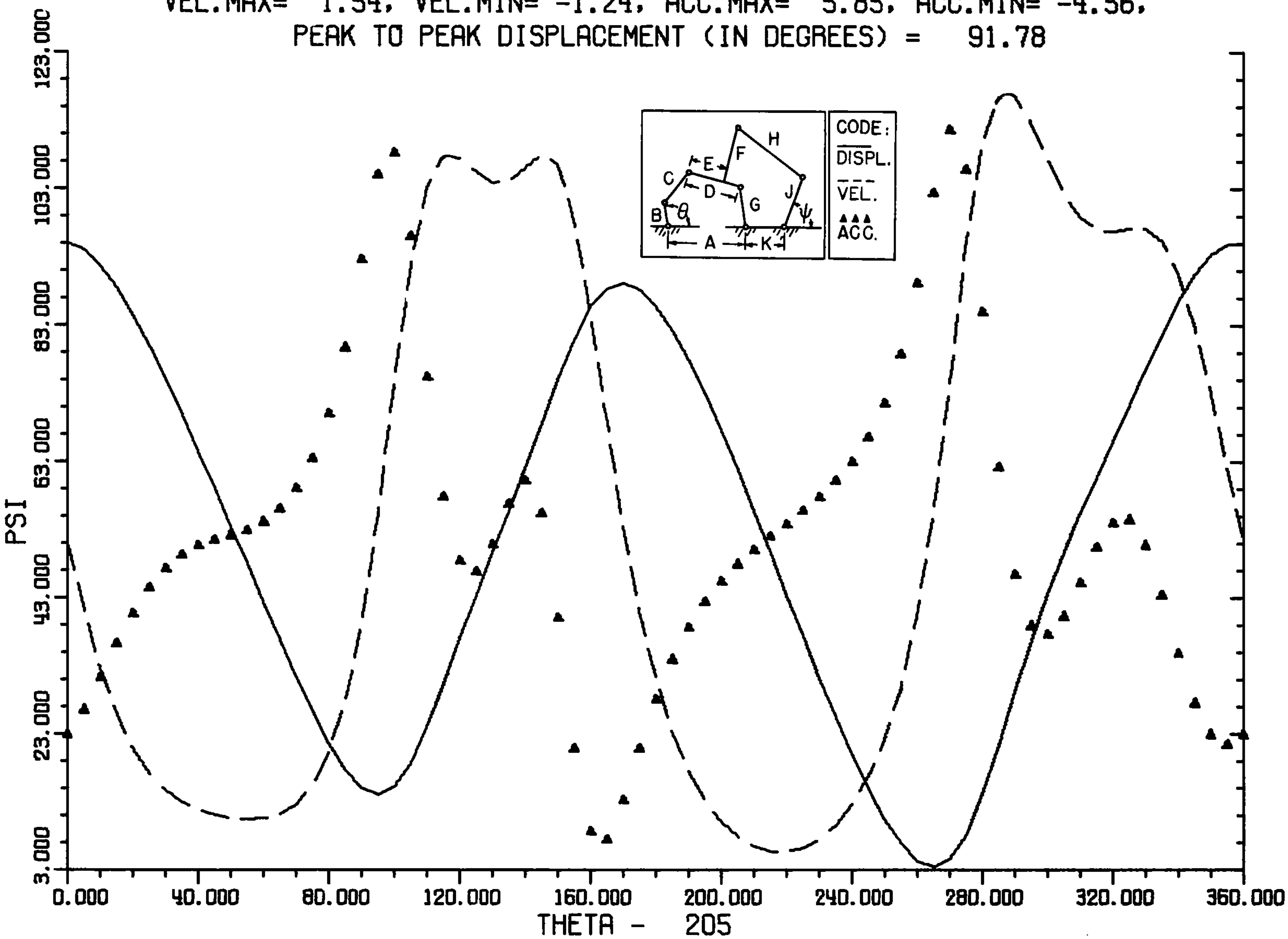


PLATE 4-23

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N =-2.00, PHIO (IN DEGREES) = 180.00,
VEL.MAX= 1.89, VEL.MIN= -1.31, ACC.MAX= 4.25, ACC.MIN= -8.22,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 95.73

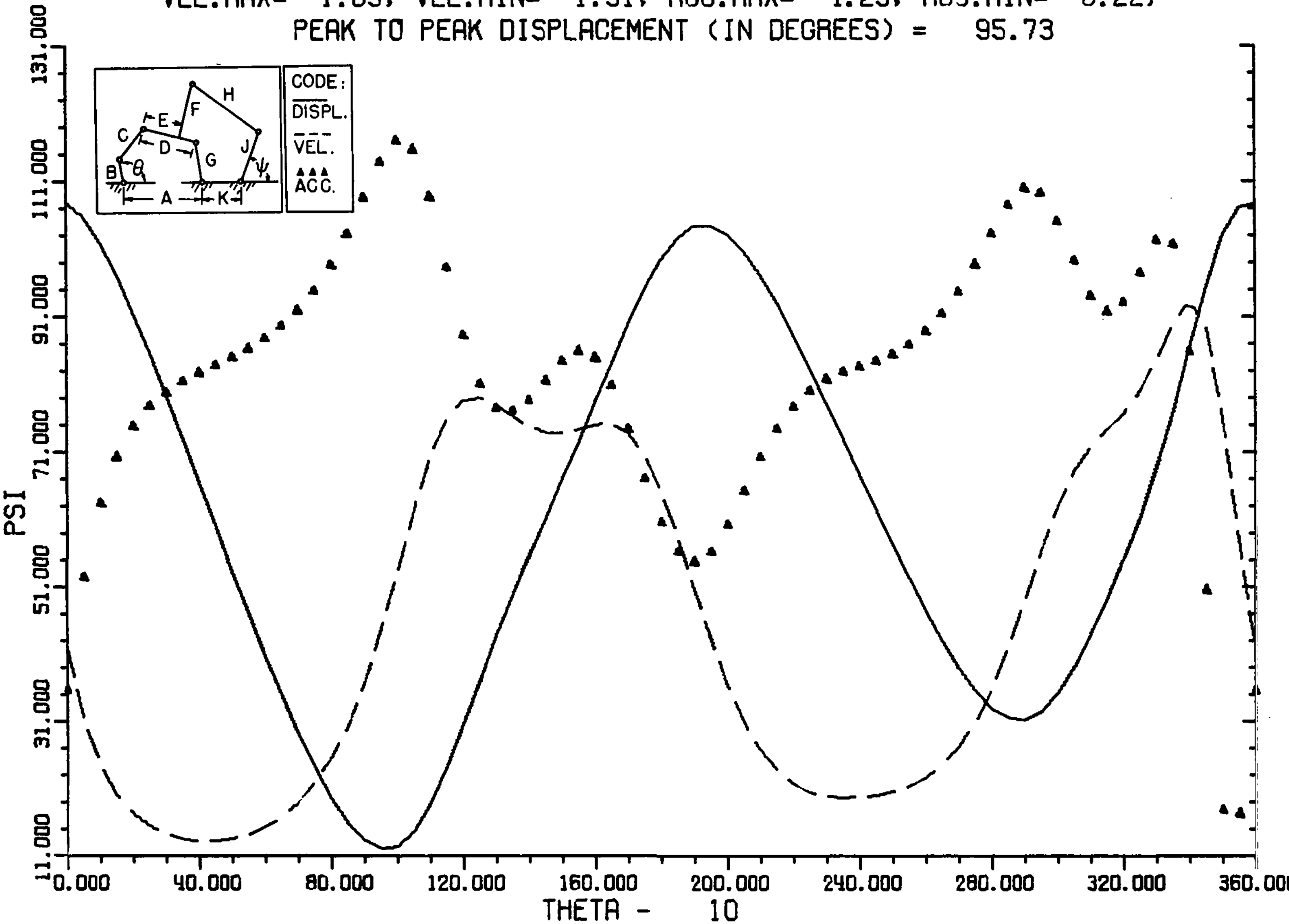


PLATE 4-24

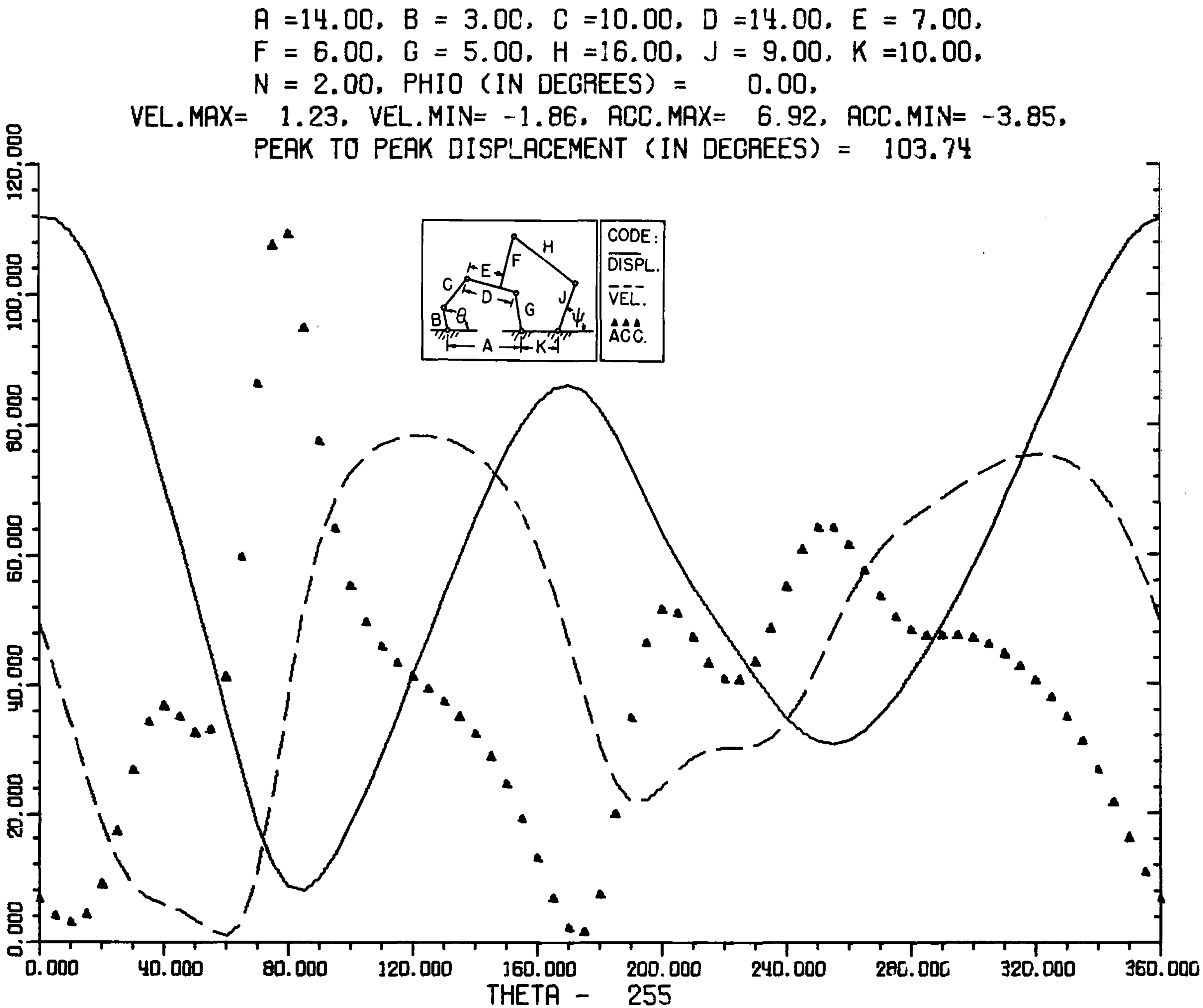

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 1.23, VEL.MIN= -1.86, ACC.MAX= 6.92, ACC.MIN= -3.85,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 103.74
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
H
J
K
PSI
THETA - 255
0.000
20.000
40.000
60.000
80.000
100.000
120.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 4-25

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 1.24, VEL.MIN= -2.04, ACC.MAX= 7.86, ACC.MIN= -3.87,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 109.63

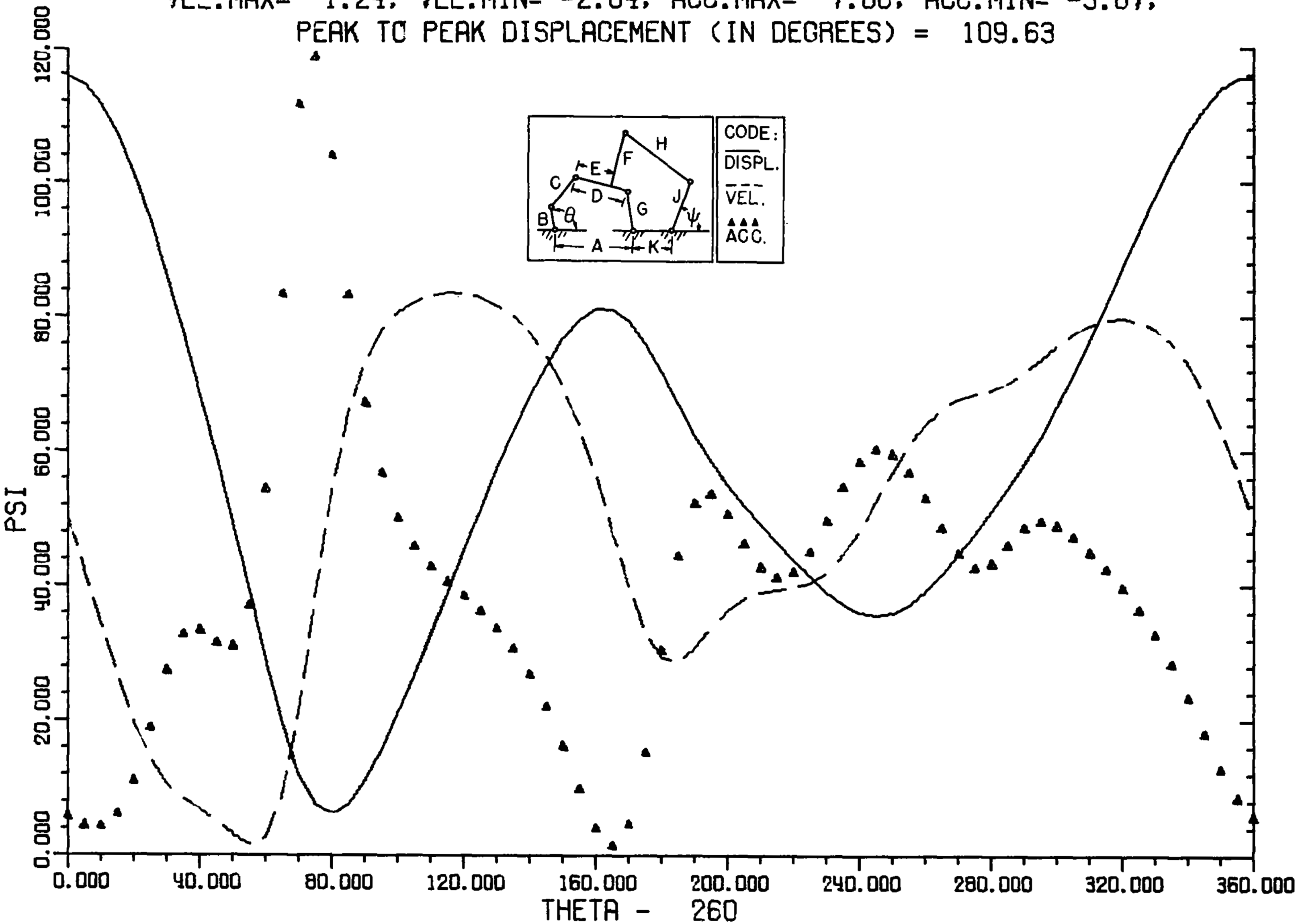


PLATE 4-26

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =12.00,
N = 2.00, PHIO (IN DEGREES) = 270.00,
VEL.MAX= 1.22, VEL.MIN= -1.90, ACC.MAX= 8.84, ACC.MIN= -5.09,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 112.51

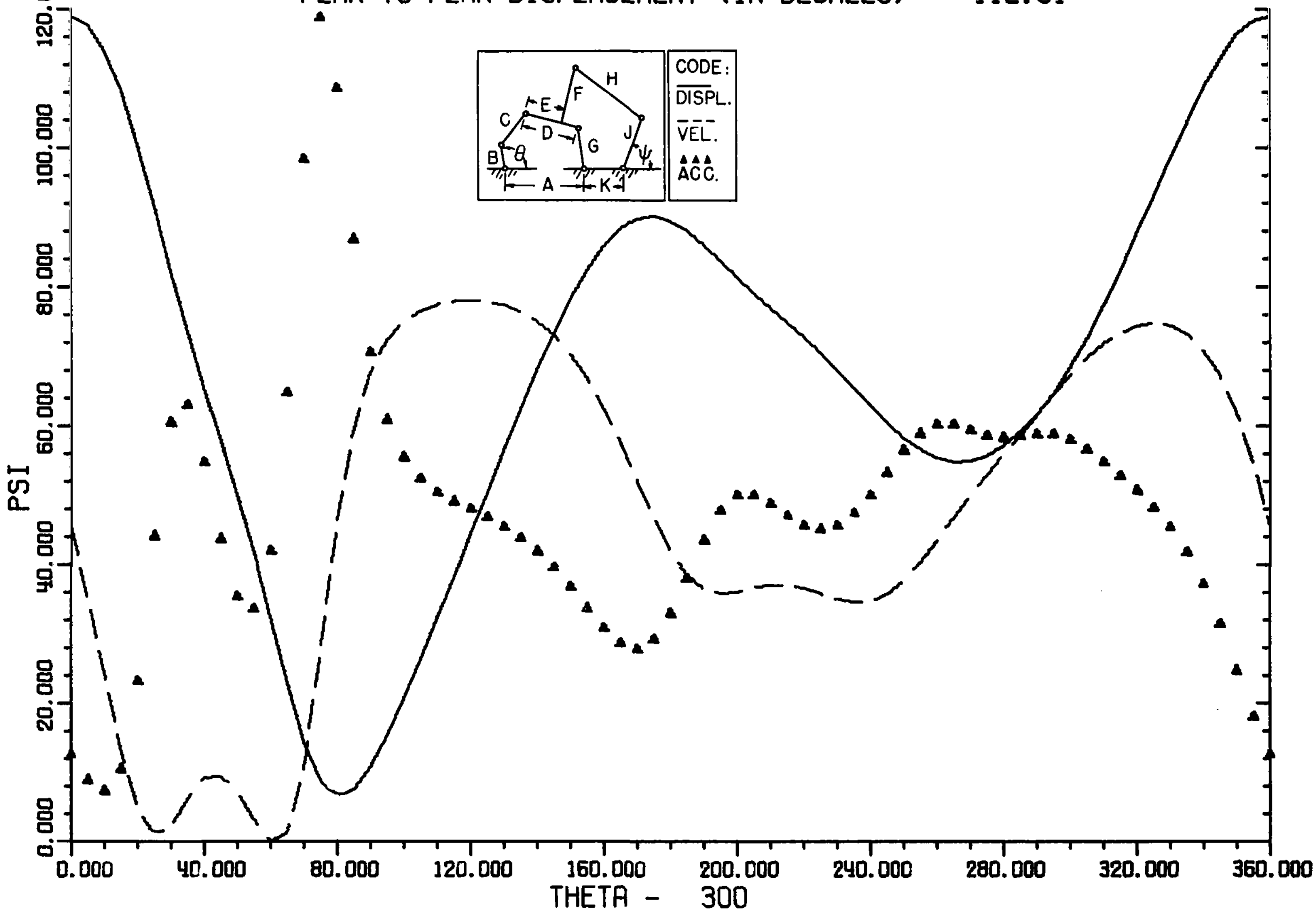
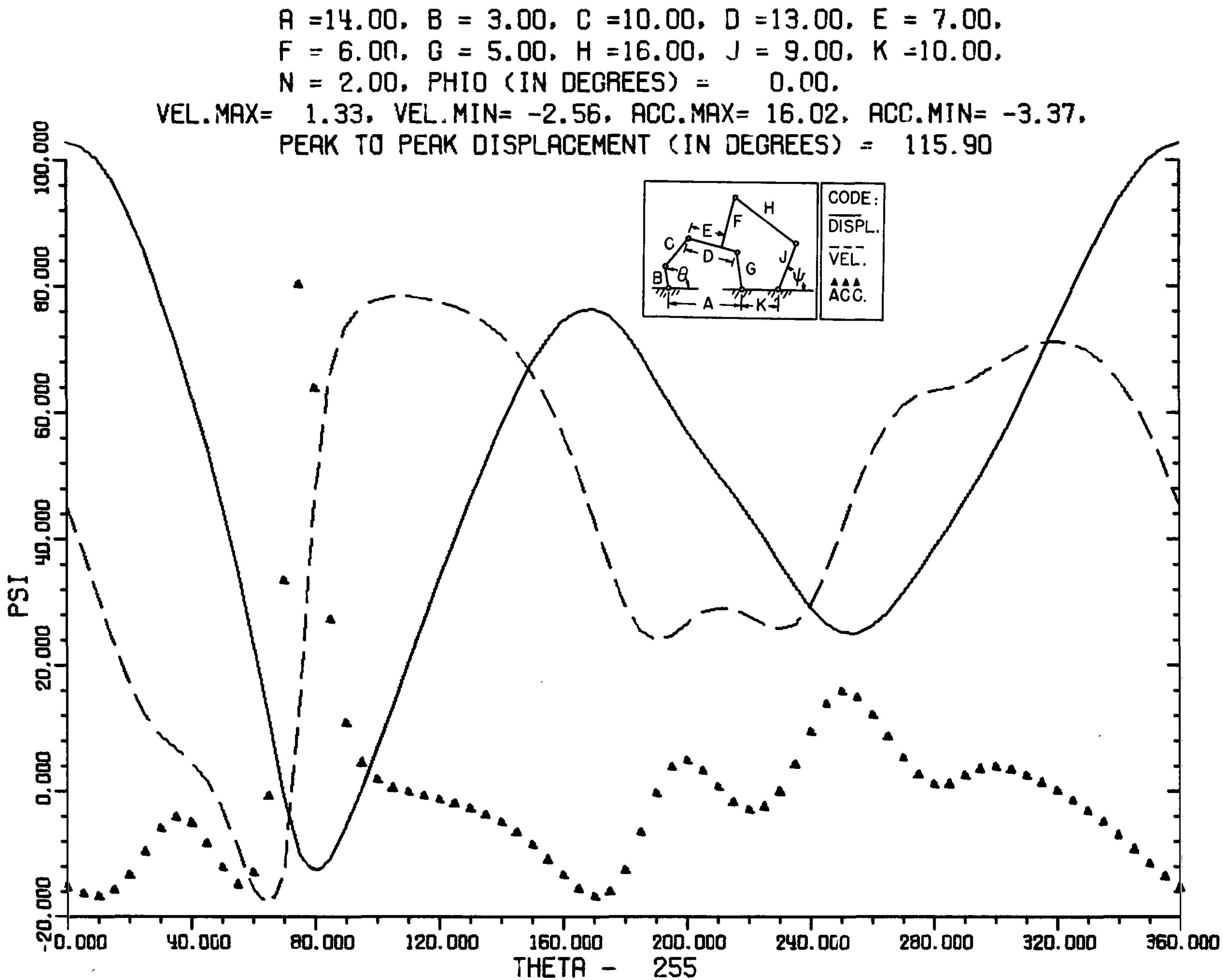


PLATE 4-27



A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 2.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 1.33, VEL.MIN= -2.56, ACC.MAX= 16.02, ACC.MIN= -3.37,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 115.90
CODE:
DISPL.
VEL.
ACC.
PSI
THETA - 255

PLATE 4-28

A =14.00, B = 3.00, C =10.00, D =13.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =12.00,
N = 3.00, PHIO (IN DEGREES) = 270.00,
VEL.MAX= 1.81, VEL.MIN= -2.81, ACC.MAX= 17.81, ACC.MIN=-12.40,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 109.17

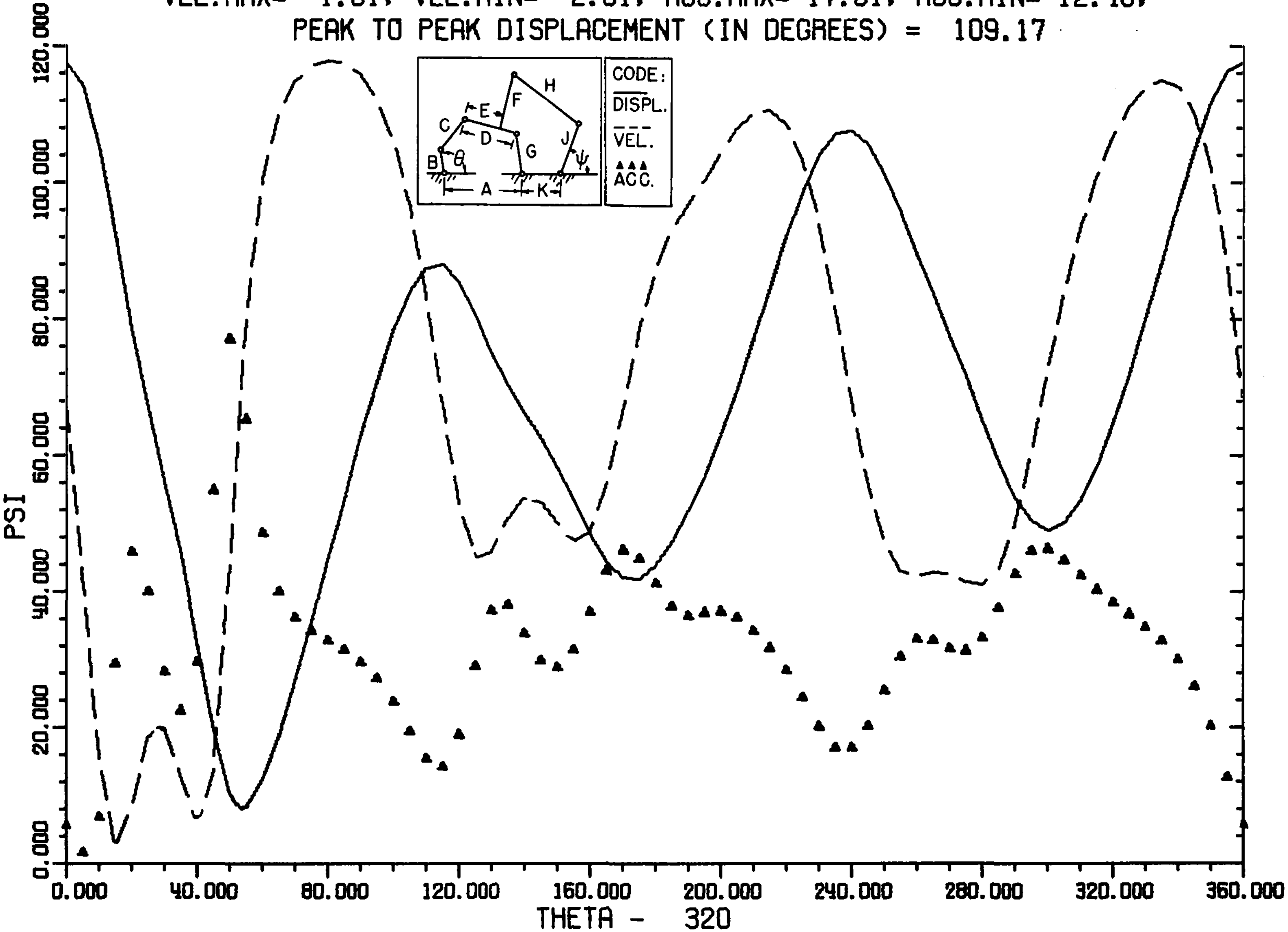


PLATE 4-29

A =14.00, B = 3.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 3.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 1.91, VEL.MIN= -2.87, ACC.MAX= 17.67, ACC.MIN=-11.48,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 110.86

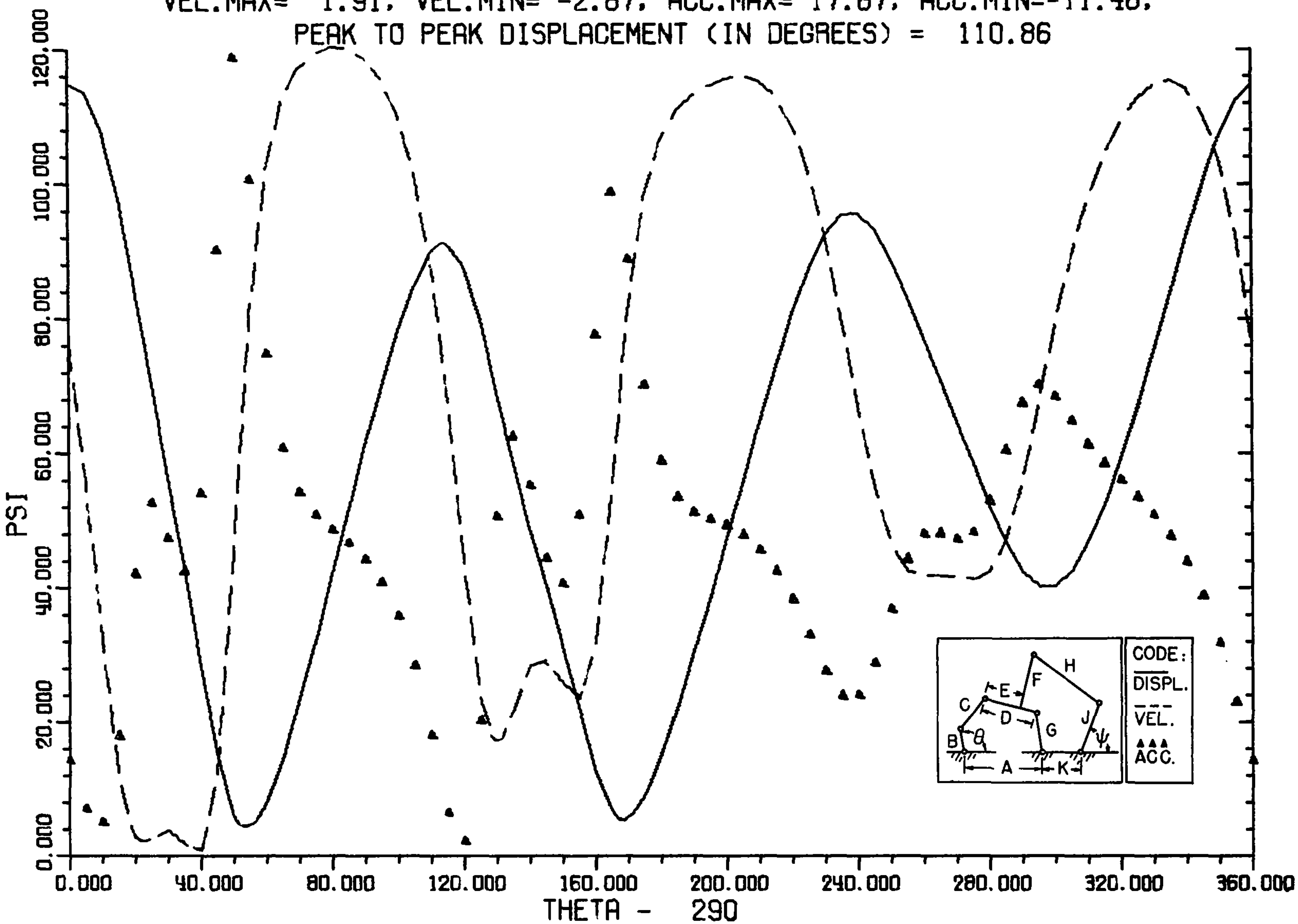


PLATE 4-30

A =14.00, B = 4.00, C =10.00, D =14.00, E = 7.00,
F = 6.00, G = 5.00, H =16.00, J = 9.00, K =10.00,
N = 3.00, PHIO (IN DEGREES) = 0.00,
VEL.MAX= 1.96, VEL.MIN= -3.24, ACC.MAX= 20.74, ACC.MIN=-12.36,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 119.24

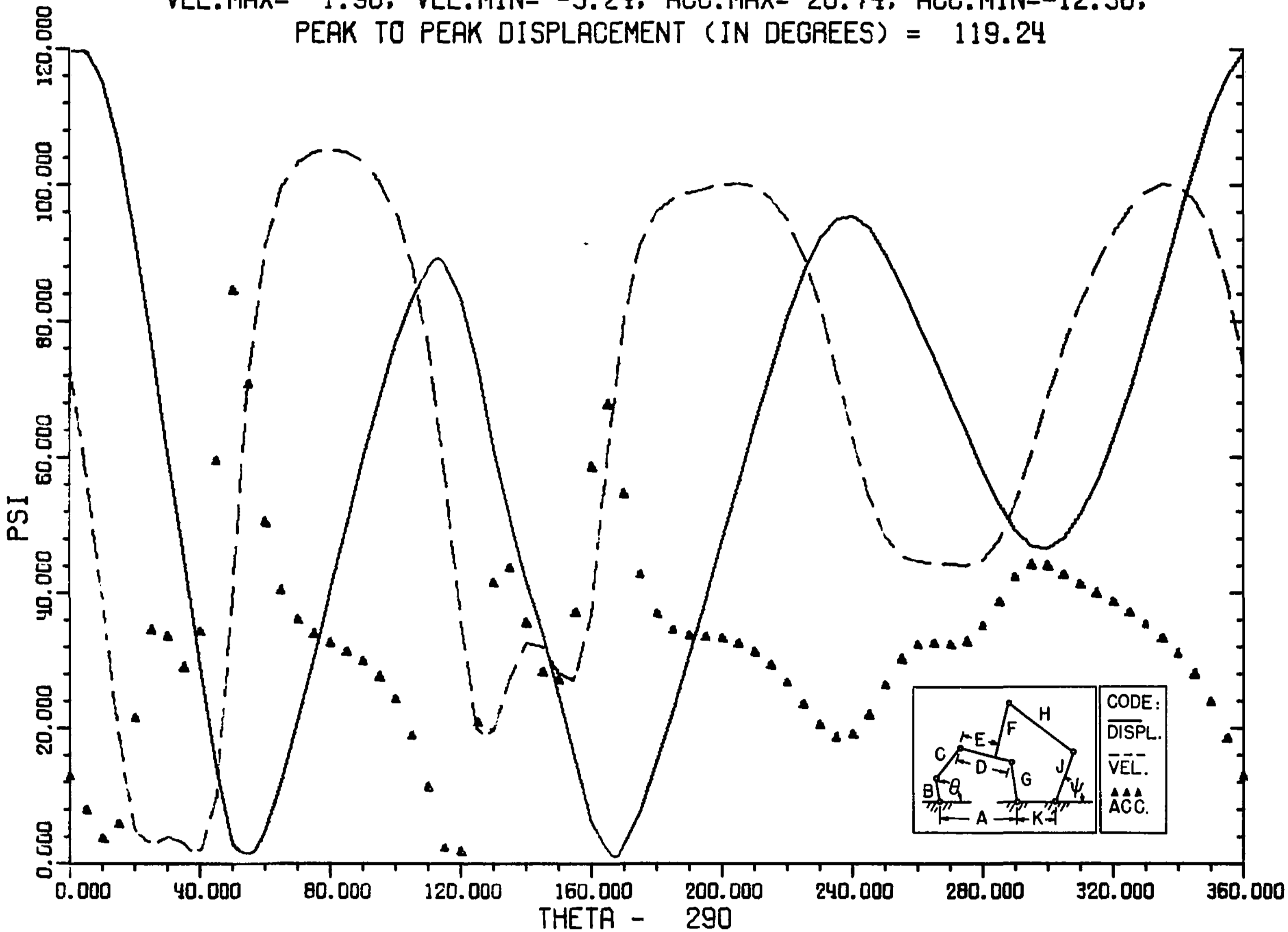


PLATE 4-31

MECHANISM #5

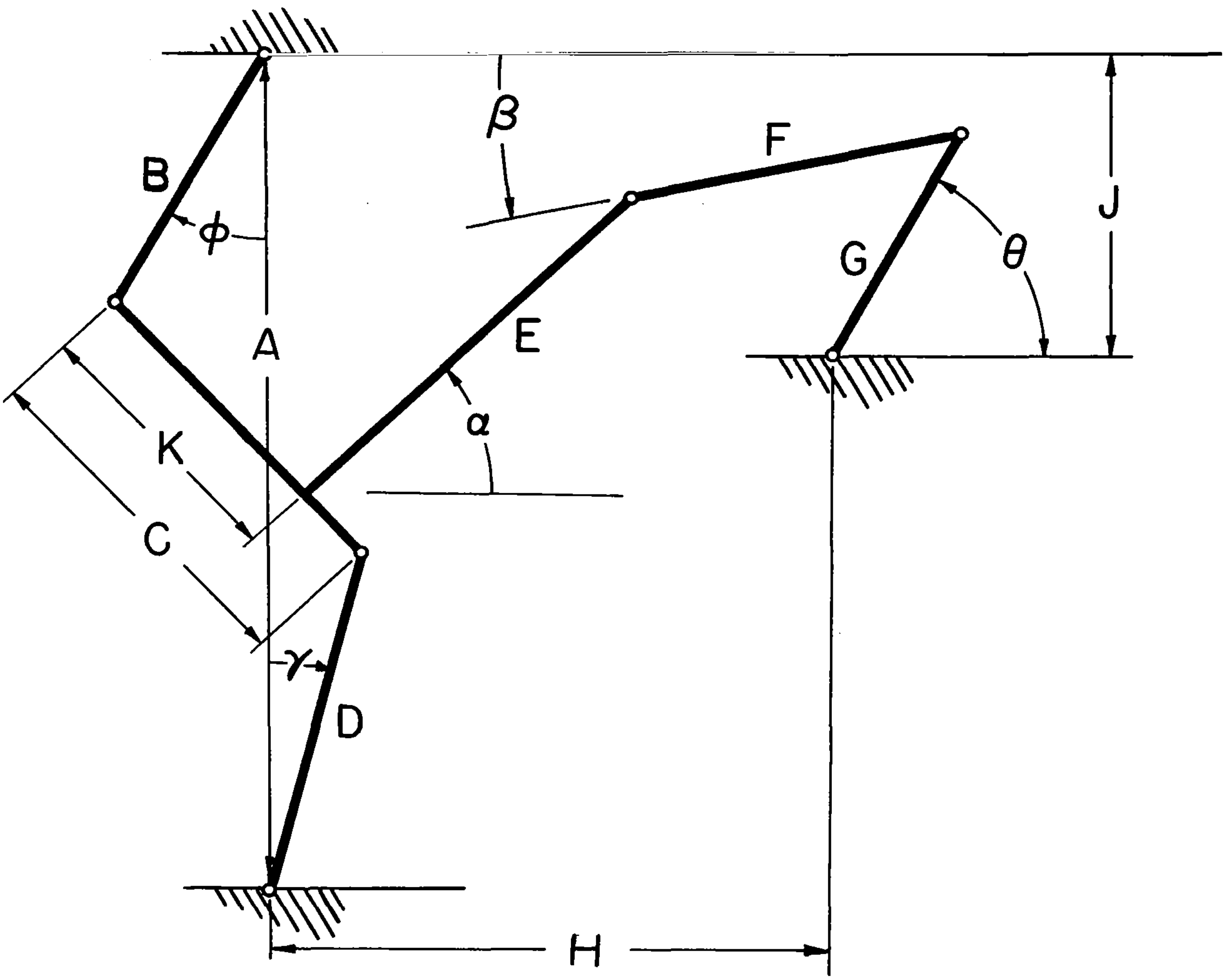


Figure 5-1

Figure 5-1 defines Mechanism #5 for which θ, the Greek letter theta, is considered as the input to the mechanism and φ, phi, is considered as the output from the mechanism. Each link of the mechanism is identified by a capital letter which is also used for specifying the length of the link. Length

E is part of link C being attached such that the length E is perpendicular to the length C at a distance K from the one end. K could be negative in which case it would be generally to the left of the joint between links B and C in the position given by Figure 5-1. The link lengths are noted as part of the title on each of the graphs which follow.

Each of the graphs for this mechanism shows ϕ versus θ as a solid line, the derivative of ϕ with respect to θ versus θ as a dashed line, and the second derivative of ϕ with respect to θ versus θ as a series of small triangles. Each curve begins with the maximum displacement of ϕ. This has been accomplished by shifting the θ axis. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. Both variables, θ and ϕ, are given in the units of degrees on each graph. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement.

Scales for the derivatives have not been presented but each graph heading includes the maximum and minimum for both the velocity and acceleration, that is, for both the first and second derivative of ϕ with respect to θ. From these data scales for the derivatives may be constructed. The units for the derivatives will be radians per radian for $d\phi/d\theta$ and radians per radian squared for $d^2\phi/d\theta^2$. A more conventional engineering unit for angular velocity may be obtained as:

$$\frac{d\phi}{dt} = \frac{d\phi}{d\theta} \times \frac{d\theta}{dt} \qquad \left[\frac{\text{radians}}{\text{second}} = \frac{\text{radians}}{\text{radian}} \times \frac{\text{radians}}{\text{second}}\right] \tag{5-1}$$

in which

$$\frac{d\theta}{dt} = \text{rpm} \times 2\pi \times \frac{1}{60} \qquad \left[\frac{\text{radians}}{\text{second}} = \frac{\text{rev}}{\text{minute}} \times \frac{\text{radians}}{\text{rev}} \times \frac{\text{minute}}{\text{seconds}}\right] \tag{5-2}$$

Substituting Eq. 5-2 into Eq. 5-1 produces:

$$\frac{d\phi}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{d\phi}{d\theta}, \qquad \frac{\text{radians}}{\text{second}} . \tag{5-3}$$

In words, the angular velocity of link B (degrees/ second) is obtained as the product of $\pi/30$ times the angular speed of link G (revolutions/ minute) and $d\phi/d\theta$ (radians/ radian). Values for this latter term may be obtained from a graph (the dashed line) or from equations which follow, or the extreme values may be obtained from the heading of a graph as VEL. MAX and VEL. MIN.

The angular acceleration of link B may be given in more conventional engineering terms as:

$$\frac{d^2\phi}{dt^2} = \frac{d}{dt}\left[\frac{d\phi}{d\theta} \times \frac{d\theta}{dt}\right]$$

$$= \frac{d\phi}{d\theta} \times \frac{d^2\theta}{dt^2} + \frac{d^2\phi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2 . \tag{5-4}$$

If the angular speed of link G remains constant then $d^2\theta/dt^2$ equals zero. The expression for the angular acceleration of the output link with the input link turning with constant speed simplifies to:

$$\frac{d^2\phi}{dt^2} = \frac{d^2\phi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2$$

$$= \frac{d^2\phi}{d\theta^2}\left[\frac{\pi}{30} \times \text{rpm}\right]^2, \qquad \frac{\text{radians}}{\text{second}^2} . \tag{5-5}$$

The term $d^2\phi/d\theta^2$ may be obtained from a graph (the series of small triangles), or from equations which follow, or the extreme values may be noted in the heading of a graph as ACC. MAX and ACC. MIN.

Referring to the figure for this mechanism the equations relating the output to the input may be derived. Considering links A, B, C, and D:

$$B \cos\phi + C \cos\alpha + D \cos\gamma = A$$

$$B \sin\phi - C \sin\alpha + D \sin\gamma = 0. \qquad (5\text{-}6)$$

Solving for $D \cos\gamma$ and $D \sin\gamma$, squaring these equations, and adding the resulting equations together will yield:

$$D^2 = A^2 + B^2 + C^2 - 2AB\cos\phi - 2\,BC\,\sin\phi\sin\alpha - 2C\,(A - B\cos\phi)\cos\alpha. \qquad (5\text{-}7)$$

Another equation involving α and ϕ is needed and may be obtained by again referring to the figure for the mechanism from which may be noted:

$$J = B\cos\phi + K\cos\alpha - E\sin\alpha - F\sin\beta + G\sin\theta$$

$$H = -B\sin\phi + K\sin\alpha + E\cos\alpha + F\cos\beta - G\cos\theta. \qquad (5\text{-}8)$$

From these equations may be solved $F \sin\beta$ and $F \cos\beta$. These resulting expressions may then be squared and added together to form:

$$M^2 + N^2 + K^2 + E^2 - F+ = 2\,(ME + NK)\sin\alpha + 2\,(NE - MK)\cos\alpha$$

$$2\,(ME + NK)\sin\alpha + 2\,(NE - MK)\cos\alpha \qquad (5\text{-}9)$$

in which

$$M = B\cos\phi + G\sin\theta - J$$

$$N = B\sin\phi + G\cos\theta + H.$$

To simplify the form of the equations, let:

$$T = A^2 + B^2 + C^2 - D^2 - 2\,AB \cos\phi$$

$$V = 2\,BC \sin\phi$$

$$W = -\,2C\,(B \cos\phi - A)$$

$$X = M^2 + N^2 + K^2 + E^2 - F^2$$

$$Y = 2\,(ME + NK)$$

$$Z = 2\,(NE - MK). \tag{5-10}$$

Eqs. 5-7 and 5-9 may be rewritten

$$T = V \sin\alpha + W \cos\alpha$$

$$X = Y \sin\alpha + Z \cos\alpha\,. \tag{5-11}$$

By simple algebraic manipulation two equations may be obtained from Eq. 5-11 one involving $\cos\alpha$ and the other involving $\sin\alpha$. By the squaring and adding of these equations α may be eliminated and a single equation involving θ and ϕ produced as:

$$U^2 + Q^2 - R^2 = 0 \tag{5-12}$$

in which

$$U = TY - VX$$

$$Q = WX - TZ$$

$$R = WY - VZ.$$

Using the Newton-Raphson method Eq. 5-12 may be solved for ϕ in terms of a given value for θ. The curve as a solid line in each of the graphs for this mechanism depicts ϕ as a function of θ and was obtained from Eq. 5-12. The units for both ϕ and θ are degrees.

Angular Velocity

The term $d\phi/d\theta$ may be obtained from Eq. 5-12 by differentiating with respect to θ yielding:

$$2U \frac{dU}{d\theta} + 2Q \frac{dQ}{d\theta} - 2R \frac{dR}{d\theta} = 0. \tag{5-13}$$

By performing the indicated operations and with considerable algebraic manipulations, the following may be determined:

$$\frac{d\phi}{d\theta} = \frac{\text{numerator}}{\text{denominator}} \tag{5-14}$$

in which

$$\begin{aligned}
\text{numerator} = {} & [U\,(TKG - VNG) + Q\,(WNG - TEG) \\
& - R\,(WKG - VEG)]\ \sin\theta \\
& - [U\,(TEG - VMG) + Q\,(WMG + TKG) \\
& - R\,(WEG + VKG)]\ \cos\theta \\
\text{denominator} = {} & [U(YAB - TEB + VMB) + Q\,(XBC\text{-}WMB - RKB - \\
& - R\,(YBC - WEB - VKB)]\ \sin\phi \\
& + [U\,(TKB - VNB - XBC) + Q\,(WNB - TEB) \\
& - R\,(WKB - VEB - ZBC)]\ \cos\phi\,.
\end{aligned}$$

With a value for $d\phi/d\theta$ from Eq. 5-14, the angular velocity of the output, link B, may be calculated using Eq. 5-3. The dashed line curves in the graphs that follow were obtained by evaluating Eq. 5-14. The unit for $d\phi/d\theta$ will be radians/radian or numerically this will be the same as the unit degrees/degree.

Angular Acceleration

An expression for $d^2\phi/d\theta^2$ may be determined by differentiating Eq. 5-13 with respect to θ producing:

$$U \frac{d^2U}{d\theta^2} + Q \frac{d^2Q}{d\theta^2} - R \frac{d^2R}{d\theta^2} + \left[\frac{dU}{d\theta}\right]^2 + \left[\frac{dQ}{d\theta}\right]^2 - \left[\frac{dR}{d\theta}\right]^2 = 0. \tag{5-15}$$

Performing the indicated differentiations and following extensive algebraic manipulations, the expression may be written:

$$\frac{d^2\phi}{d\theta^2} = \frac{-\text{Number}}{\text{Denomin}} \tag{5-16}$$

in which

$$\begin{aligned}
\text{Number} = {} & \left[\frac{d\phi}{d\theta}\right]^2 \Big\{ U[2(\sin\phi)(VNB - TKB + XBC) + 2(\cos\phi)(YAB - TEB + VMB)] \\
& + Q[2(\sin\phi)(TEB - WNB) + 2(\cos\phi)(XBC - WMB - TKB - ZAB)] \\
& - R[2(\sin\phi)(VEB - WKB + ZBC) + 2(\cos\phi)(YBC - WEB - VKB)]\Big\} \\
& + \Big\{ U[2(\sin\theta)(VMG - TEG) + 2(\cos\theta)(VNG - TKG) - 2V\left[\frac{dM}{d\theta}\right]^2 \\
& - 2V\left[\frac{dN}{d\theta}\right]^2 + 2\frac{dT}{d\theta}\times\frac{dY}{d\theta} - 2\frac{dV}{d\theta}\times\frac{dX}{d\theta}\Big] + Q\Big[2(\sin\theta)(-WMG - TKG) \\
& + 2(\cos\theta)(TEG - WNG) + 2W\left[\frac{dM}{d\theta}\right]^2 + 2W\left[\frac{dN}{d\theta}\right]^2 + 2\frac{dW}{d\theta}\times\frac{dX}{d\theta}
\end{aligned}$$

$$- 2 \frac{dT}{d\theta} \times \frac{dZ}{d\theta}\Bigg] - R \Bigg[2(\sin\theta)(- WEG - VKG) + 2(\cos\theta)(VEG - WKG)$$

$$+ 2 \frac{dW}{d\theta} \times \frac{dY}{d\theta} - 2 \frac{dV}{d\theta} \times \frac{dZ}{d\theta}\Bigg] + \left[\frac{dU}{d\theta}\right]^2 + \left[\frac{dQ}{d\theta}\right]^2 - \left[\frac{dR}{d\theta}\right]^2 \Bigg\}$$

and in which

$$\text{Denomin} = U[2\sin\phi(YAB - TEB + VMB) + 2\cos\phi(TKB - VNB - XBC)]$$
$$+ Q[2\sin\phi(XBC - WMB - TKB - ZAB) + 2\cos\phi(WNB - TEB)]$$
$$- R[2\sin\phi(YBC - WEB - VKB) + 2\cos\phi(WKB - VEB - ZBC)] .$$

In these expressions several terms need to be derived and are as follows:

$$\frac{dT}{d\theta} = 2\,AB\sin\phi\,\frac{d\phi}{d\theta}, \quad \frac{dW}{d\theta} = 2\,BC\sin\phi\,\frac{d\phi}{d\theta}$$

$$\frac{dV}{d\theta} = 2\,BC\cos\phi\,\frac{d\phi}{d\theta}, \quad \frac{dX}{d\theta} = 2M\frac{dM}{d\theta} + 2N\frac{dN}{d\theta}$$

$$\frac{dY}{d\theta} = 2E\frac{dM}{d\theta} + 2K\frac{dN}{d\theta}, \quad \frac{dZ}{d\theta} = 2E\frac{dN}{d\theta} - 2K\frac{dM}{d\theta}$$

$$\frac{dM}{d\theta} = - B\sin\phi\frac{d\phi}{d\theta} + G\cos\theta, \quad \frac{dN}{d\theta} = B\cos\phi\frac{d\phi}{d\theta} - G\sin\theta$$

$$\frac{dU}{d\theta} = T\frac{dY}{d\theta} + Y\frac{dT}{d\theta} - V\frac{dX}{d\theta} - X\frac{dV}{d\theta}$$

$$\frac{dQ}{d\theta} = W\frac{dX}{d\theta} + X\frac{dW}{d\theta} - T\frac{dZ}{d\theta} - Z\frac{dT}{d\theta}$$

$$\frac{dR}{d\theta} = W\frac{dY}{d\theta} + Y\frac{dW}{d\theta} - V\frac{dZ}{d\theta} - Z\frac{dV}{d\theta} .$$

With these several expressions $d^2\phi/d\theta^2$ may be calculated. This quantity has been evaluated for several different configurations and is presented in the graphs for this mechanism as a series of small triangles. As noted previously a more common engineering expression for angular acceleration is given by Eq. 5-5 in which $d^2\phi/d\theta^2$ may be determined by Eq. 5-16 and the related equations.

A= 1.00, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.00, G= 0.40, H= 1.40, J= 1.40, K= 0.60,
VEL.MAX= 0.08, VEL.MIN= -0.09, ACC.MAX= 0.28, ACC.MIN= -0.20,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 5.64

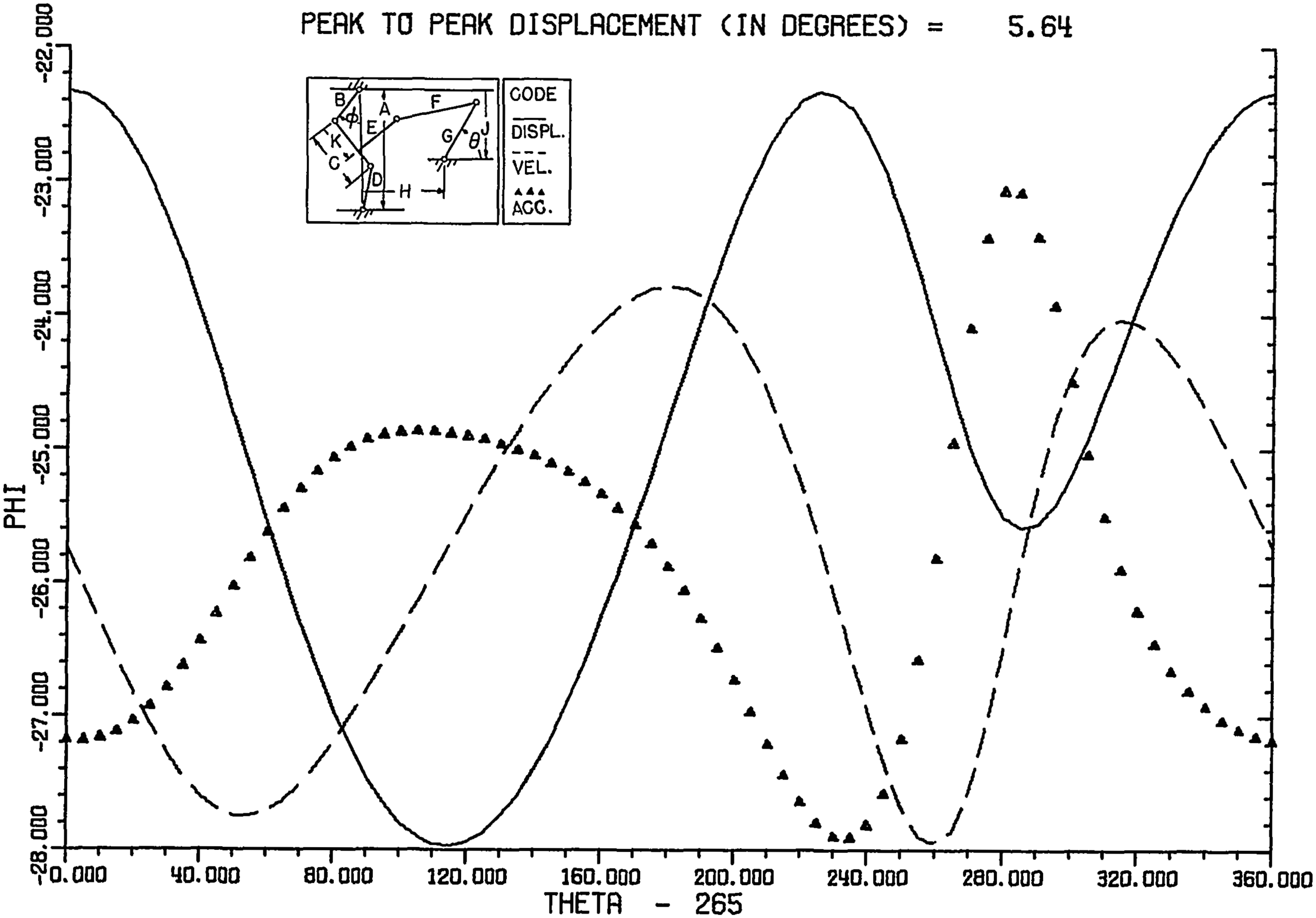


PLATE 5-1

A= 0.80, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.00, G= 0.40, H= 1.20, J= 1.40, K= 0.60.
VEL.MAX= 0.10, VEL.MIN= -0.08, ACC.MAX= 0.30, ACC.MIN= -0.26.
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 7.29

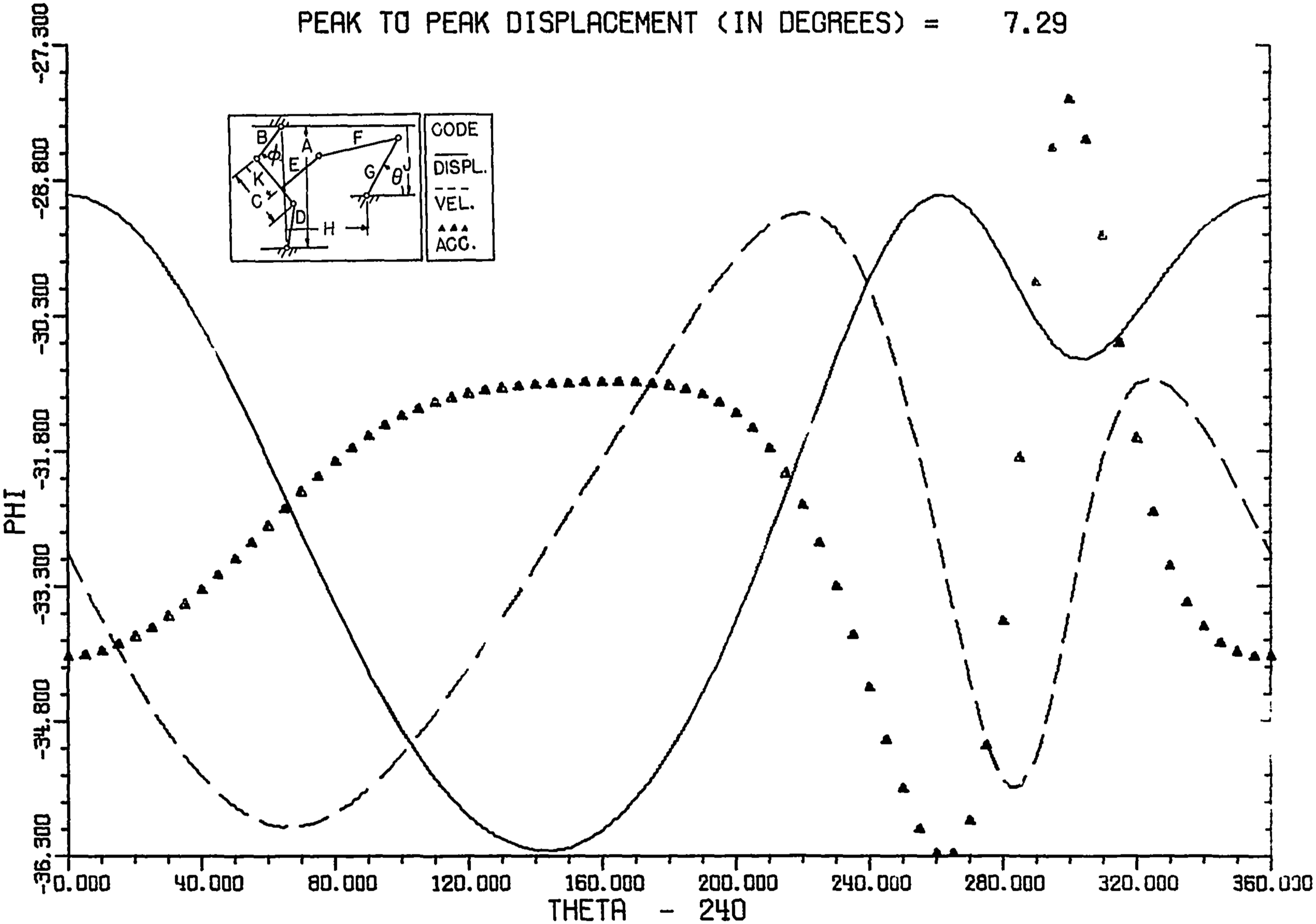


PLATE 5-2

A= 1.00, B= 0.60, C= 1.40, D= 1.00, E= 1.10,
F= 1.20, G= 0.50, H= 1.90, J= 1.40, K= 0.60,
VEL.MAX= 0.11, VEL.MIN= -0.12, ACC.MAX= 0.15, ACC.MIN= -0.15,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 8.87

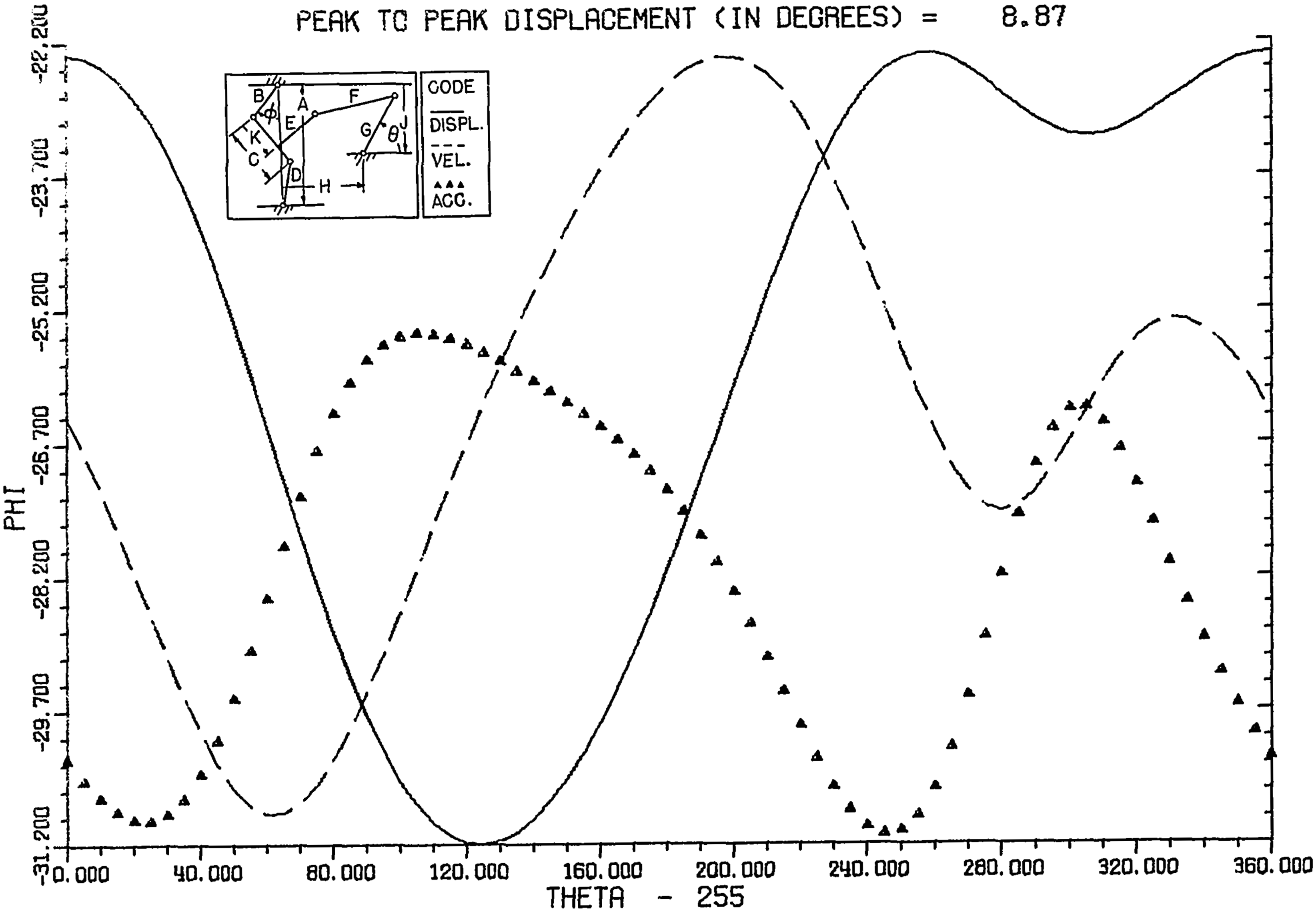


PLATE 5-3

A= 1.00, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.00, G= 0.40, H= 1.60, J= 1.40, K= 0.60,
VEL.MAX= 0.13, VEL.MIN= -0.15, ACC.MAX= 0.18, ACC.MIN= -0.17,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 11.60

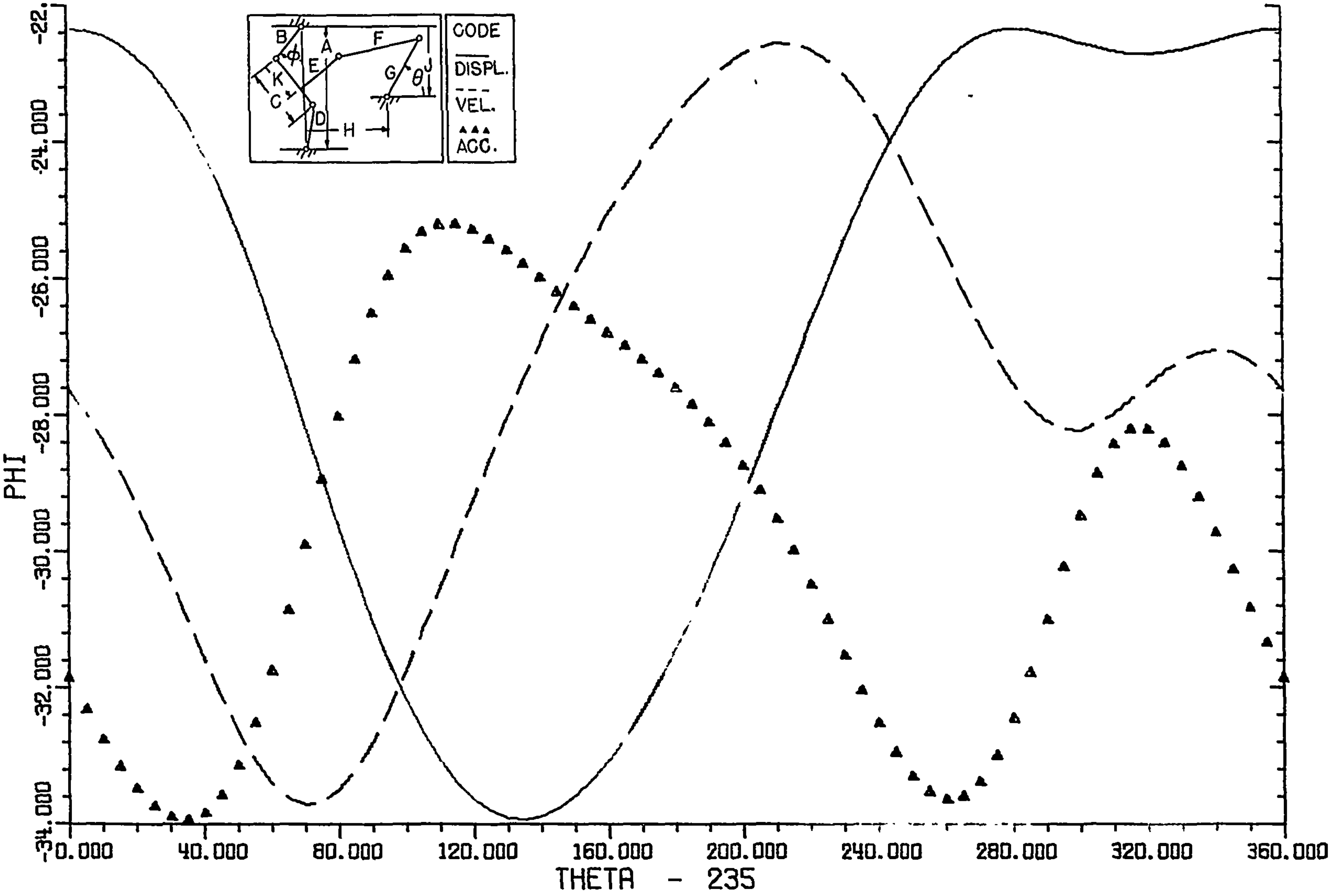


PLATE 5-4

A= 0.80, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.00, G= 0.40, H= 1.40, J= 1.00, K= 0.60,
VEL.MAX= 0.21, VEL.MIN= -0.15, ACC.MAX= 0.17, ACC.MIN= -0.24,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 17.62

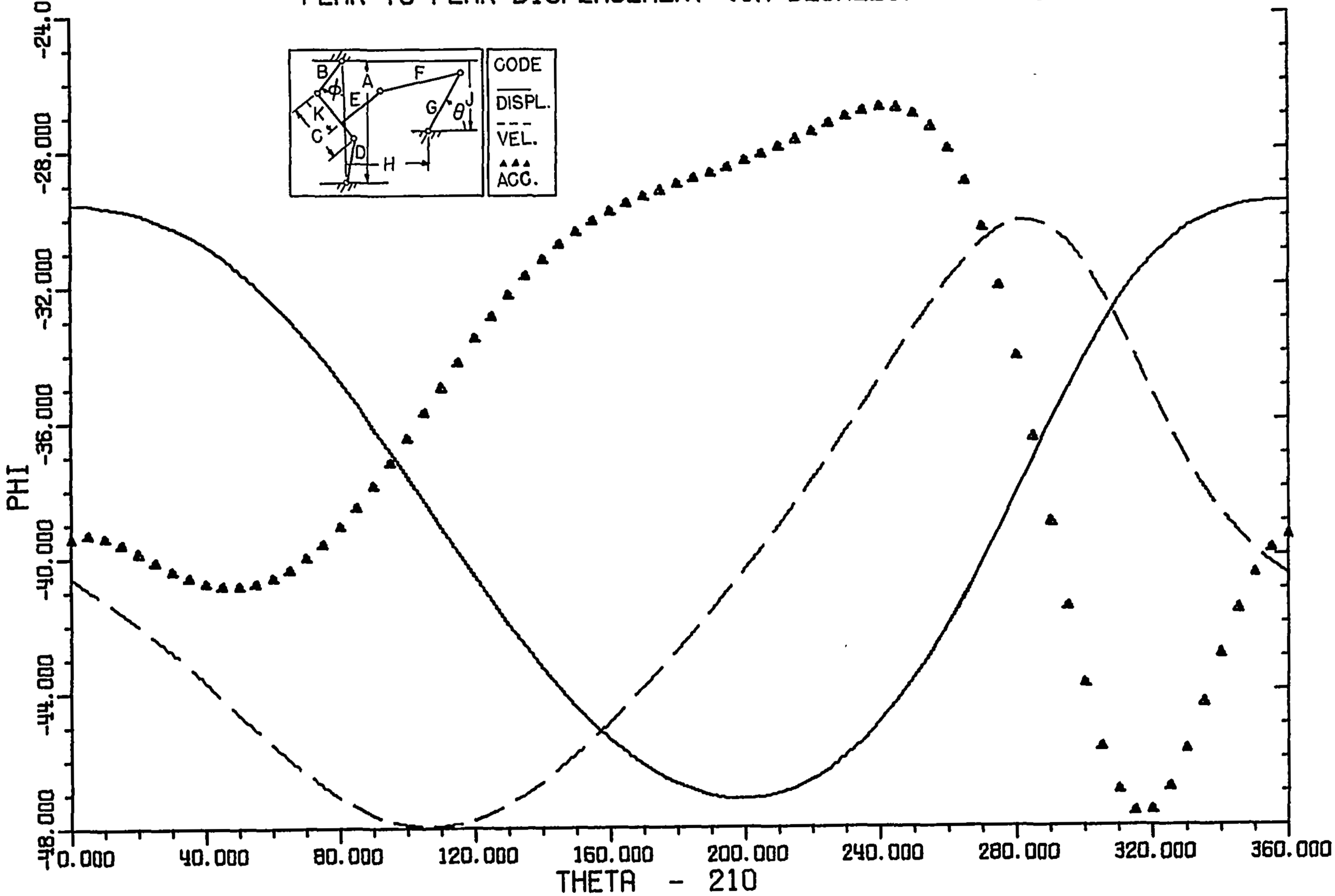


PLATE 5 - 5

A= 0.80, B= 0.80, C= 1.50, D= 1.00, E= 1.00,
F= 1.00, G= 0.50, H= 1.80, J= 1.40, K= 0.70,
VEL.MAX= 0.19, VEL.MIN= -0.22, ACC.MAX= 0.24, ACC.MIN= -0.20,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 20.26

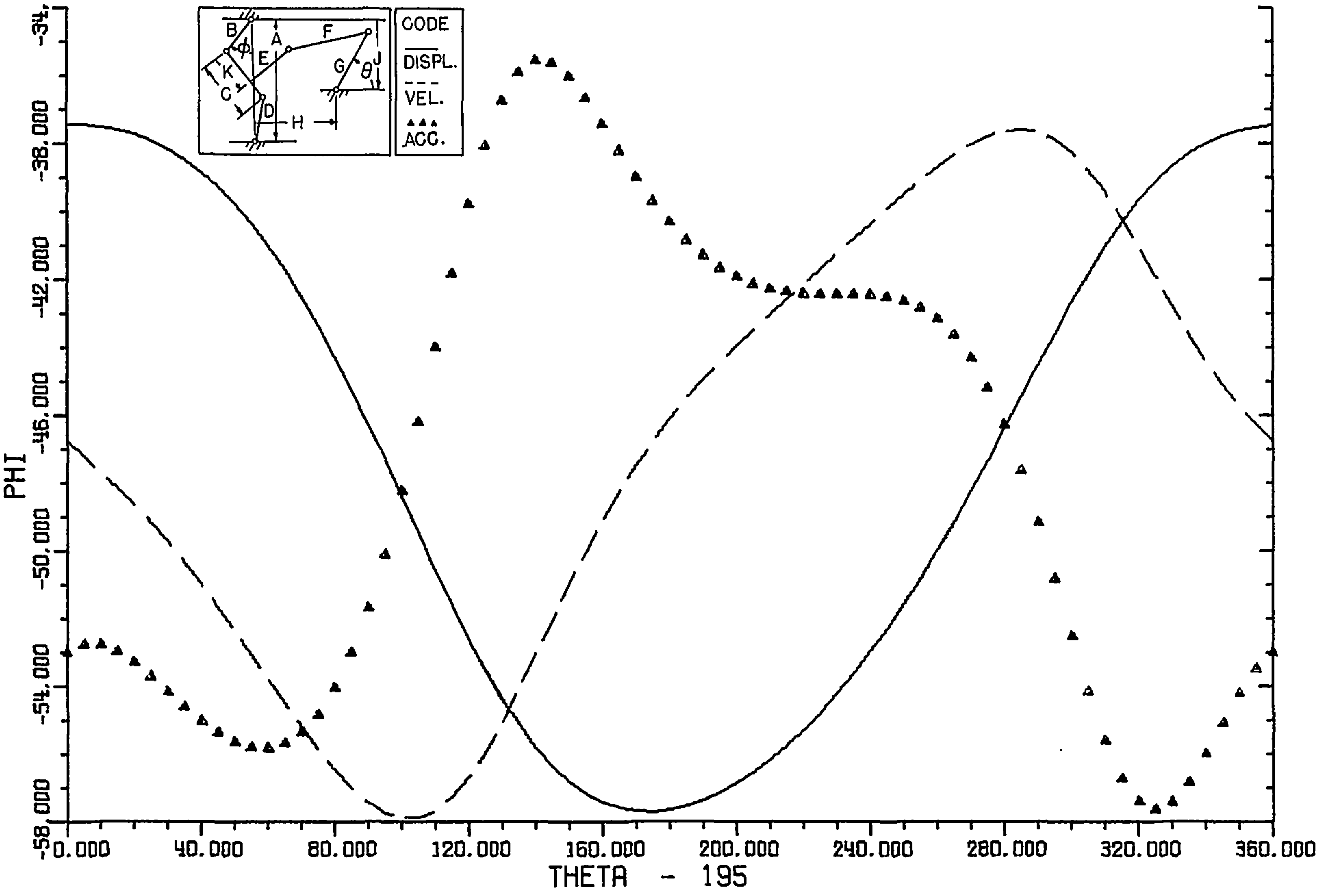


PLATE 5 - 6

A= 0.80, B= 0.80, C= 1.60, D= 0.90, E= 0.80,
F= 1.00, G= 0.50, H= 1.70, J= 1.30, K= 0.70,
VEL.MAX= 0.20, VEL.MIN= -0.30, ACC.MAX= 0.52, ACC.MIN= -0.42,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 21.25

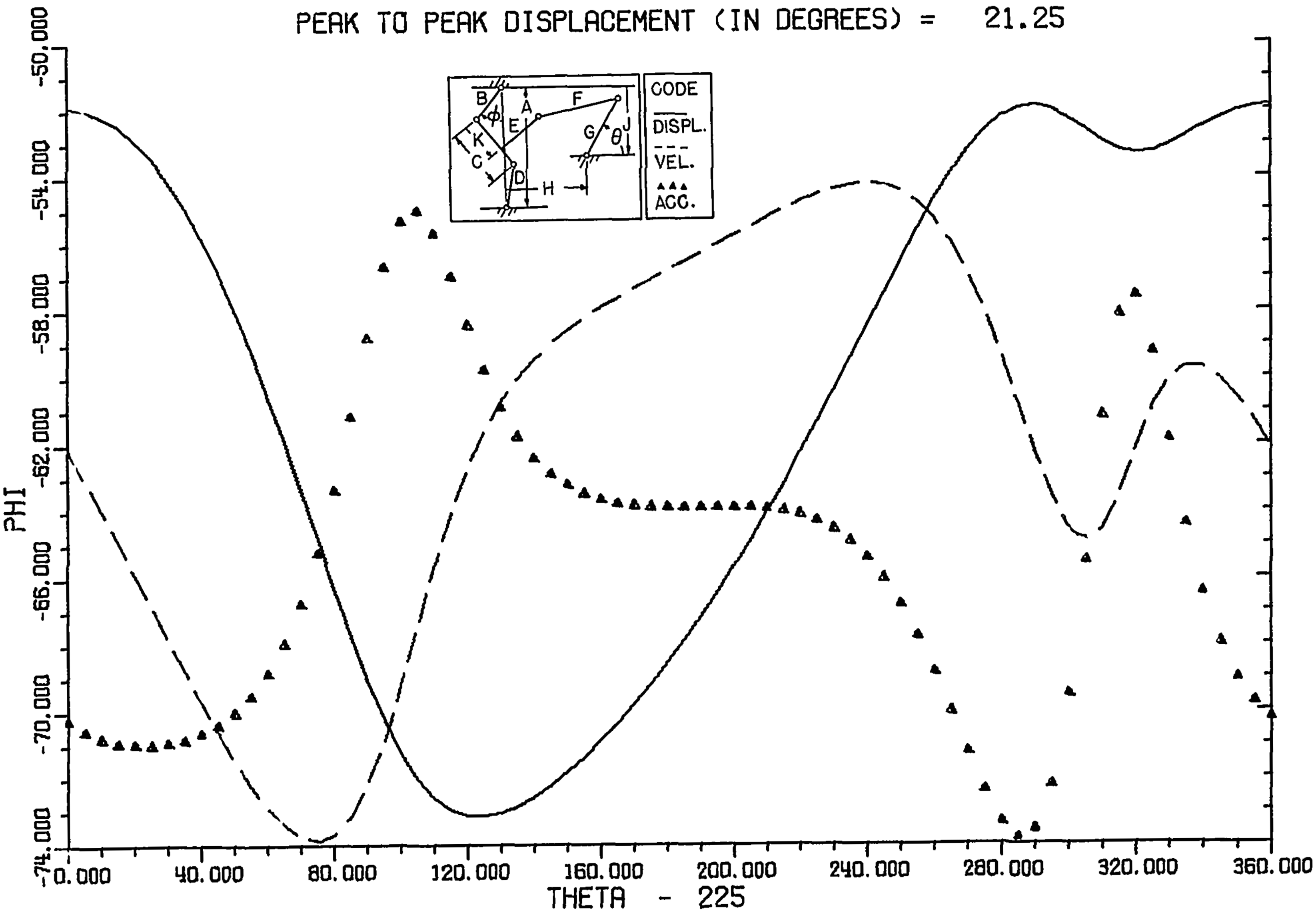


PLATE 5-7

A= 0.80, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.00, G= 0.50, H= 1.60, J= 1.40, K= 0.60,
VEL.MAX= 0.20, VEL.MIN= -0.26, ACC.MAX= 0.33, ACC.MIN= -0.22,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 22.02

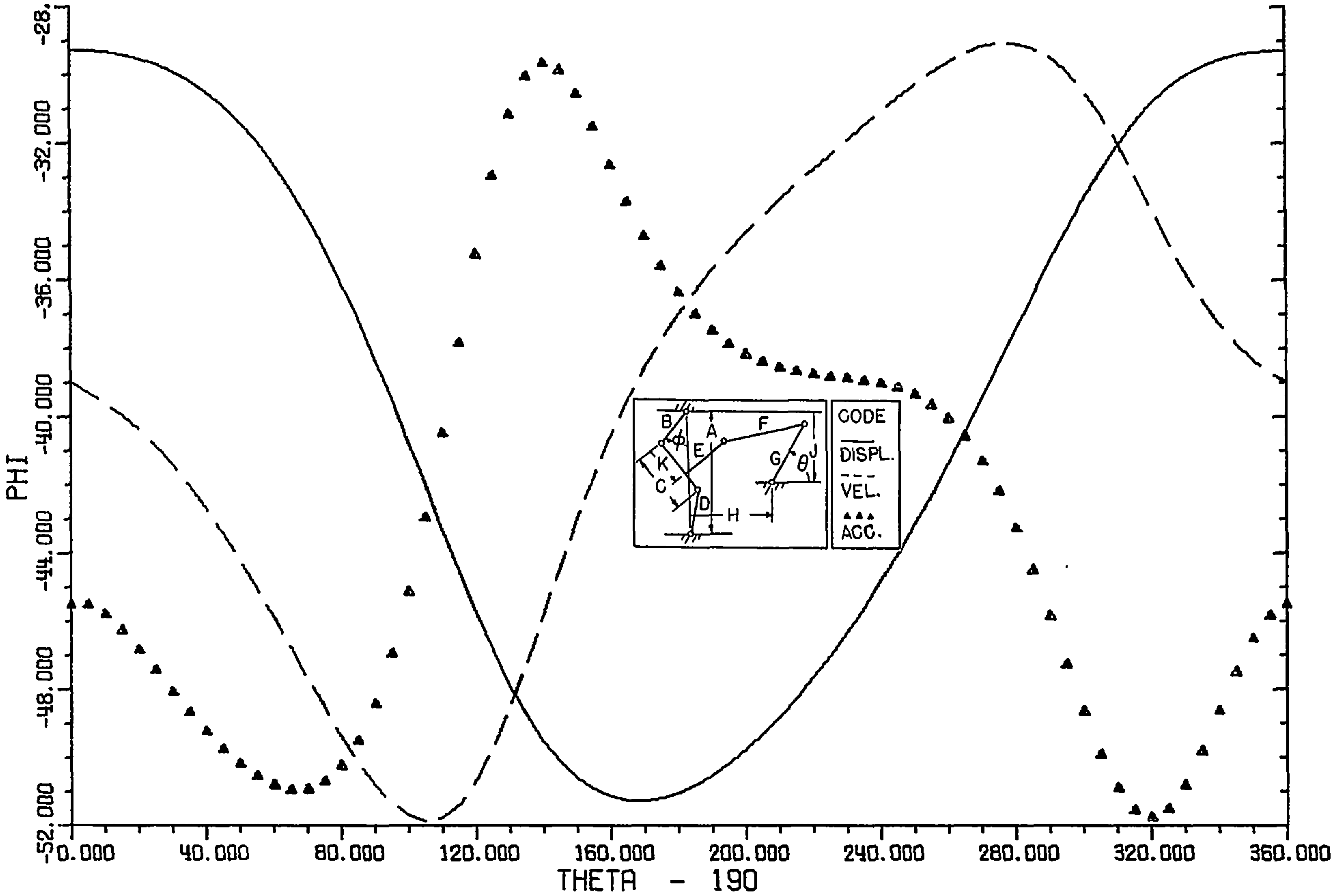


PLATE 5-8

A= 0.60, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.20, G= 0.60, H= 1.40, J= 1.20, K= 0.60,
VEL.MAX= 0.31, VEL.MIN= -0.22, ACC.MAX= 0.23, ACC.MIN= -0.55,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 24.37

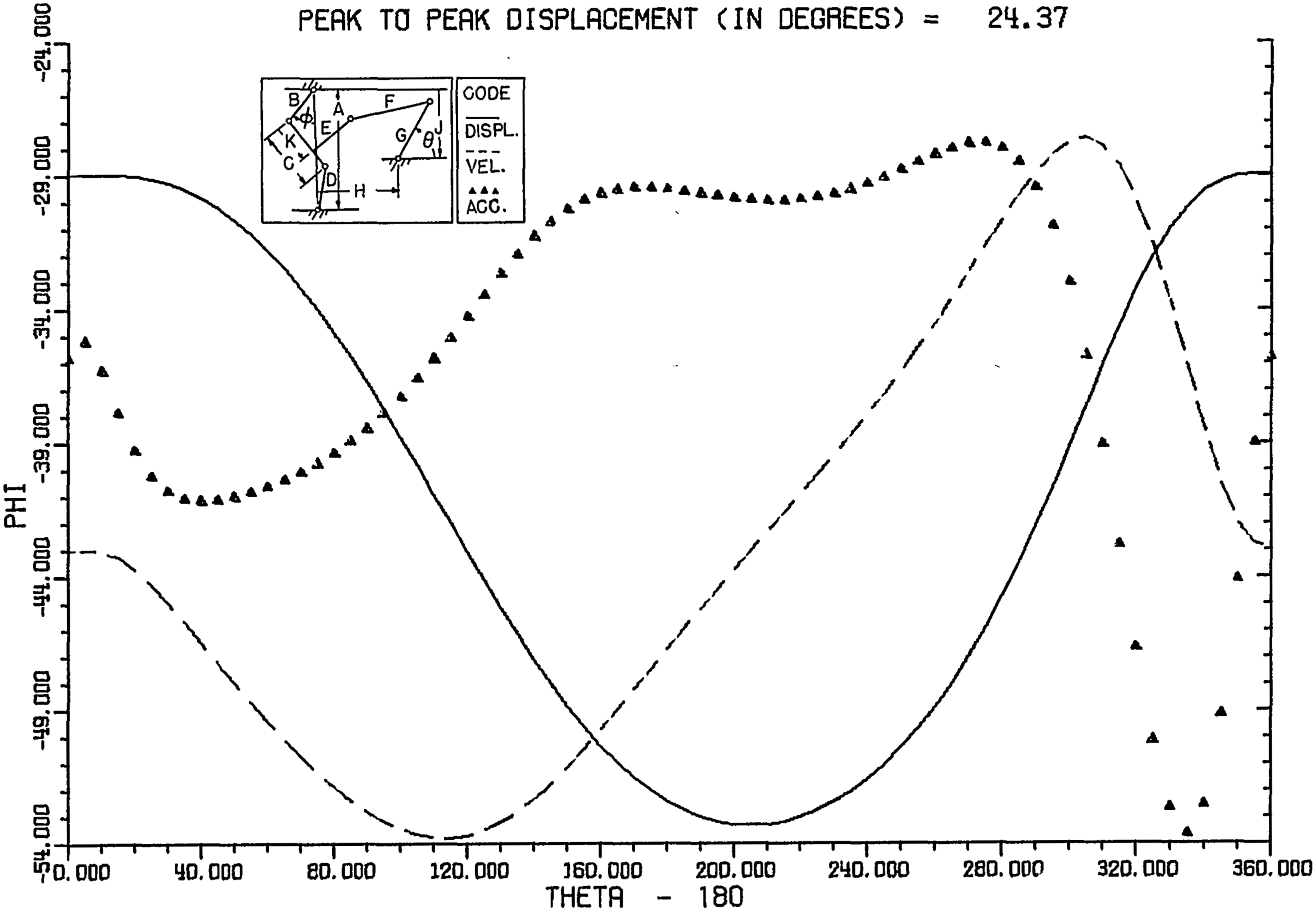


PLATE 5-9

A= 1.80, B= 1.00, C= 1.50, D= 1.00, E= 1.00,
F= 1.00, G= 0.50, H= 1.40, J= 1.40, K= 0.70,
VEL.MAX= 0.24, VEL.MIN= -0.31, ACC.MAX= 0.21, ACC.MIN= -0.49,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 29.07

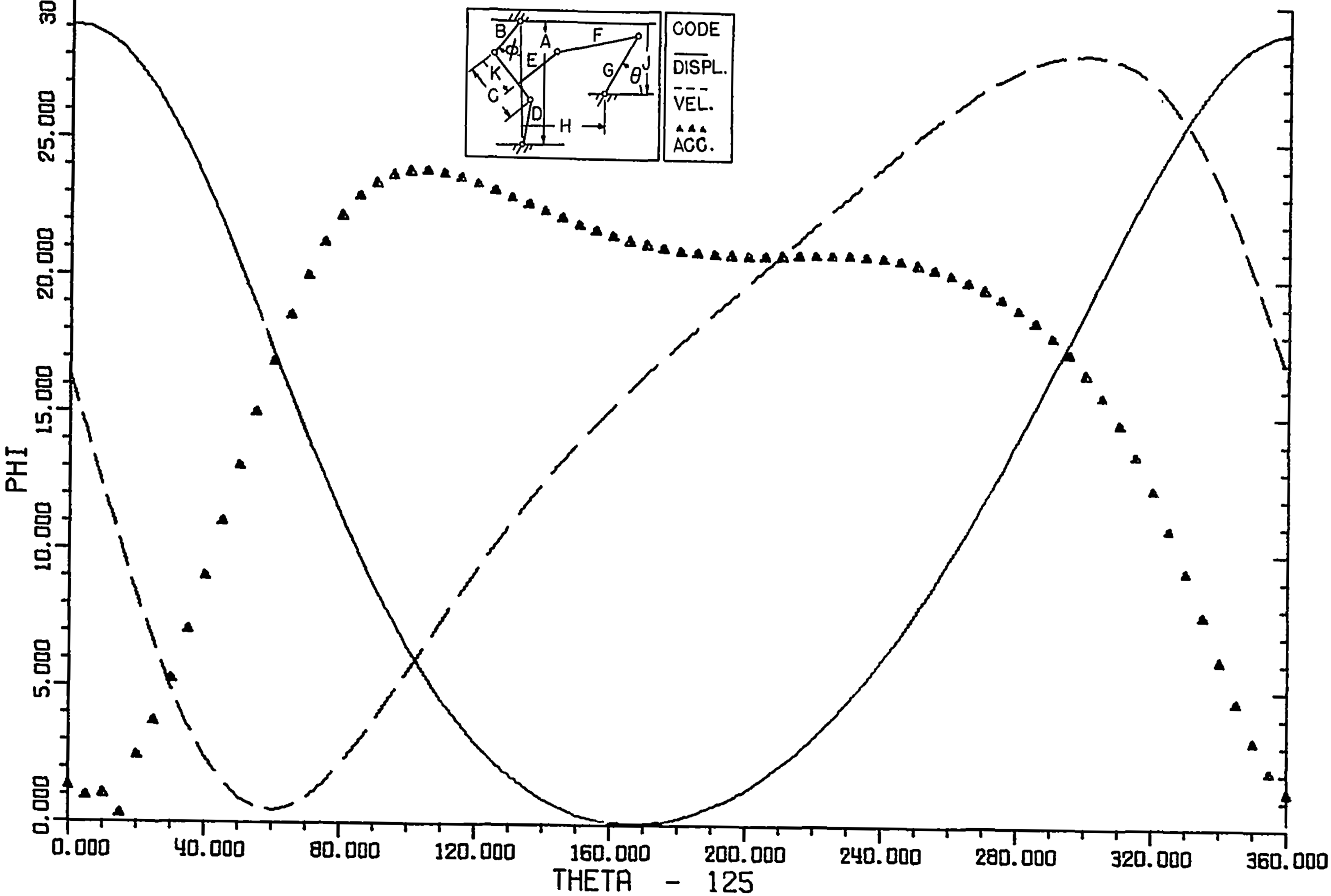


PLATE 5-10

A= 2.20, B= 0.80, C= 1.20, D= 0.80, E= 0.80,
F= 1.00, G= 0.50, H= 1.80, J= 1.00, K= 0.70,
VEL.MAX= 0.43, VEL.MIN= -0.46, ACC.MAX= 0.54, ACC.MIN= -0.94,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 32.88

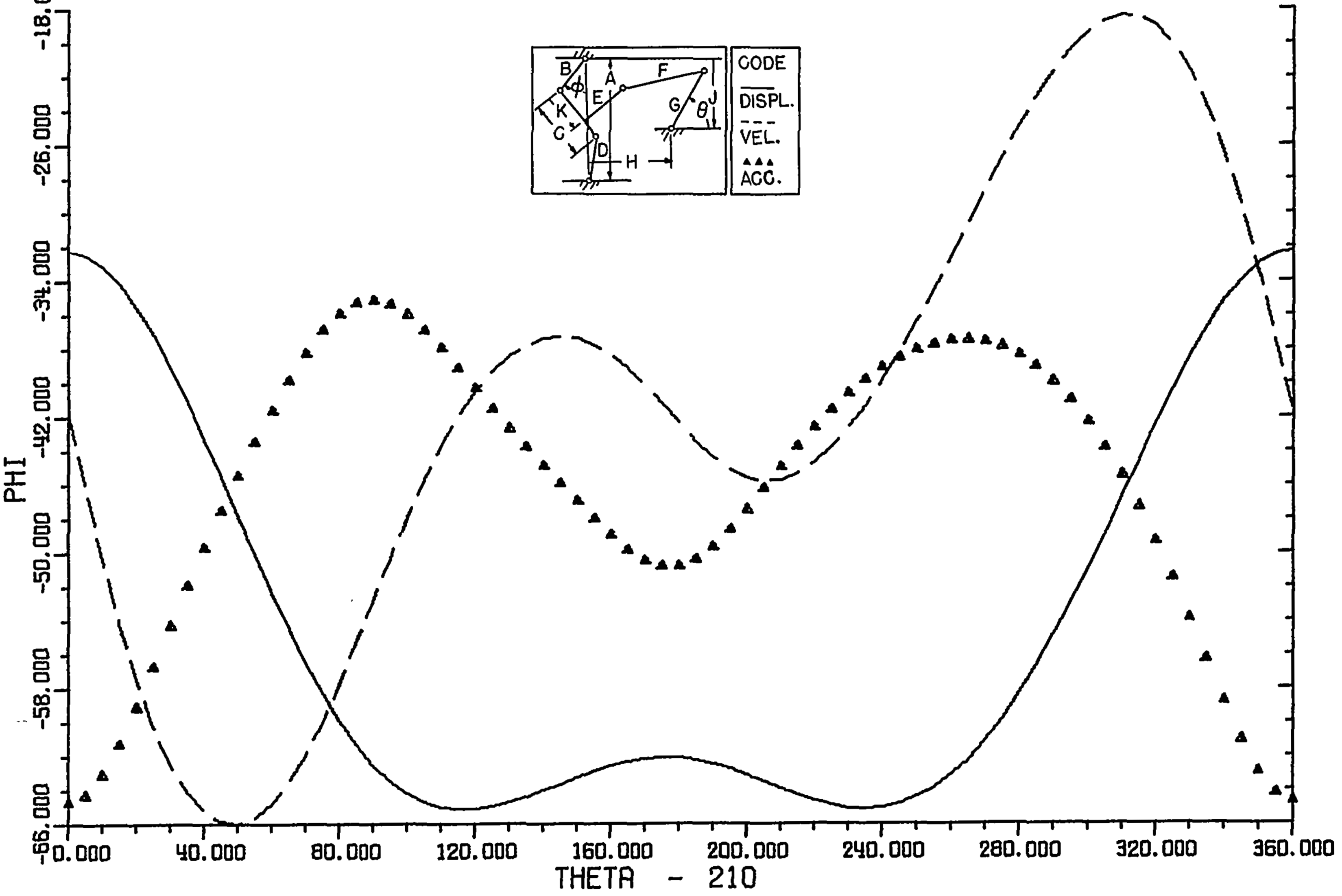


PLATE 5-11

A= 0.20, B= 0.80, C= 1.50, D= 0.80, E= 0.80,
F= 1.00, G= 0.50, H= 1.20, J= 1.00, K= 0.70,
VEL.MAX= 0.42, VEL.MIN= -0.32, ACC.MAX= 0.28, ACC.MIN= -0.74,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 36.04

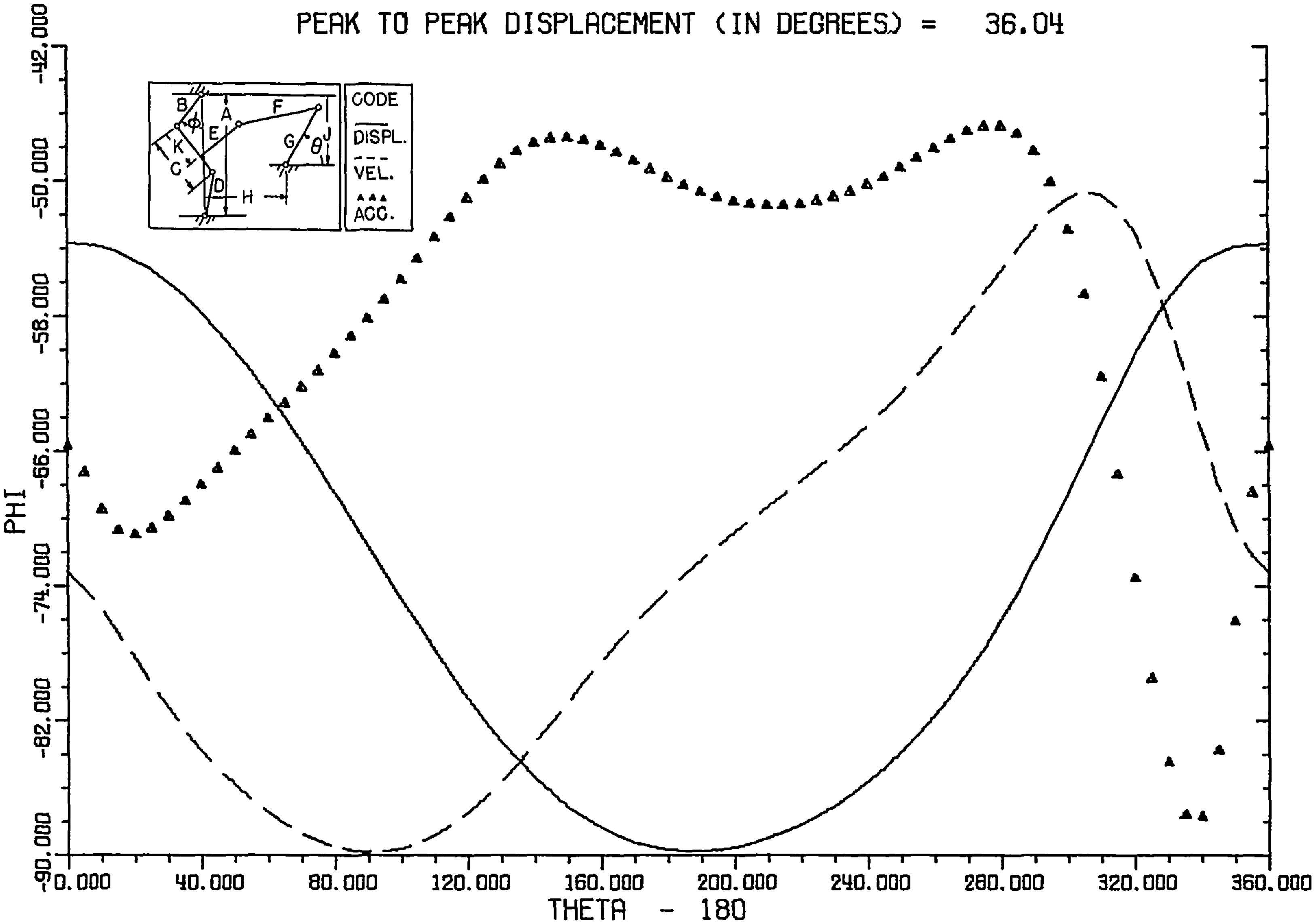


PLATE 5-12

A= 3.00, B= 1.20, C= 1.40, D= 1.00, E= 0.80,
F= 1.00, G= 0.60, H= 1.80, J= 1.40, K= 0.70,
VEL.MAX= 0.51, VEL.MIN= -0.46, ACC.MAX= 0.54, ACC.MIN= -1.26,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 36.14

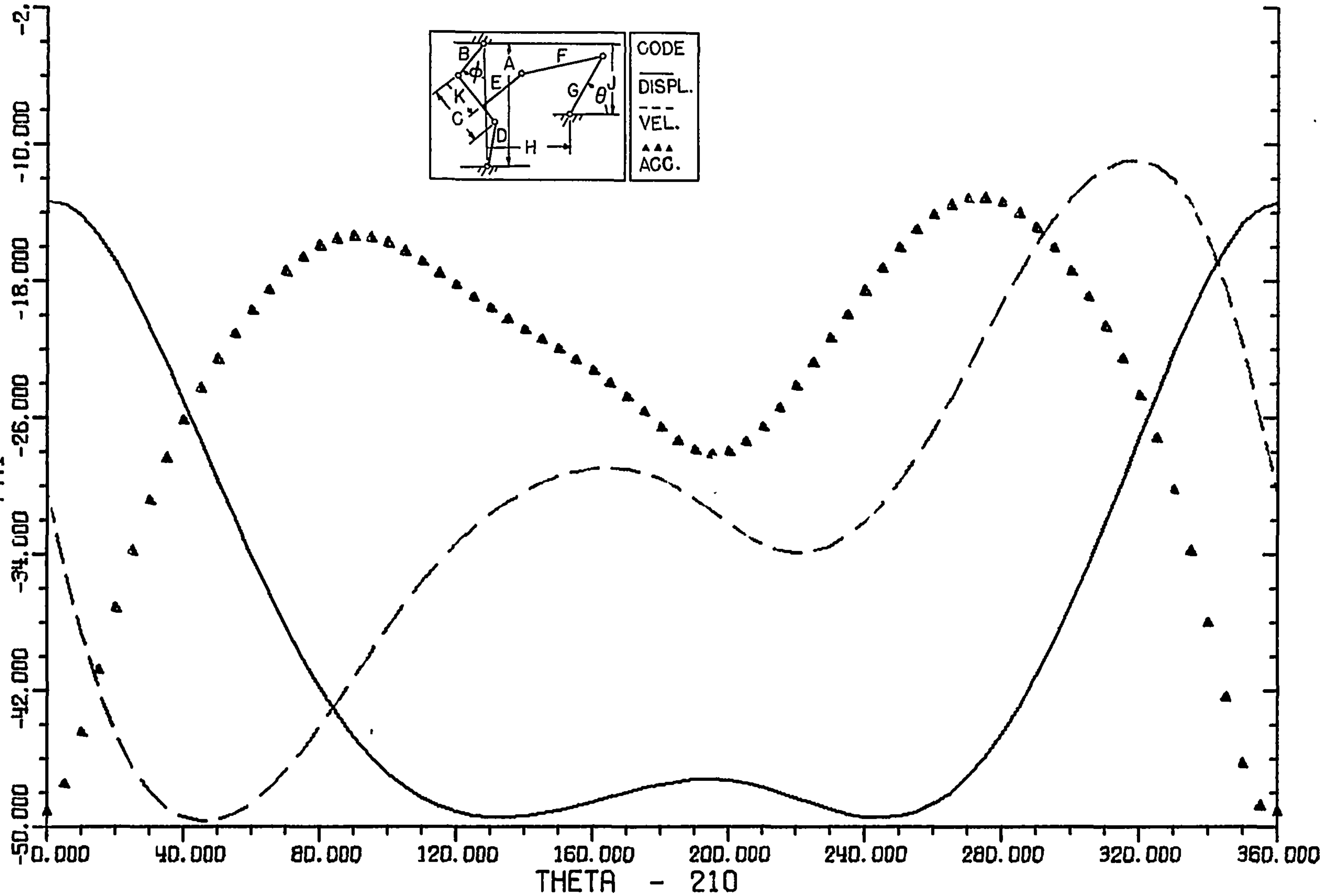


PLATE 5-13

A= 2.20, B= 1.00, C= 1.50, D= 1.00, E= 1.00,
F= 1.00, G= 0.50, H= 1.60, J= 1.00, K= 0.70,
VEL.MAX= 0.43, VEL.MIN= -0.40, ACC.MAX= 0.29, ACC.MIN= -0.87,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 42.58

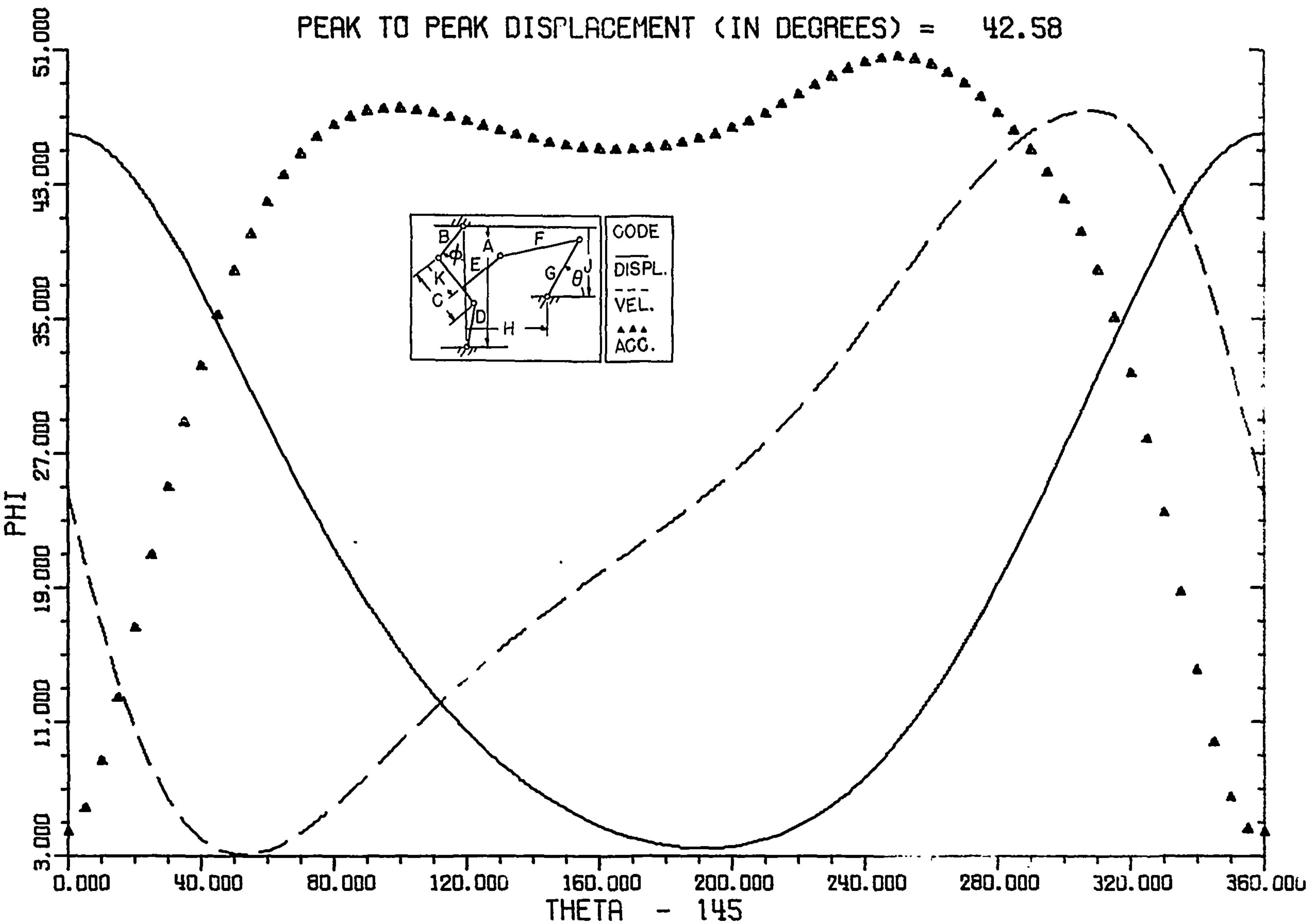


PLATE 5-14

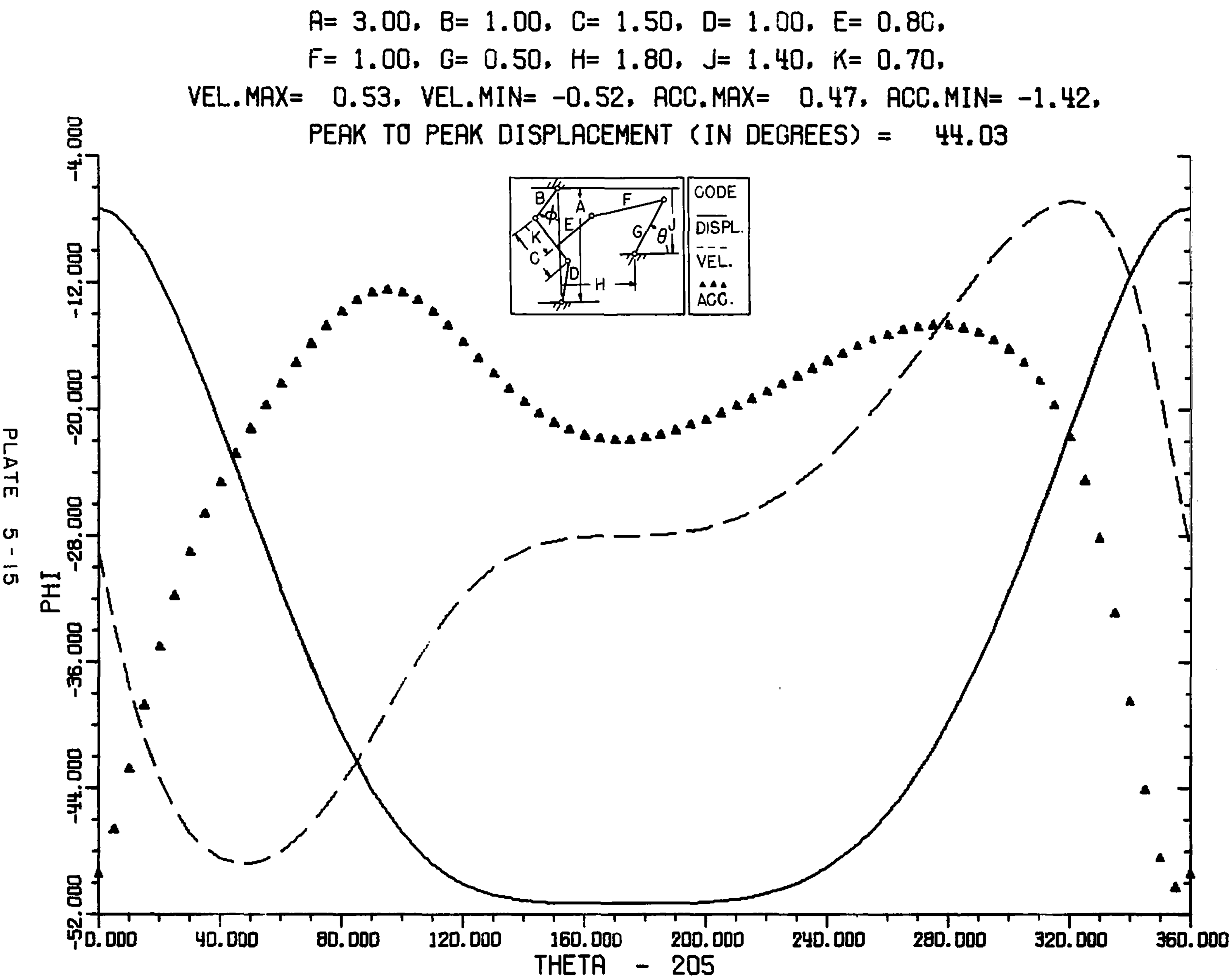

A= 3.00, B= 1.00, C= 1.50, D= 1.00, E= 0.80,
F= 1.00, G= 0.50, H= 1.80, J= 1.40, K= 0.70,
VEL.MAX= 0.53, VEL.MIN= -0.52, ACC.MAX= 0.47, ACC.MIN= -1.42,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 44.03
CODE
DISPL.
VEL.
ACC.
PHI
THETA - 205
-4.000
-12.000
-20.000
-28.000
-36.000
-44.000
-52.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 5-15

A= 2.00, B= 0.90, C= 1.50, D= 1.10, E= 0.80,
F= 1.20, G= 0.50, H= 1.90, J= 1.50, K= 0.70,
VEL.MAX= 0.39, VEL.MIN= -0.39, ACC.MAX= 0.39, ACC.MIN= -0.55,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 46.70

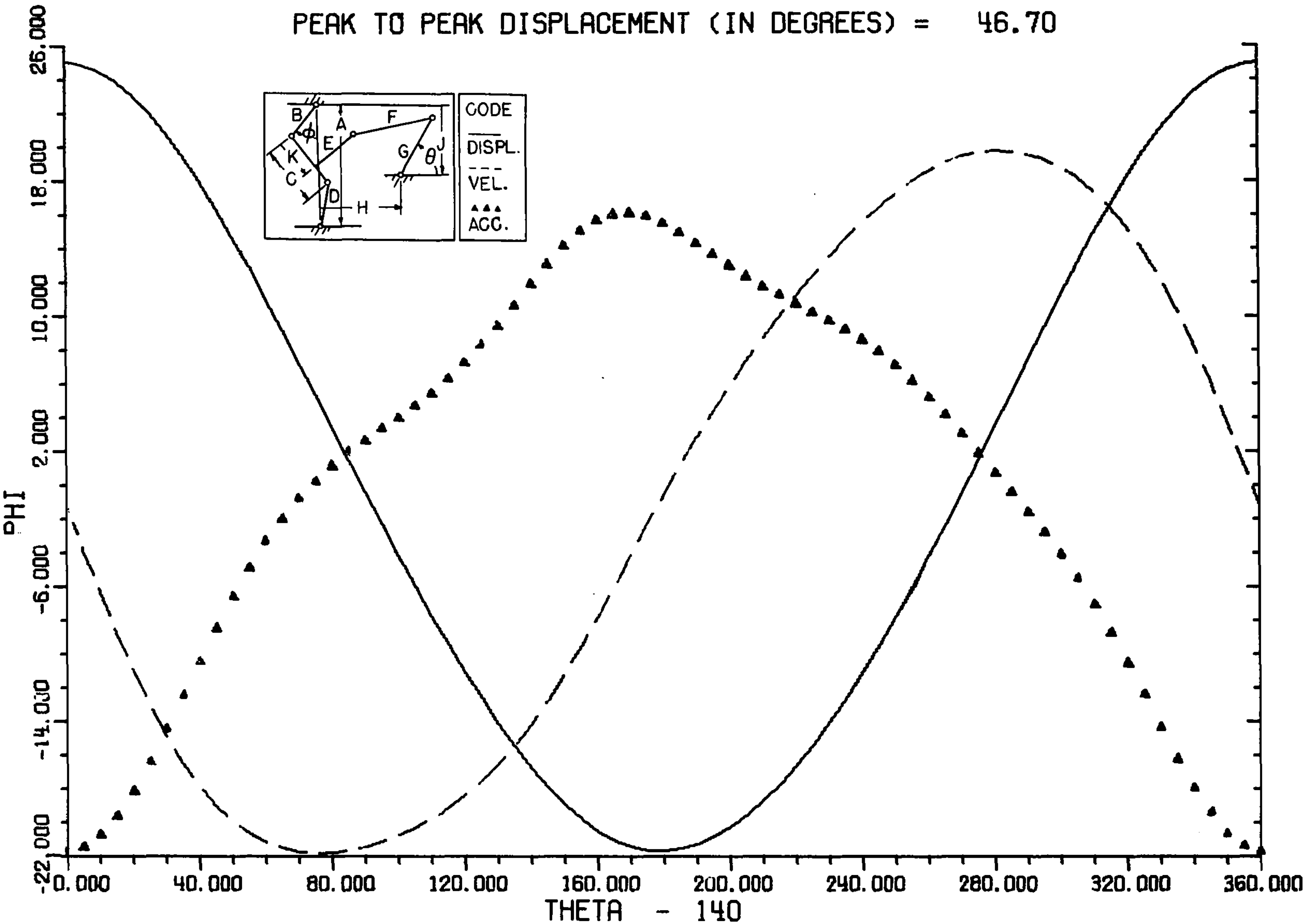


PLATE 5-16

A= 1.50, B= 0.80, C= 0.90, D= 0.90, E= 0.80,
F= 1.00, G= 0.50, H= 1.70, J= 1.40, K= 0.70,
VEL.MAX= 0.34, VEL.MIN= -0.45, ACC.MAX= 0.98, ACC.MIN= -0.49,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 50.61

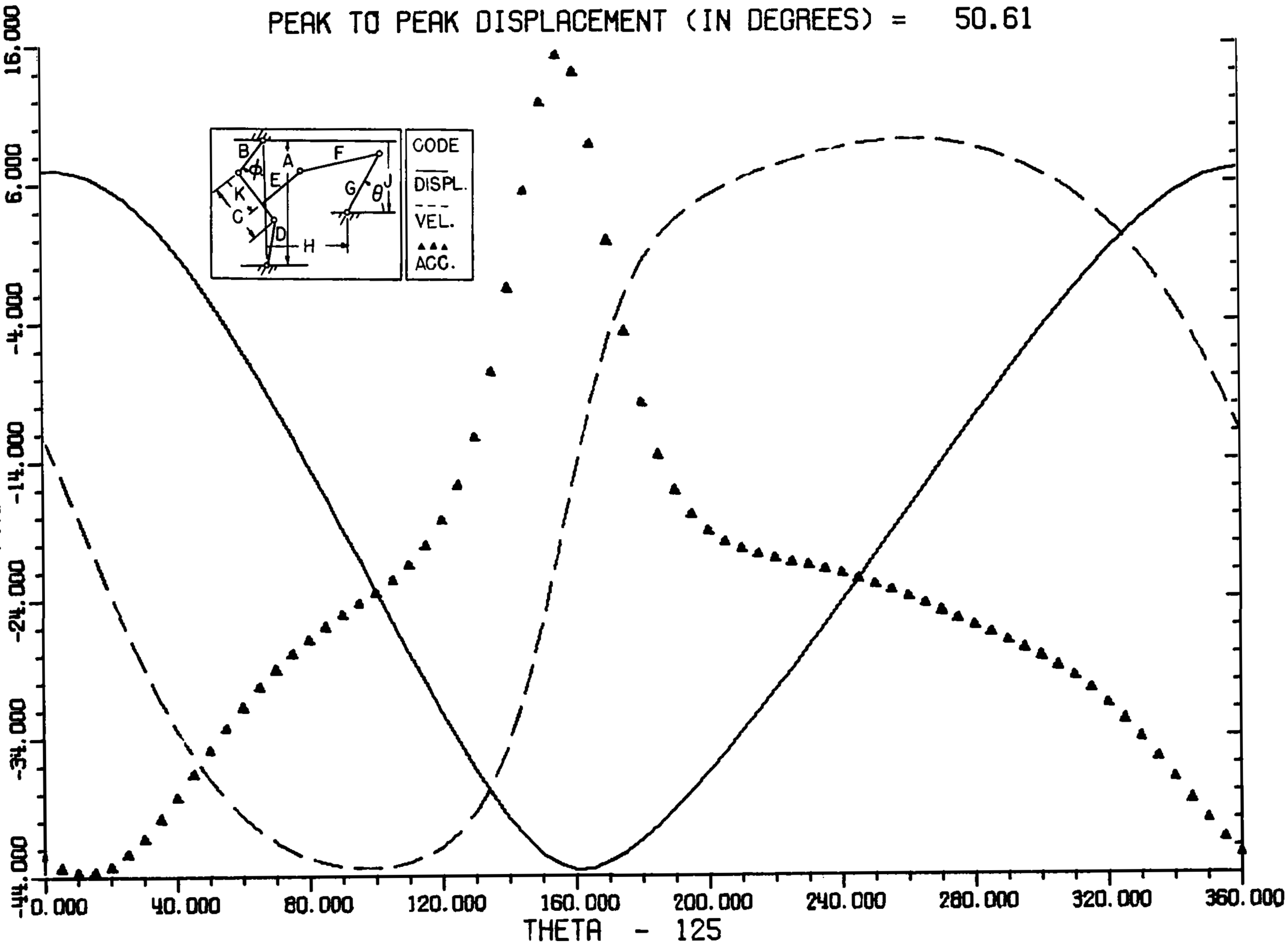


PLATE 5-17

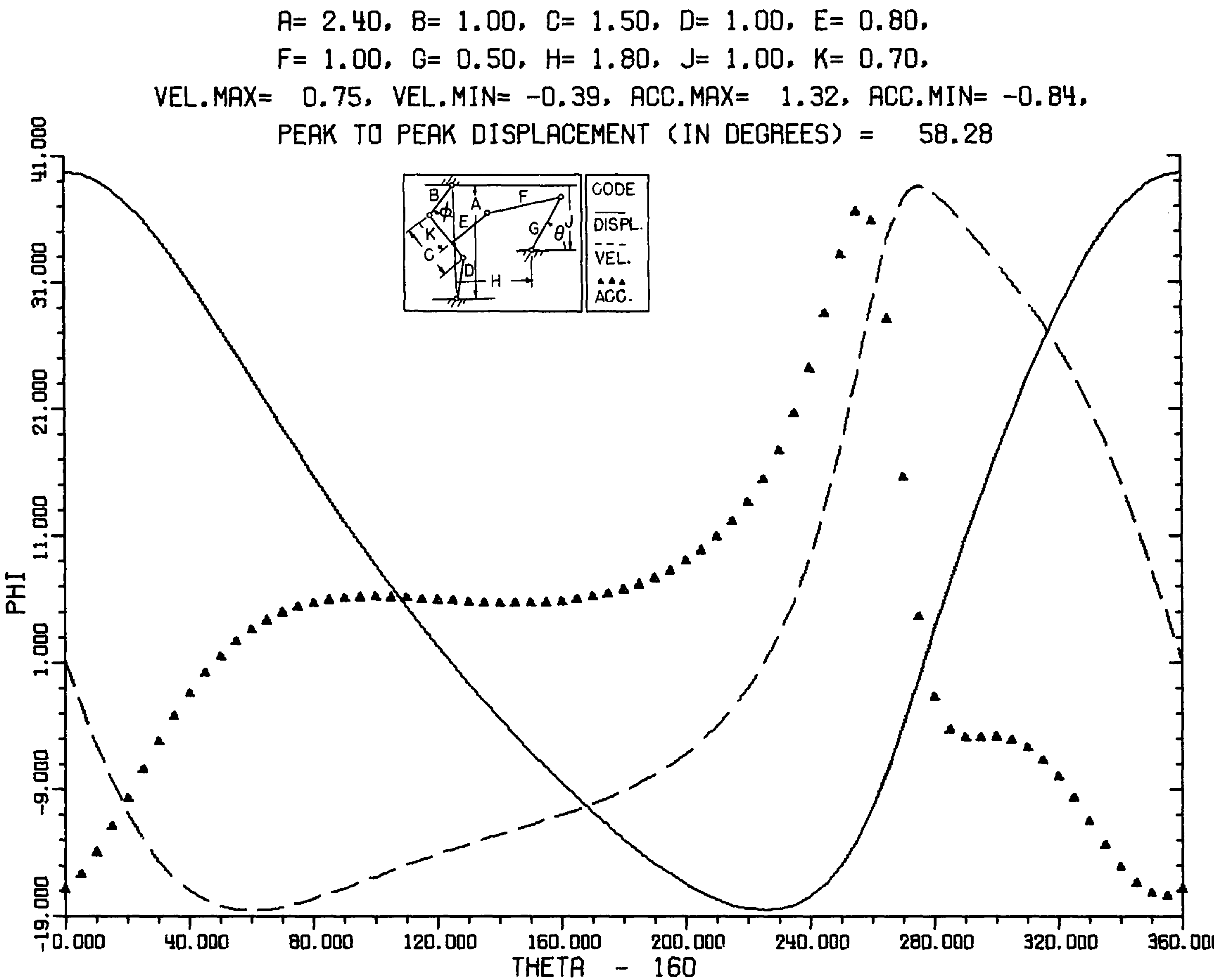

A= 2.40, B= 1.00, C= 1.50, D= 1.00, E= 0.80,
F= 1.00, G= 0.50, H= 1.80, J= 1.00, K= 0.70,
VEL.MAX= 0.75, VEL.MIN= -0.39, ACC.MAX= 1.32, ACC.MIN= -0.84,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 58.28
B
A
F
K
E
G
J
C
D
H
CODE
DISPL.
VEL.
ACC.
PHI
41.000
31.000
21.000
11.000
1.000
-9.000
-19.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 160

PLATE 5-18

MECHANISM #6

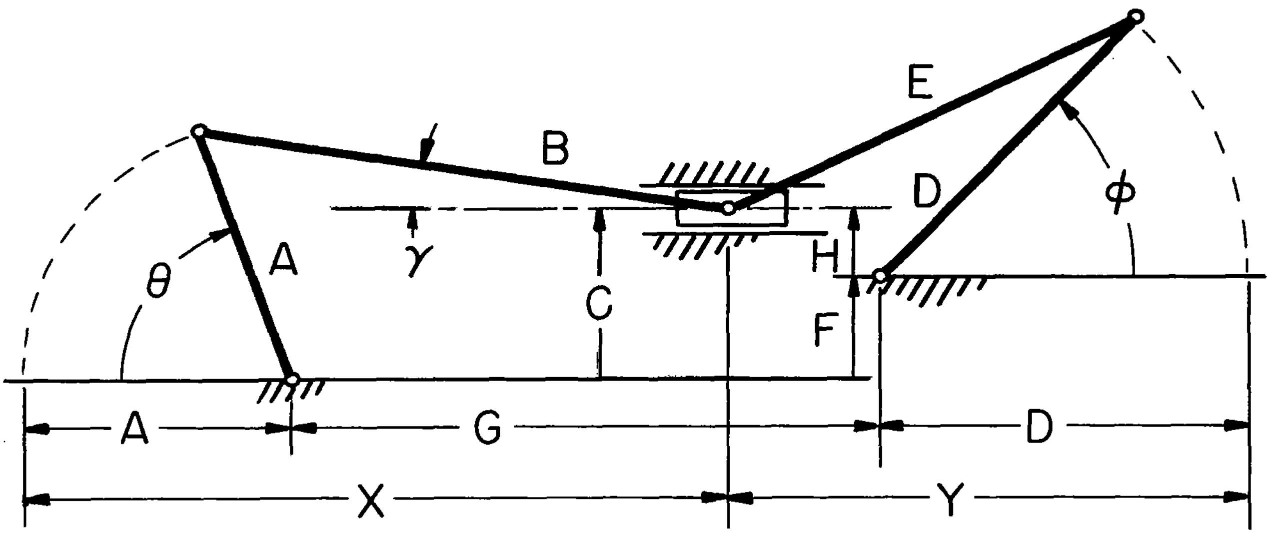


Figure 6-1

Figure 6-1 defines Mechanism #6 which may be viewed as a pair of slider-crank mechanisms joined back-to-back. The input is considered to be angular position of link A and the output the angular position of link D. Thus θ, the Greek letter theta, is the input and ϕ, phi, is the output and the solid lines in the series of graphs for this mechanism depict ϕ as a function of θ for specific combinations of parameters A, B, C, D, E, F, and G (the identity as well as the length of a link is noted by a capital letter).

Each graph shows ϕ versus θ as a solid line, the derivative of ϕ with respect to θ versus θ as a dashed line, and the second derivative of ϕ with respect to θ versus θ as a series of small triangles. Note that each graph has been shifted so that the displacement begins at its maximum value. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. Both ϕ and θ are presented in the units degrees.

Scales have not been presented for the derivatives but each graph heading includes the maximum and minimum angular velocities and accelerations which are usually most important to designers. Scales for the derivatives can be produced easily from the heading data. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement.

The points about which links A and D rotate are not necessarily on a line parallel to the direction of motion of the slider. The offset of these two points is shown as a distance F. One may consider this differently and say that the slider does not necessarily have motion parallel to the line of centers of the links A and D. The one viewpoint may be translated into the other at will.

Referring to Figure 6-1 of this mechanism, the following equations may be noted:

$$X = A\,(1 - \cos\theta) + B\cos\gamma$$

$$= A - A\cos\theta + [B^2 - (A\sin\theta - C)^2]^{1/2} \tag{6-1}$$

$$Y = D\,(1 - \cos\phi) + [E^2 - (D\sin\phi - H)^2]^{1/2}. \tag{6-2}$$

The center-to-center distance between the axes of rotation of links A and D, measured parallel to the direction of motion of the slider, is G and may be written:

$$G = X - A + Y - D. \tag{6-3}$$

Let $J = X - A - G$, which in terms of θ may be written

$$J = [B^2 - (A\sin\theta - C)^2]^{1/2} - A\cos\theta - G. \tag{6-4}$$

Combining the four equations yields:

$$D\cos\phi - J = [E^2 - (D\sin\phi - H)^2]^{1/2}. \tag{6-5}$$

Squaring Eq. 6-5 (and grouping terms calling them K) will produce:

$$K - J \cos \phi - H \sin \phi = 0 \tag{6-6}$$

in which

$$K = (D^2 + H^2 + J^2 - E^2)/(2D) . \tag{6-7}$$

Given the parameters which define the mechanism (A, B, C, D, E, F, and G) and a value of θ then the solution of Eq. 6-6 will produce a value for ϕ. If Eq. 6-6 is rewritten as

$$S = K - J \cos \phi - H \sin \phi \tag{6-8}$$

then ϕ which corresponds to a particular value of θ will imply that S equals zero. The solution to Eq. 6-8 (i.e. S = 0) may be obtained by Newton's method which reduces to the successive application of:

$$\phi_1 = \phi_0 - \left[\frac{K - J \cos \phi_0 - H \sin \phi_0}{J \sin \phi_0 - H \cos \phi_0}\right] . \tag{6-9}$$

In this equation ϕ_0 is an approximate solution of Eq. 6-8 (that makes S = 0) and ϕ_1 is a more exact value. If the original value, ϕ_0, is very close to the solution, Eq. 6-9 may need to be applied only a few times to obtain the value of ϕ which corresponds to θ.

Angular Velocity

To obtain the angular velocity, Eq. 6-6 may be differentiated with respect to θ:

$$\frac{dK}{d\theta} - \cos \phi \frac{dJ}{d\theta} + J \sin \phi \frac{d\phi}{d\theta} - H \cos \phi \frac{d\phi}{d\theta} = 0 . \tag{6-10}$$

From Eq. 6-10:

$$\frac{d\phi}{d\theta} = \frac{\cos\phi \dfrac{dJ}{d\theta} - \dfrac{dK}{d\theta}}{J \sin\phi - H\cos\phi} . \tag{6-11}$$

The appropriate derivatives may be determined from Eqs. 6-4 and 6-7:

$$\frac{dJ}{d\theta} = \frac{-(A\cos\theta)(A\sin\theta - C)}{[B^2 - (A\sin\theta - C)^2]^{1/2}} + A\sin\theta$$

$$\frac{dK}{d\theta} = \frac{J}{D} \times \frac{dJ}{d\theta}.$$

Eq. 6-11 with the two equations above and Eq. 6-4, which defines J, determine the angular velocity of the output, $d\phi/d\theta$, in the units of degrees/degree. A more conventional expression for angular velocity is obtained as:

$$\frac{d\phi}{dt} = \frac{d\phi}{d\theta} \times \frac{d\theta}{dt} \qquad \left[\frac{\text{radians}}{\text{second}} = \frac{\text{radians}}{\text{radian}} \times \frac{\text{radians}}{\text{second}}\right] \tag{6-12}$$

in which

$$\frac{d\theta}{dt} = \text{rpm} \times 2\pi \times \frac{1}{60}$$

$$\left[\frac{\text{radians}}{\text{second}} = \frac{\text{rev}}{\text{minute}} \times \frac{\text{radians}}{\text{rev}} \times \frac{\text{minute}}{\text{seconds}}\right].$$

Equation 6-12 may be written:

$$\frac{d\phi}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{d\phi}{d\theta}, \qquad \frac{\text{radians}}{\text{second}}. \tag{6-13}$$

In words, the angular velocity of link D (radians/second) is obtained as the product of $\pi/30$ times the angular speed of link A (revolutions/minute) and

$d\phi/d\theta$ (radians/ radian). Values for this latter term may be obtained from a graph (the dashed line), or from Eq. 6-11, or from the heading of a graph as VEL. MAX or VEL. MIN.

Angular Acceleration

The angular acceleration of the output link D may be obtained by differentiating Eq. 6-10 with respect to θ with the result:

$$\frac{d^2K}{d\theta^2} - \cos\phi \frac{d^2J}{d\theta^2} + \sin\phi \frac{d\phi}{d\theta} \times \frac{dJ}{d\theta} + J \sin\phi \frac{d^2\phi}{d\theta^2}$$

$$+ J \cos\phi \left[\frac{d\phi}{d\theta}\right]^2 + \sin\phi \frac{dJ}{d\theta} \times \frac{d\phi}{d\theta} + H \sin\phi \left[\frac{d\phi}{d\theta}\right]^2$$

$$- H \cos\phi \frac{d^2\phi}{d\theta^2} = 0. \qquad (6\text{-}14)$$

This may be rearranged as:

$$\frac{d^2\phi}{d\theta^2} = \left\{ \cos\phi \frac{d^2J}{d\theta^2} - \frac{d^2K}{d\theta^2} - 2 \sin\phi \frac{d\phi}{d\theta} \frac{dJ}{d\theta} \right.$$

$$\left. - \left[J \cos\phi + H \sin\phi\right] \left[\frac{d\phi}{d\theta}\right]^2 \right\} \Big/ (J \sin\phi - H \cos\phi) \qquad (6\text{-}15)$$

in which

$$\frac{d^2K}{d\theta^2} = \frac{1}{D} \left[J \frac{d^2J}{d\theta^2} + \left(\frac{dJ}{d\theta}\right)^2 \right]$$

$$\frac{d^2J}{d\theta^2} = \left[\frac{A^2}{2} \sin 2\theta - A \cos\theta\right] \left[\frac{M}{N^2}\right] + A \cos\theta$$

$$- \left[A^2 \cos 2\theta - AC \sin\theta\right] \left[\frac{1}{N}\right]$$

$$M = \frac{(A \cos \theta)(C - A \sin \theta)}{N}$$

$$N = [B^2 - (A \sin \theta - C)^2]^{1/2} .$$

After ϕ has been found for a particular value of θ (using several iterations of Eq. 6-9) then the angular velocity of link D may be found from Eq. 6-11. Following the evaluation of this equation the angular acceleration of the output link, D, may be obtained by means of Eq. 6-15. The evaluation of these equations has produced the several graphs which follow.

The angular acceleration of link D may be given in more conventional engineering terms as:

$$\frac{d^2\phi}{dt^2} = \frac{d}{dt}\left[\frac{d\phi}{d\theta} \times \frac{d\theta}{dt}\right]$$

$$= \frac{d\phi}{d\theta} \times \frac{d^2\theta}{dt^2} + \frac{d^2\phi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2 . \qquad (6\text{-}16)$$

If the angular speed of link A remains constant, then $d^2\theta/dt^2$ equals zero. The expression for the angular acceleration of the output link, with the input link turning with constant speed, simplifies to:

$$\frac{d^2\phi}{dt^2} = \frac{d^2\phi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2$$

$$= \frac{d^2\phi}{d\theta^2}\left[\frac{\pi}{30} \times \text{rpm}\right]^2 , \quad \frac{\text{radians}}{\text{second}^2} . \qquad (6\text{-}17)$$

The term $d^2\phi/d\theta^2$ may be obtained from a graph (the series of small triangles), or from Eq. 6-15, or the extreme values may be noted in the heading of a graph as ACC. MAX or ACC. MIN.

A= 1.00, B=16.00, C= 4.00, D= 4.00,
E= 6.00, F= 1.00, G=10.00,
VEL.MAX= 0.27, VEL.MIN= -0.26, ACC.MAX= 0.27, ACC.MIN= -0.28,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 30.65

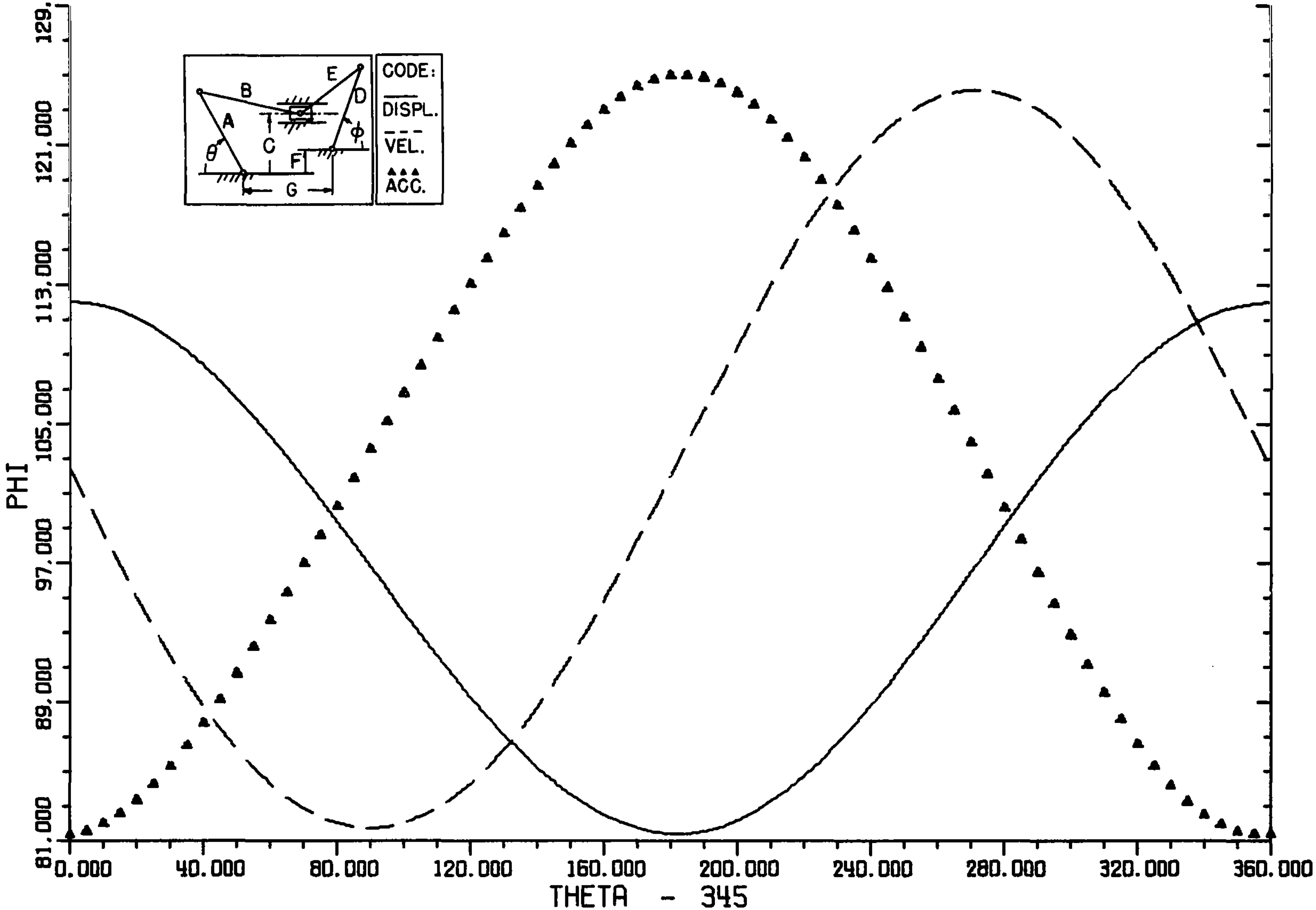


PLATE 6-1

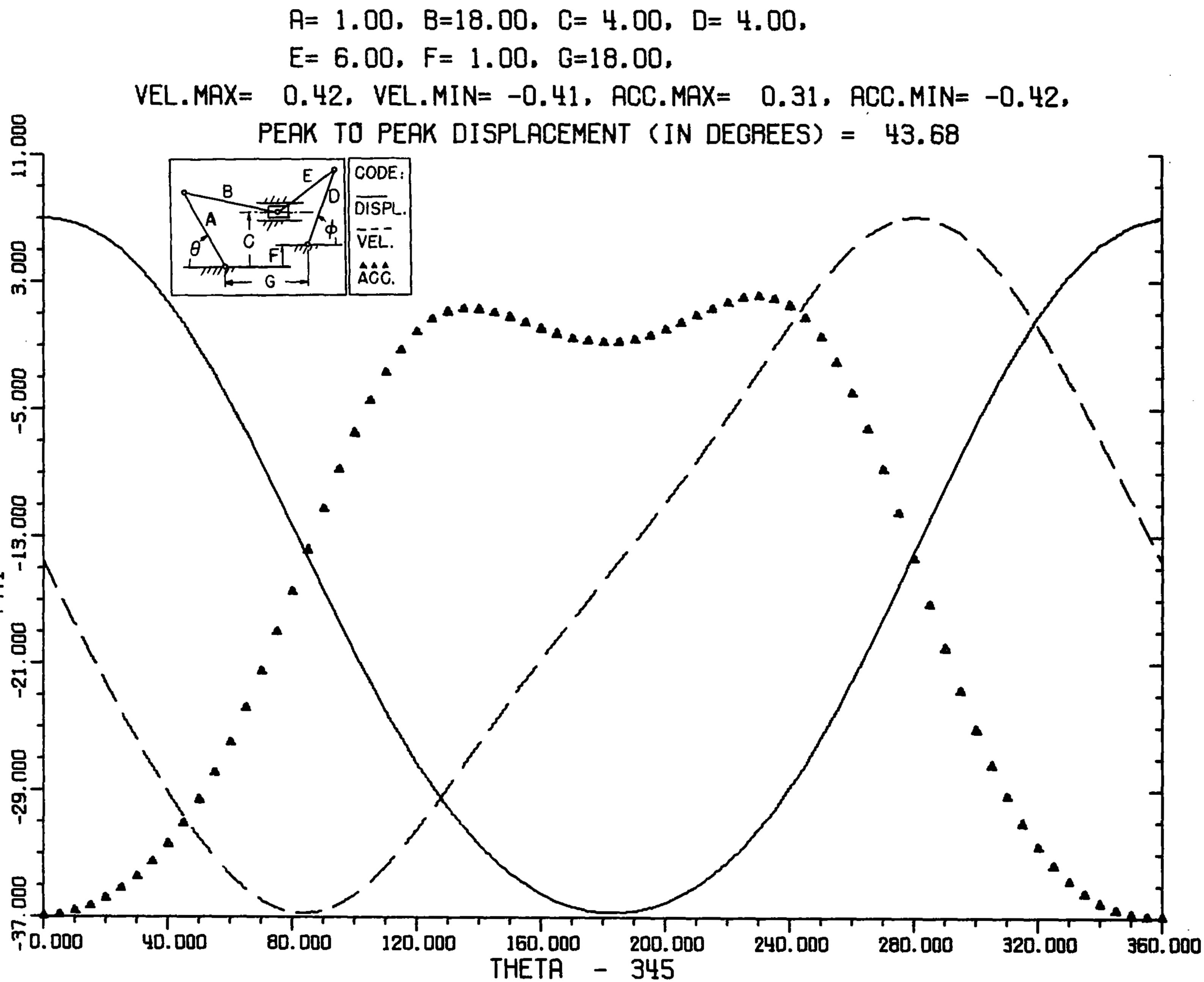

A= 1.00, B=18.00, C= 4.00, D= 4.00,
E= 6.00, F= 1.00, G=18.00,
VEL.MAX= 0.42, VEL.MIN= -0.41, ACC.MAX= 0.31, ACC.MIN= -0.42,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 43.68
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
THETA - 345
11.000
3.000
-5.000
-13.000
-21.000
-29.000
-37.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 6-2

A= 1.00, B=10.00, C= 8.00, D= 4.00,
E= 8.00, F= 2.00, G=10.00,
VEL.MAX= 0.49, VEL.MIN= -0.37, ACC.MAX= 0.42, ACC.MIN= -0.52,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 48.40

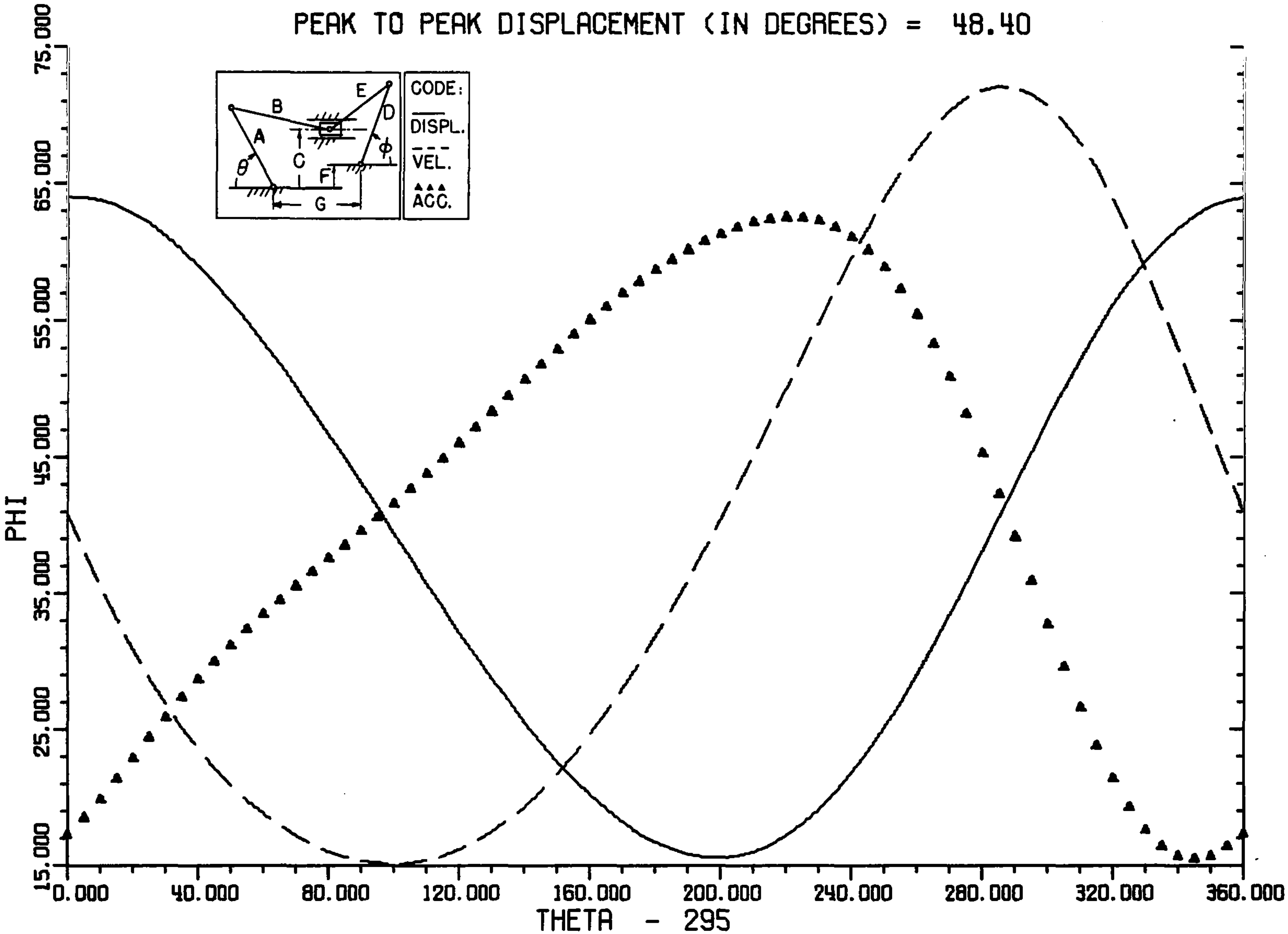


PLATE 6-3

A= 2.00, B= 6.00, C= 2.00, D= 4.00,
E= 4.00, F= 1.00, G=10.00,
VEL.MAX= 0.50, VEL.MIN= -0.41, ACC.MAX= 0.79, ACC.MIN= -0.43,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 53.01

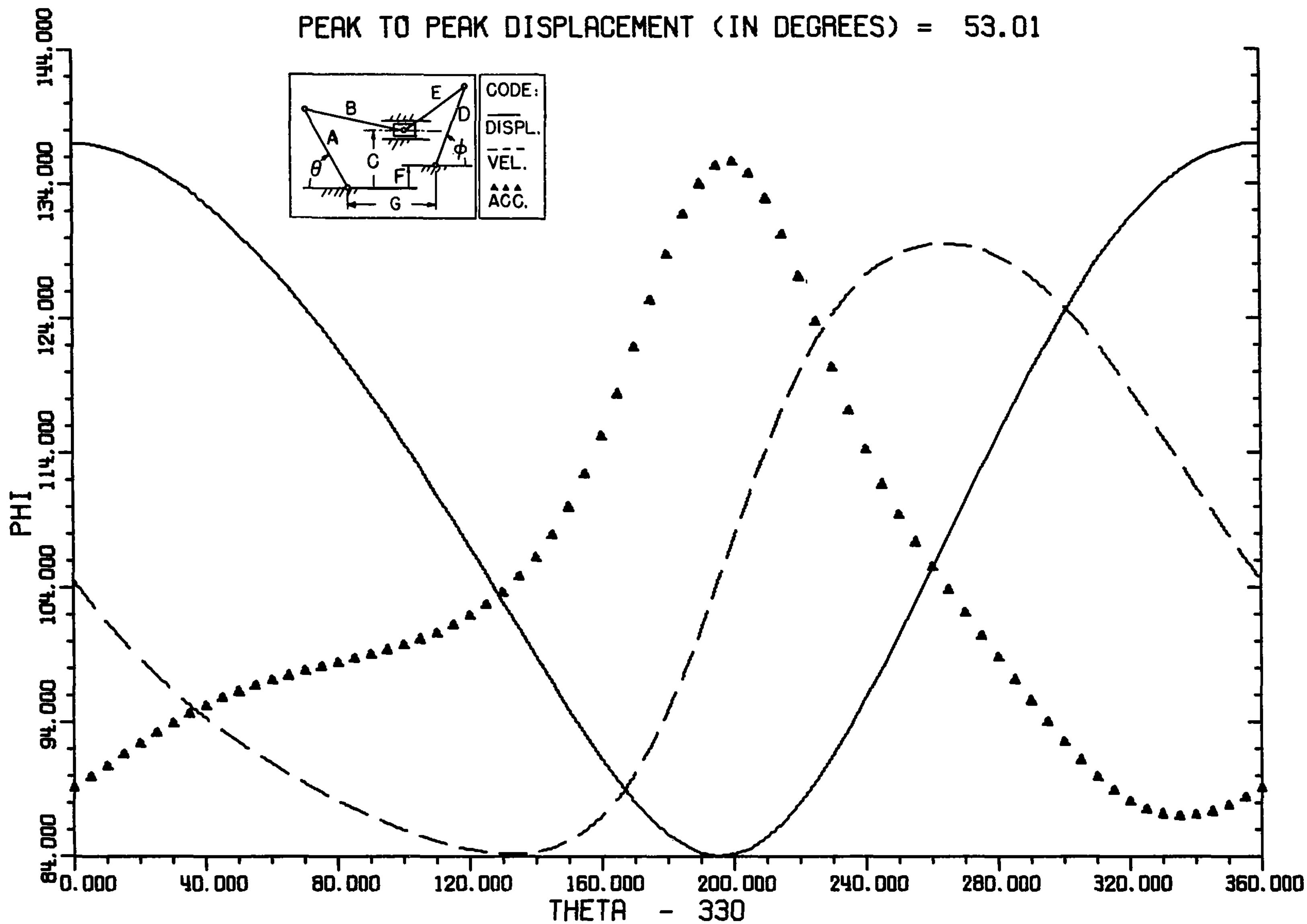


PLATE 6-4

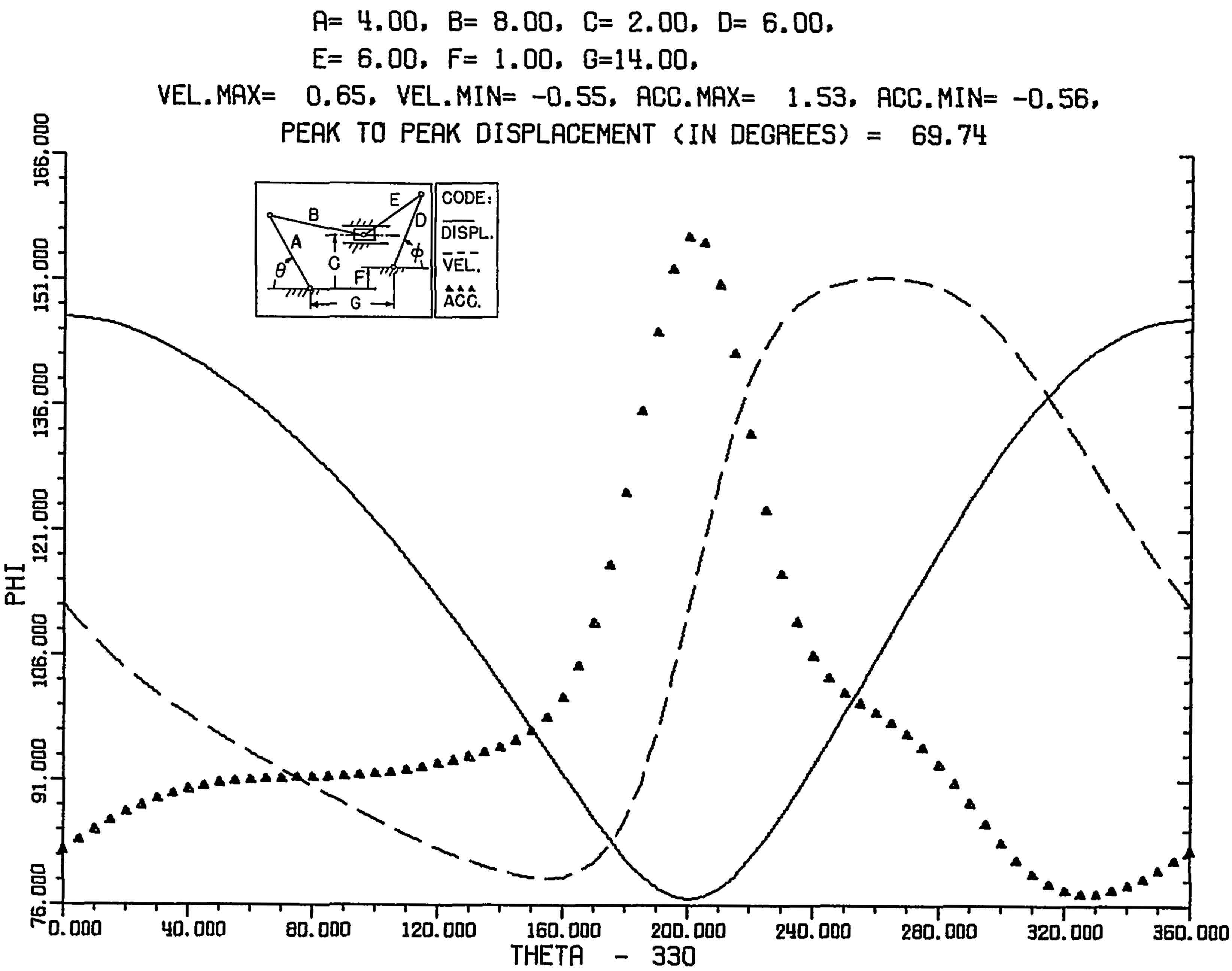

A= 4.00, B= 8.00, C= 2.00, D= 6.00,
E= 6.00, F= 1.00, G=14.00,
VEL.MAX= 0.65, VEL.MIN= -0.55, ACC.MAX= 1.53, ACC.MIN= -0.56,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 69.74
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
76.000
91.000
106.000
121.000
136.000
151.000
166.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 330

PLATE 6-5

A= 5.00, B=14.00, C= 4.00, D= 9.00,
E=11.00, F= 1.00, G=10.00,
VEL.MAX= 1.44, VEL.MIN= -1.12, ACC.MAX= 2.82, ACC.MIN= -2.05,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 70.48

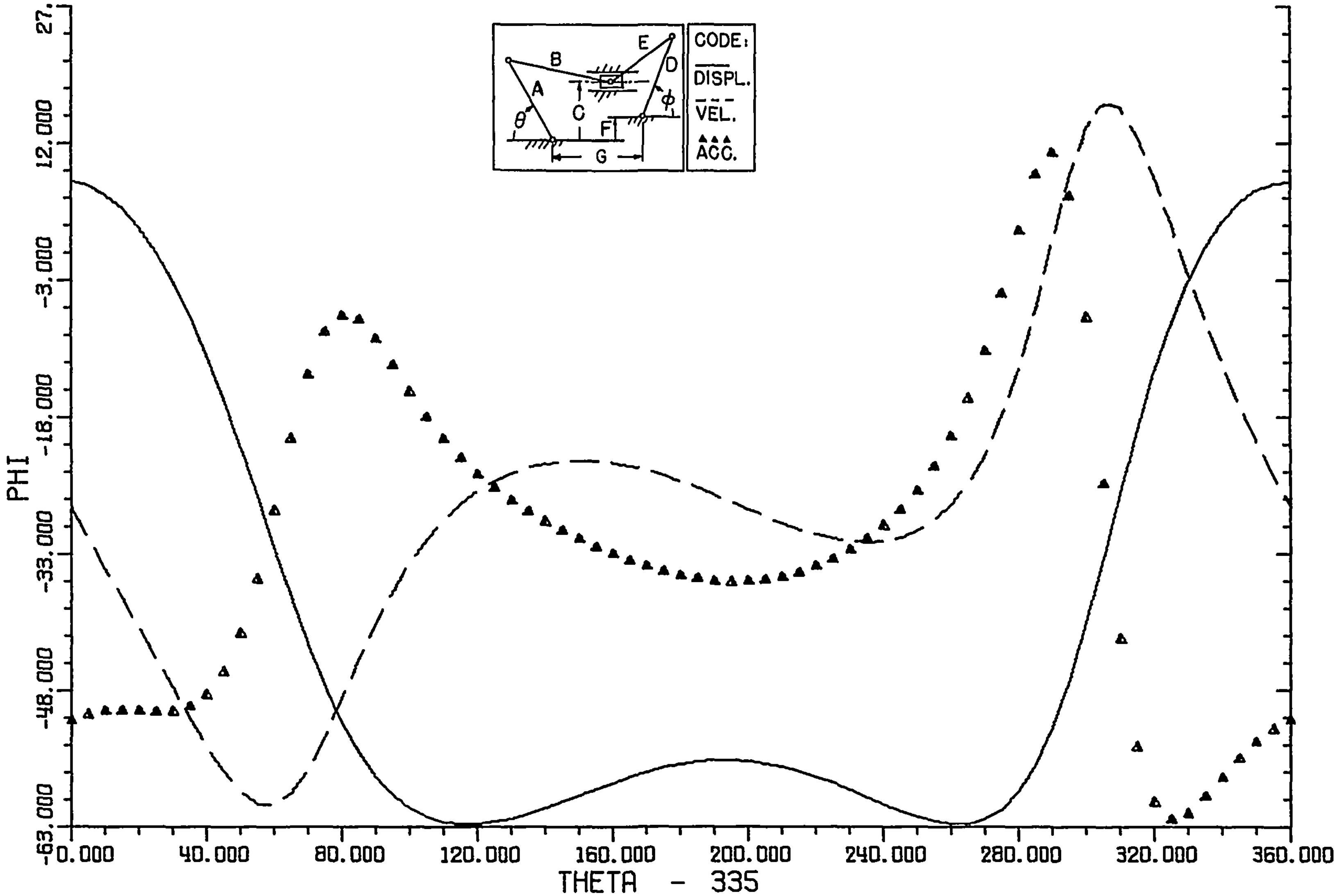
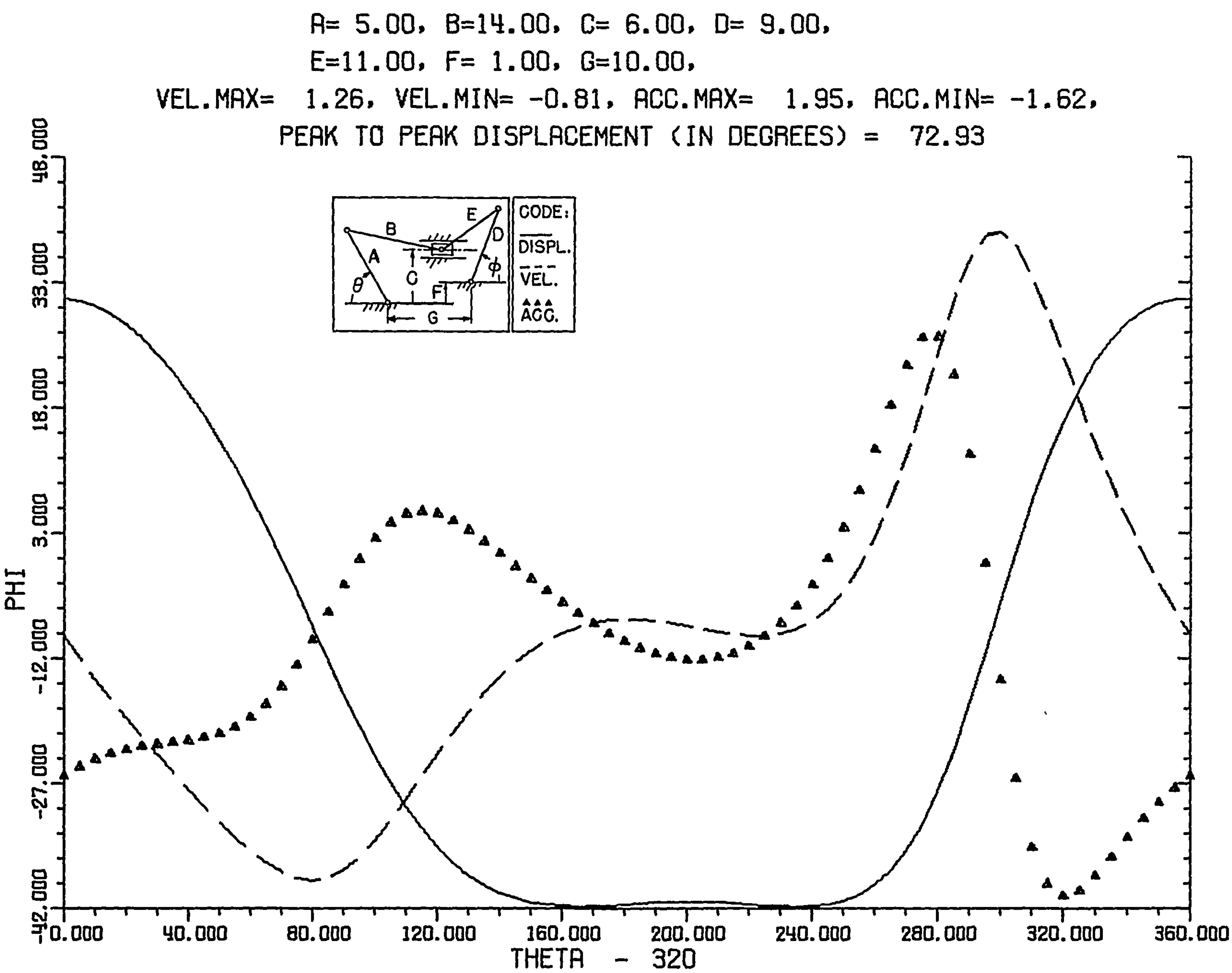


PLATE 6-6



A= 5.00, B=14.00, C= 6.00, D= 9.00,
E=11.00, F= 1.00, G=10.00,
VEL.MAX= 1.26, VEL.MIN= -0.81, ACC.MAX= 1.95, ACC.MIN= -1.62,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 72.93
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
48.000
33.000
18.000
3.000
-12.000
-27.000
-42.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 320

PLATE 6-7

A= 6.00, B=15.00, C= 8.00, D= 9.00,
E=13.00, F= 1.00, G=10.00,
VEL.MAX= 1.74, VEL.MIN= -0.79, ACC.MAX= 3.27, ACC.MIN= -2.93,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 80.21

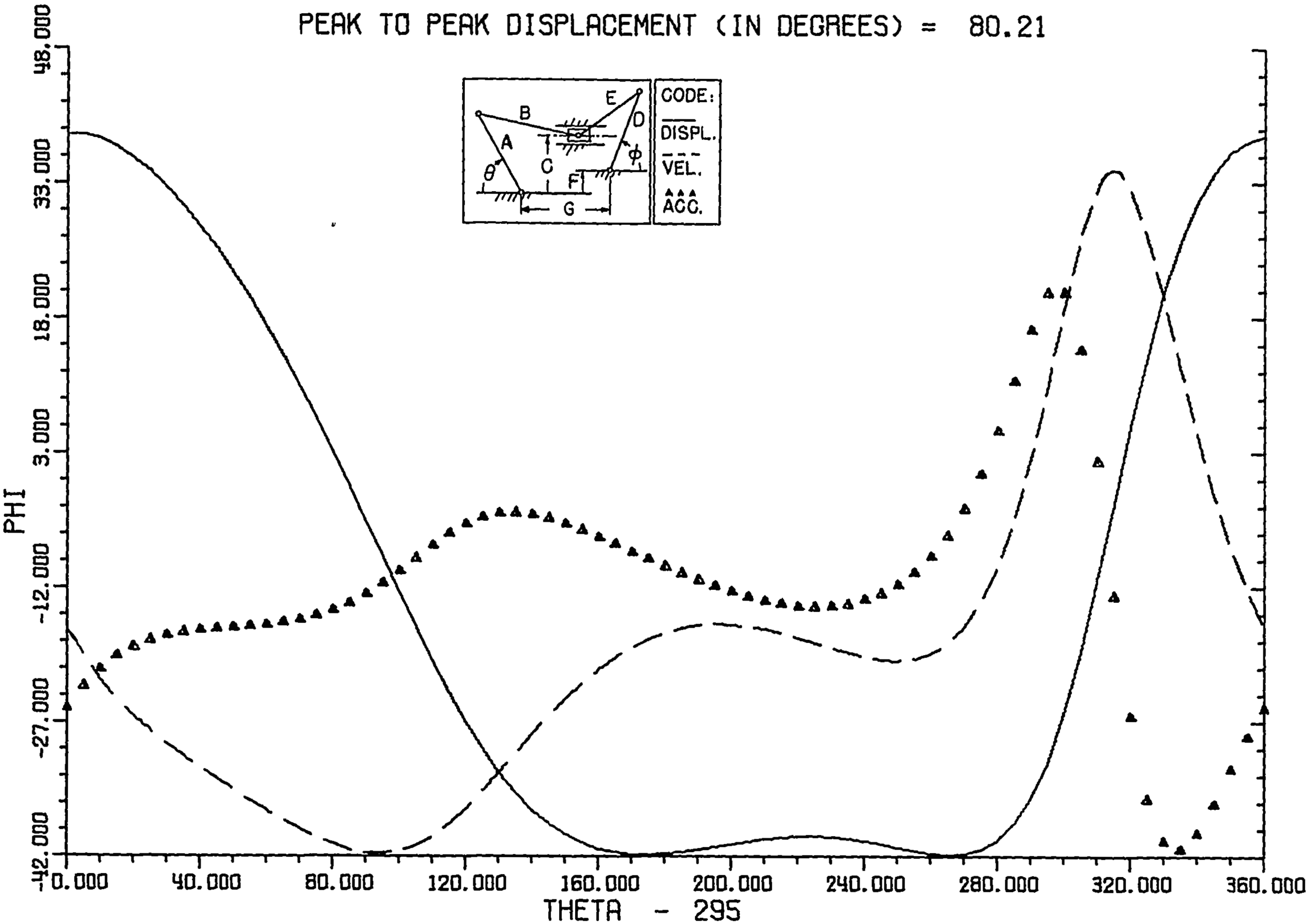


PLATE 6-8

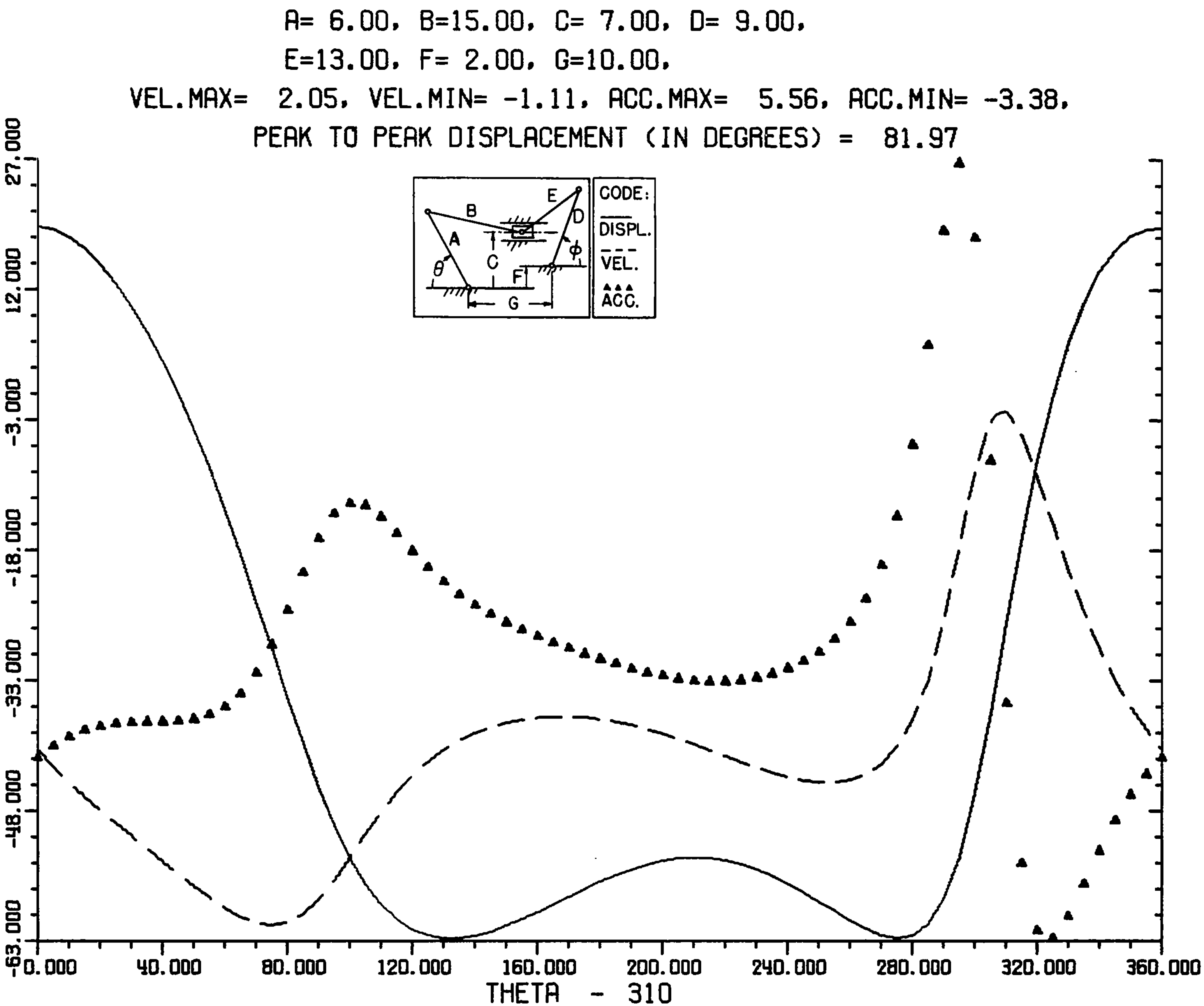

A= 6.00, B=15.00, C= 7.00, D= 9.00,
E=13.00, F= 2.00, G=10.00,
VEL.MAX= 2.05, VEL.MIN= -1.11, ACC.MAX= 5.56, ACC.MIN= -3.38,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 81.97
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
27.000
12.000
-3.000
-18.000
-33.000
-48.000
-63.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 310

PLATE 6-9

A= 6.00, B=15.00, C= 8.00, D= 9.00,
E=13.00, F= 2.00, G=10.00,
VEL.MAX= 2.12, VEL.MIN= -0.96, ACC.MAX= 4.83, ACC.MIN= -3.63,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 88.75

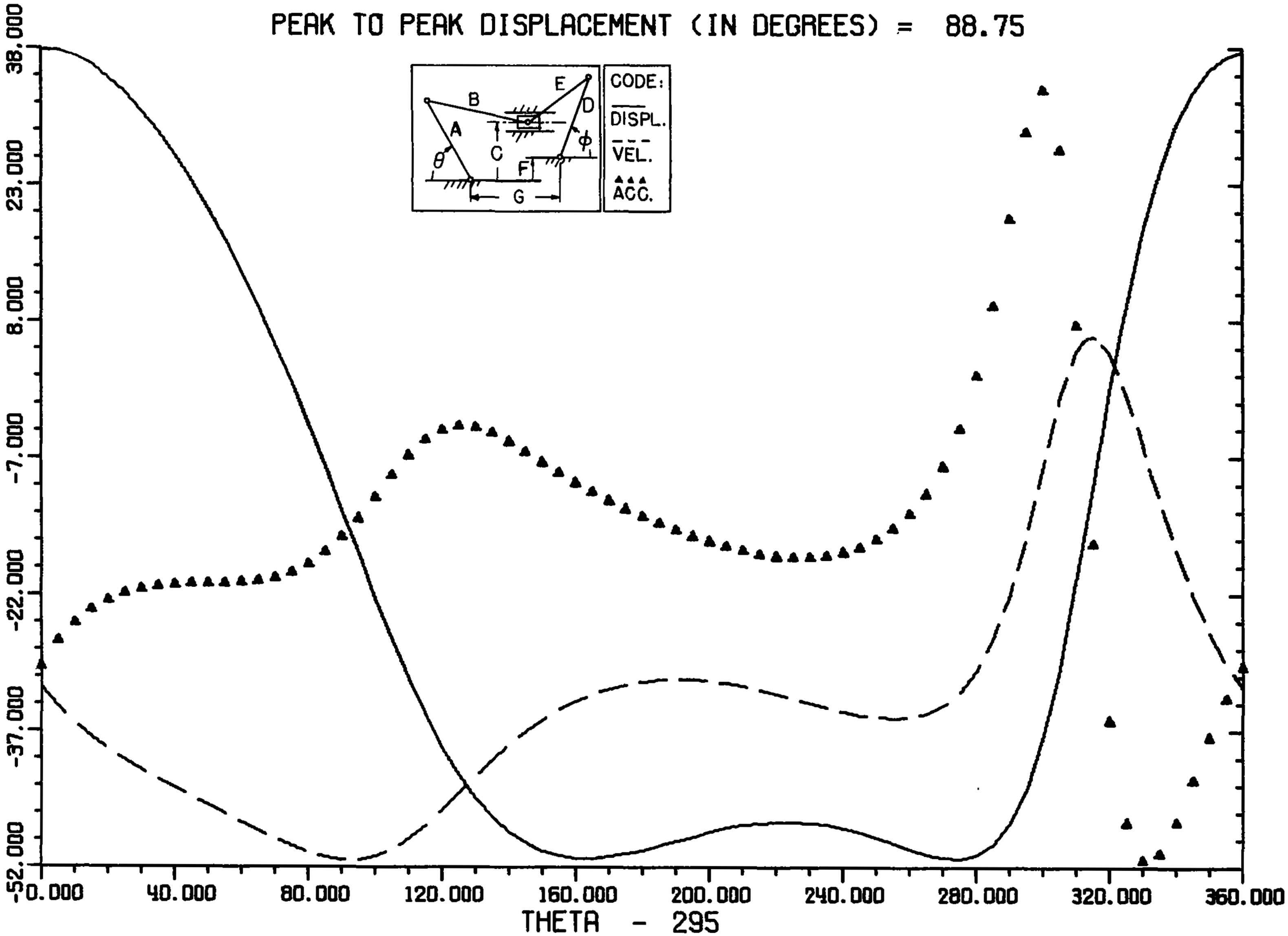


PLATE 6-10

A= 4.00, B=12.00, C= 7.00, D= 7.00,
E= 9.00, F= 1.00, G=10.00,
VEL.MAX= 1.34, VEL.MIN= -0.74, ACC.MAX= 1.73, ACC.MIN= -1.71,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 89.10

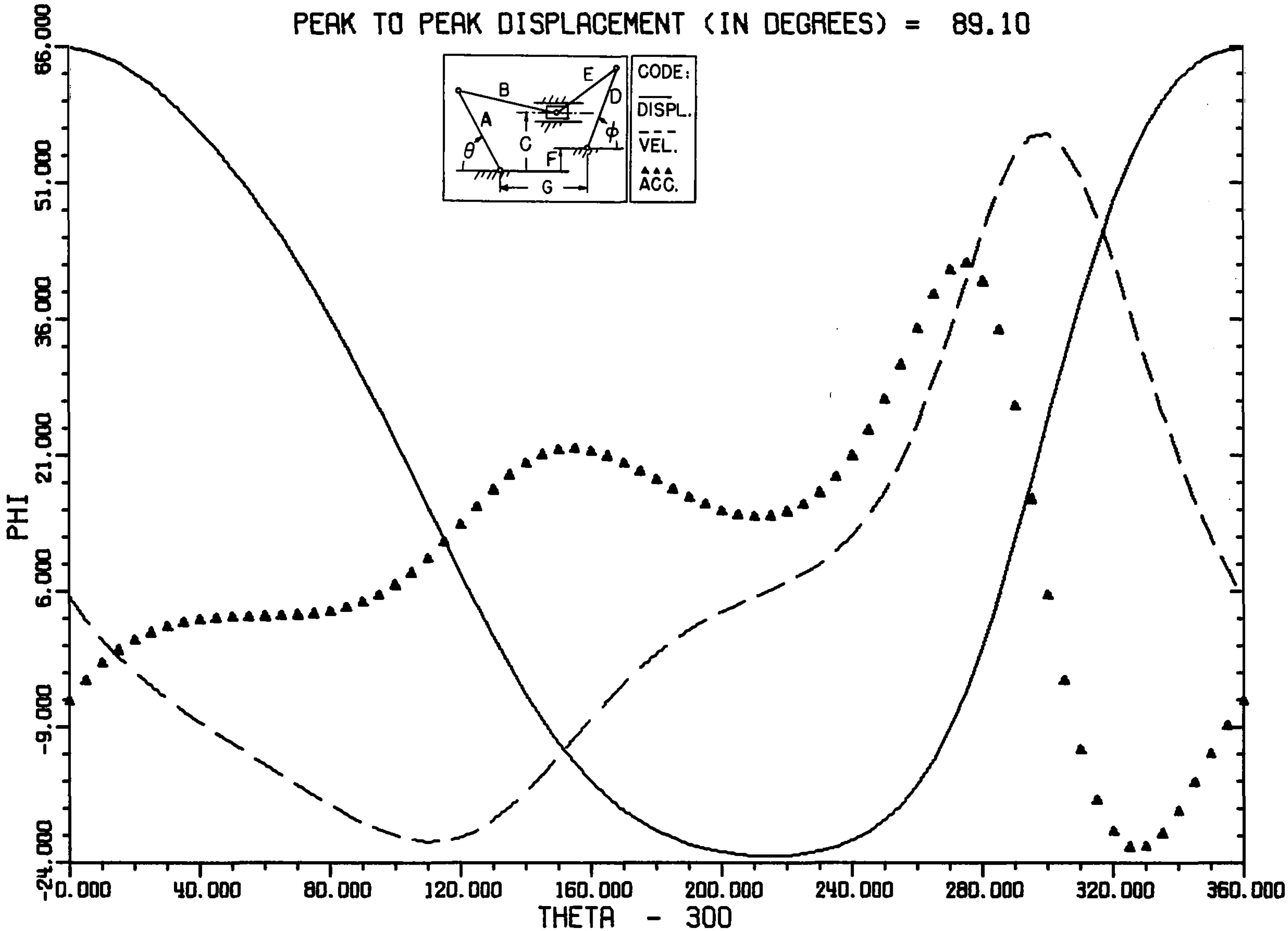


PLATE 6-11

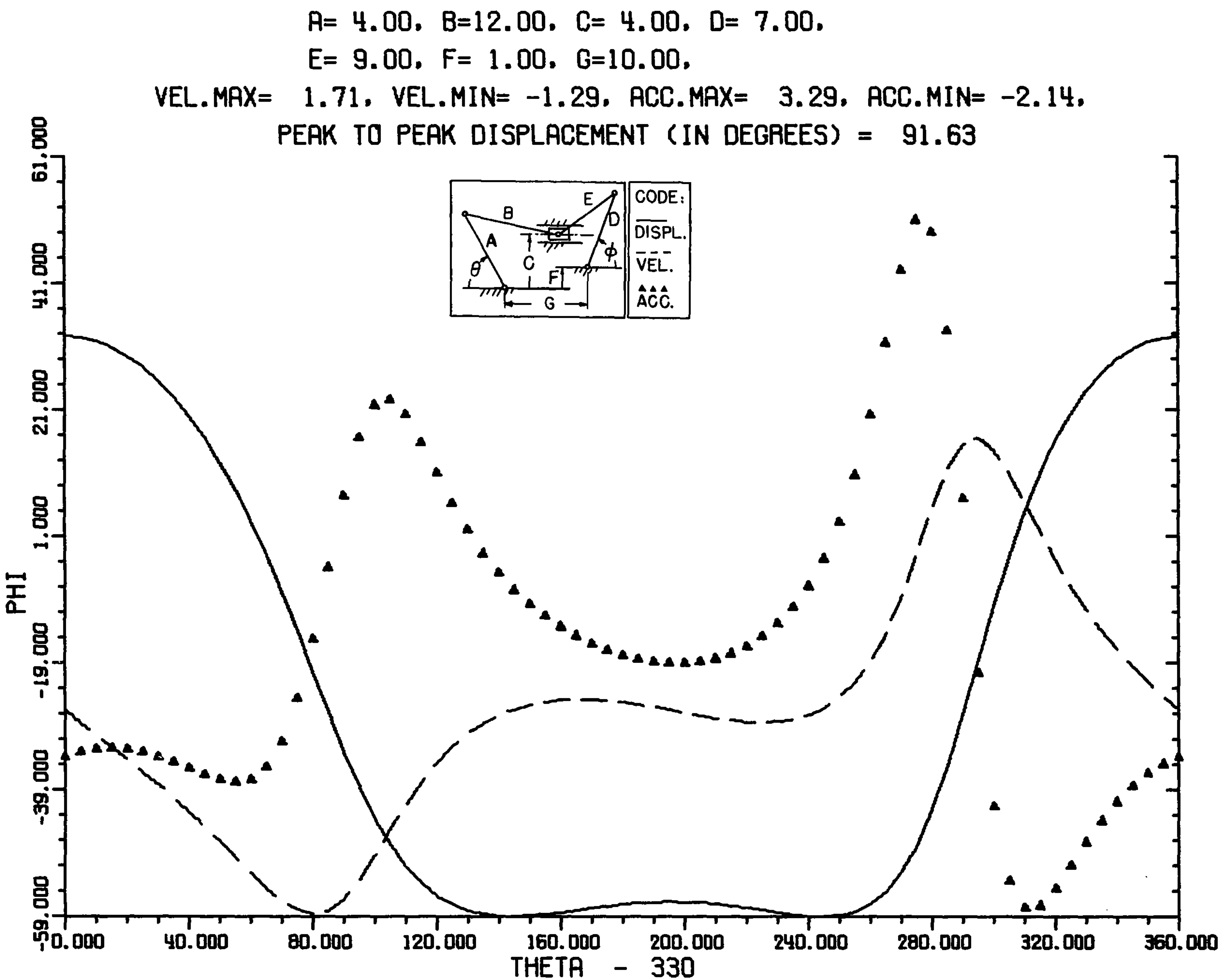

A= 4.00, B=12.00, C= 4.00, D= 7.00,
E= 9.00, F= 1.00, G=10.00,
VEL.MAX= 1.71, VEL.MIN= -1.29, ACC.MAX= 3.29, ACC.MIN= -2.14,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 91.63
CODE:
DISPL.
VEL.
ACC.
PHI
THETA - 330
61.000
41.000
21.000
1.000
-19.000
-39.000
-59.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 6 - 12

A= 2.00, B= 8.00, C= 2.00, D= 4.00,
E= 4.00, F= 1.00, G=10.00,
VEL.MAX= 1.13, VEL.MIN= -1.06, ACC.MAX= 2.52, ACC.MIN= -0.84,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 92.09

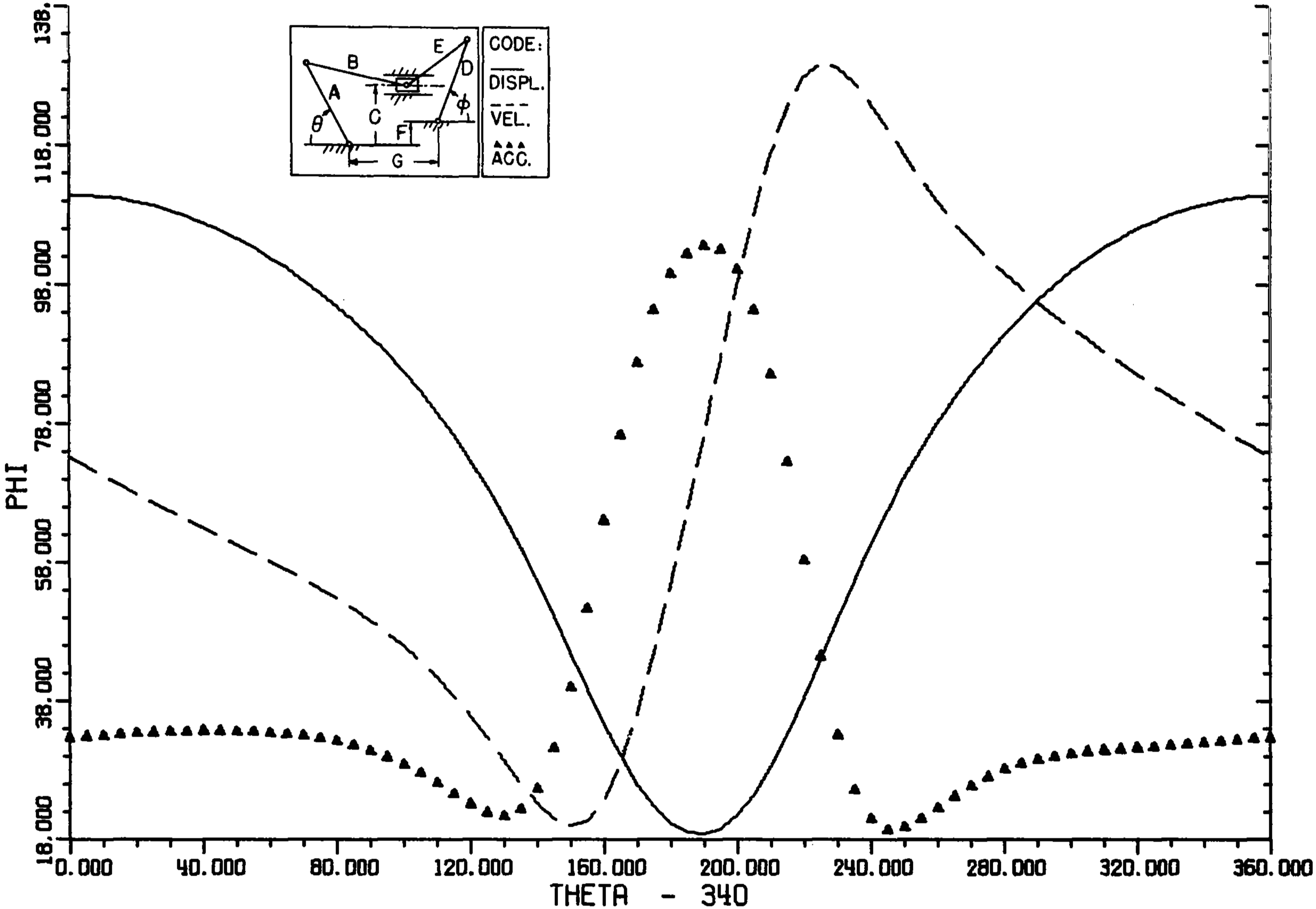


PLATE 6 - 13

A= 5.00, B=14.00, C= 6.00, D= 9.00,
E=11.00, F= 3.00, G=10.00,
VEL.MAX= 2.24, VEL.MIN= -1.44, ACC.MAX= 5.57, ACC.MIN= -3.62,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 95.77

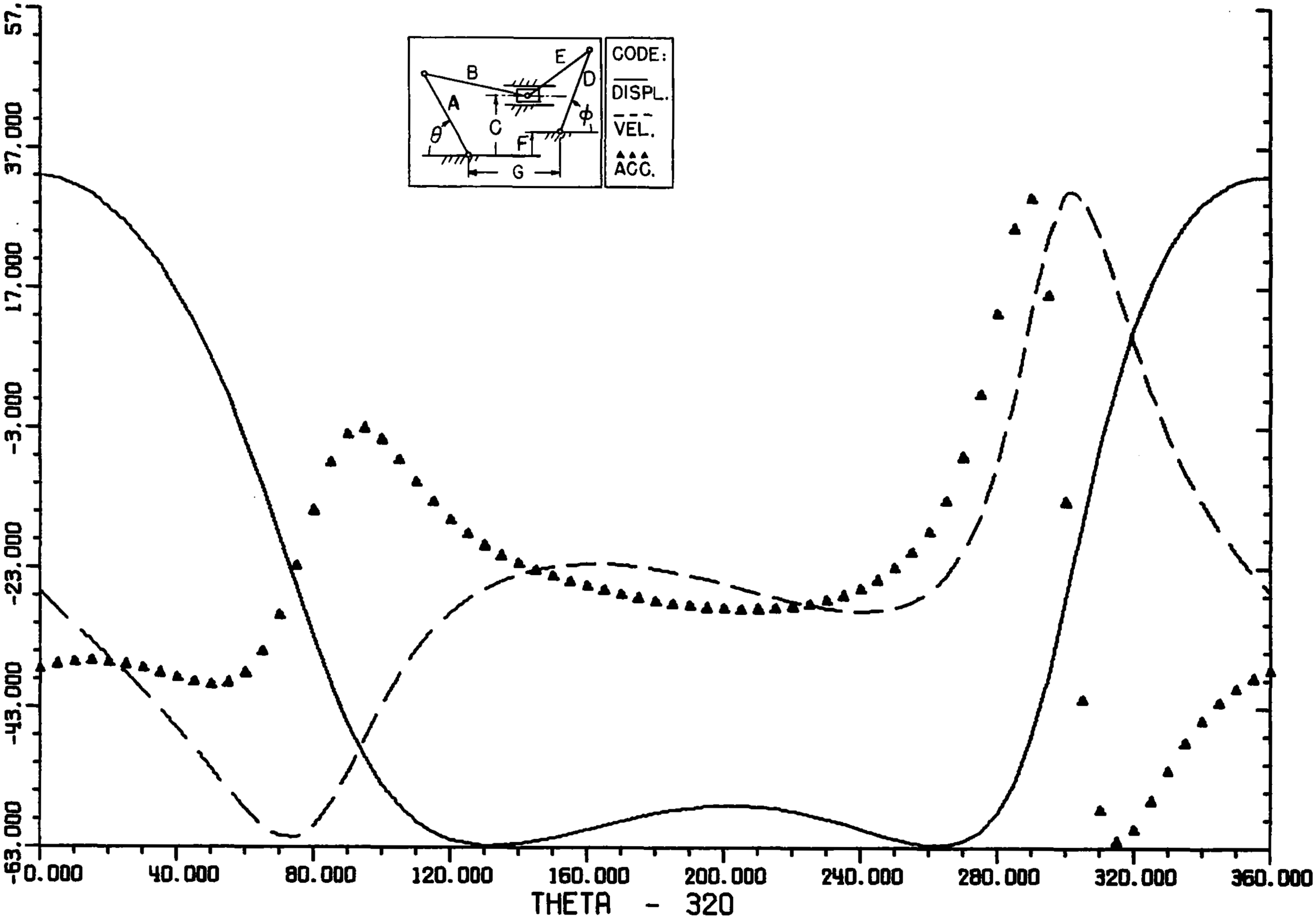


PLATE 6-14

A= 6.00, B=15.00, C= 8.00, D= 9.00,
E=13.00, F= 1.00, G=12.00,
VEL.MAX= 1.87, VEL.MIN= -0.92, ACC.MAX= 3.36, ACC.MIN= -2.78,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 97.11

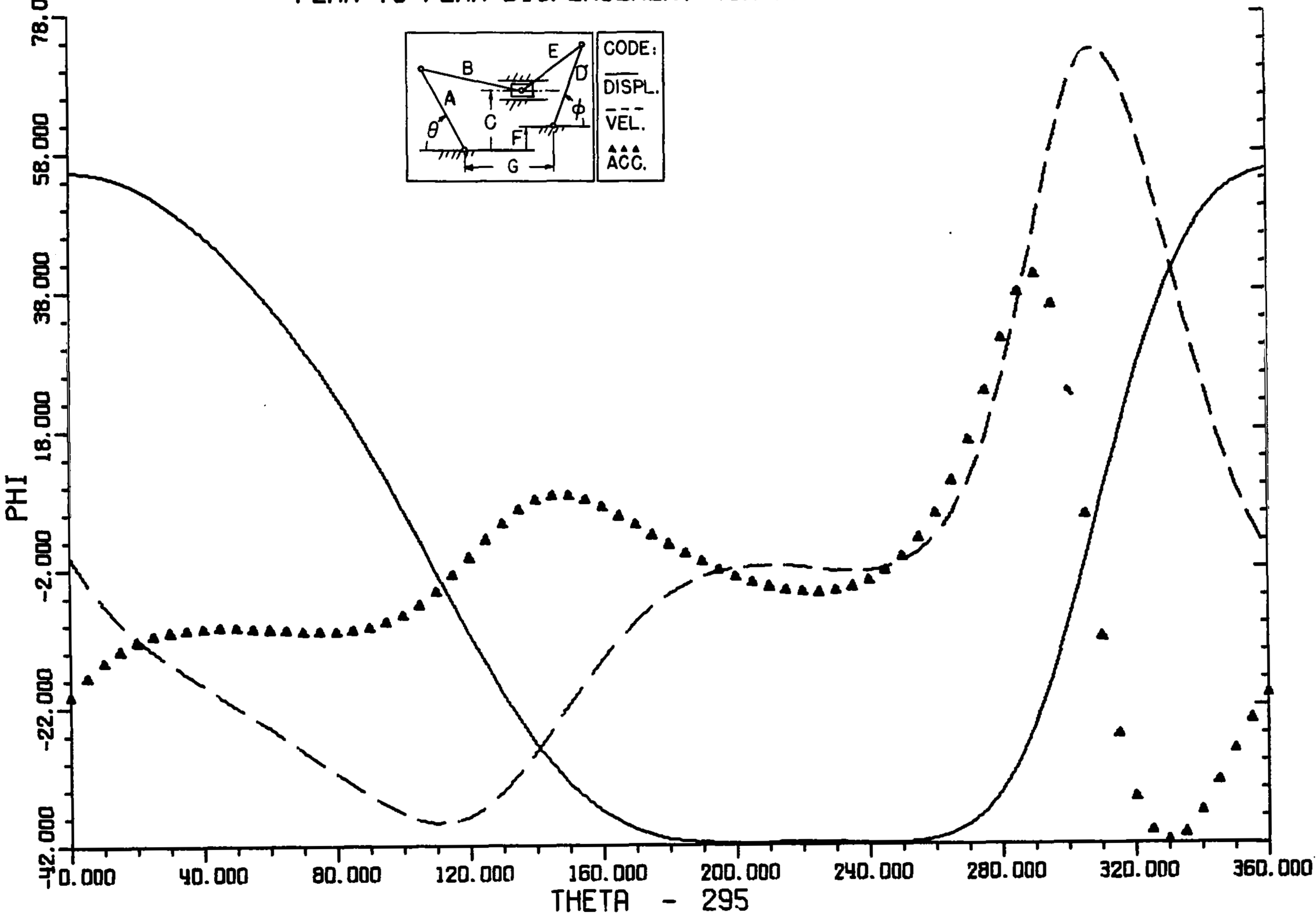


PLATE 6-15

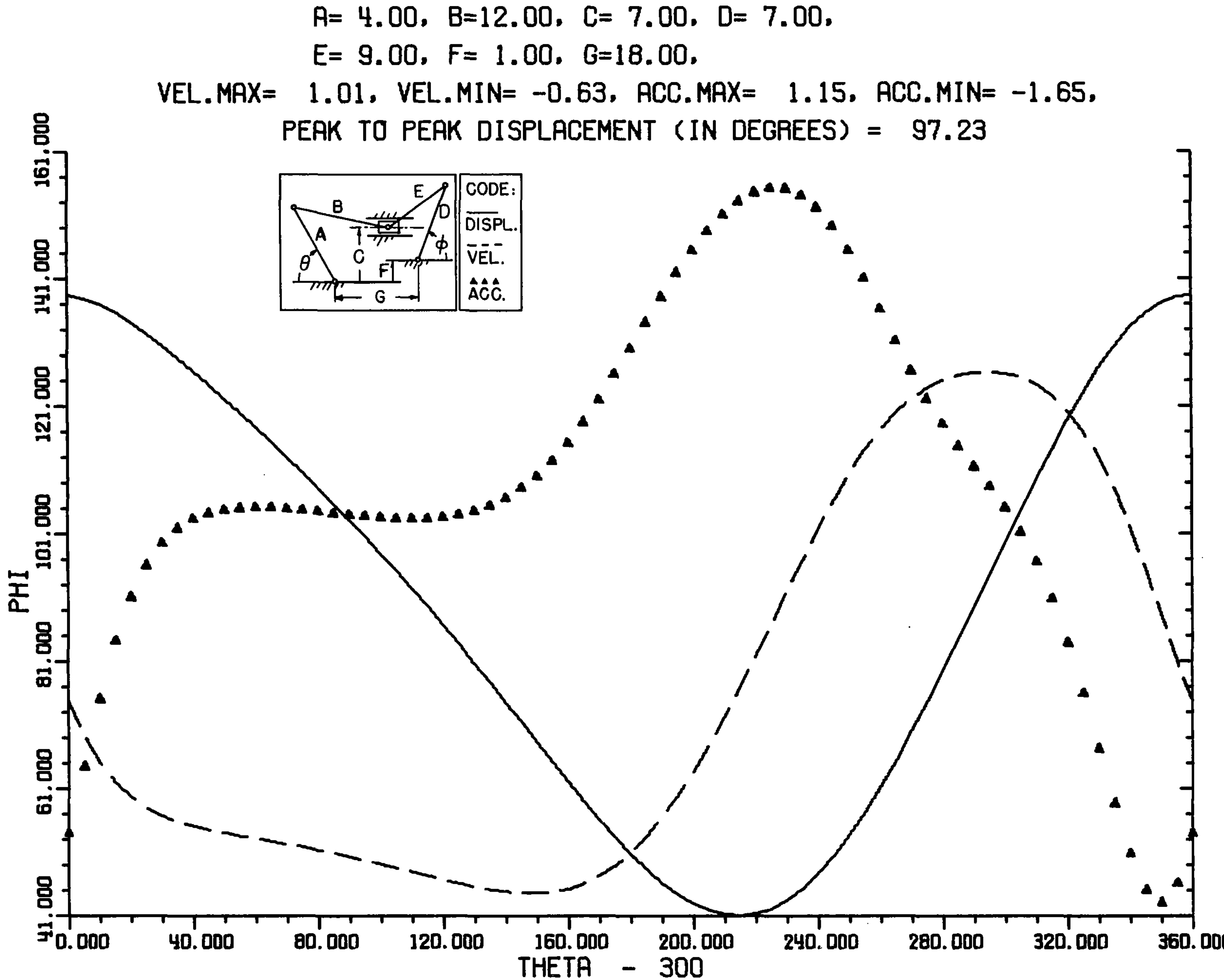

A= 4.00, B=12.00, C= 7.00, D= 7.00,
E= 9.00, F= 1.00, G=18.00,
VEL.MAX= 1.01, VEL.MIN= -0.63, ACC.MAX= 1.15, ACC.MIN= -1.65,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 97.23
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
161.000
141.000
121.000
101.000
81.000
61.000
41.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 300

PLATE 6-16

A= 5.00, B=12.00, C= 6.00, D= 7.00,
E= 9.00, F= 1.00, G=10.00,
VEL.MAX= 1.95, VEL.MIN= -1.02, ACC.MAX= 3.43, ACC.MIN= -2.64,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 104.17

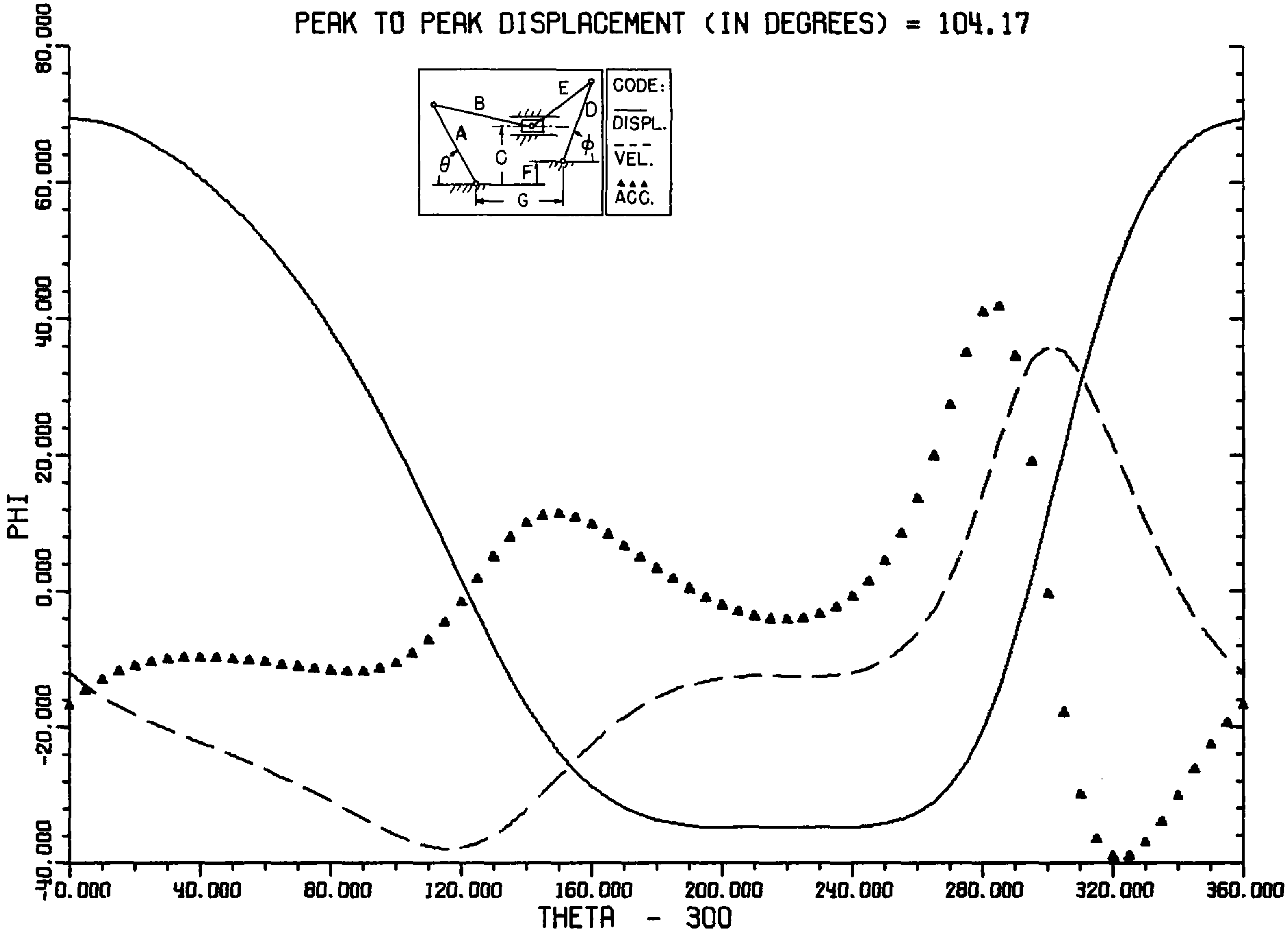


PLATE 6-17

A= 6.00, B=15.00, C= 7.00, D= 9.00,
E=13.00, F= 2.00, G=12.00,
VEL.MAX= 2.45, VEL.MIN= -1.37, ACC.MAX= 6.62, ACC.MIN= -3.66,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 105.46

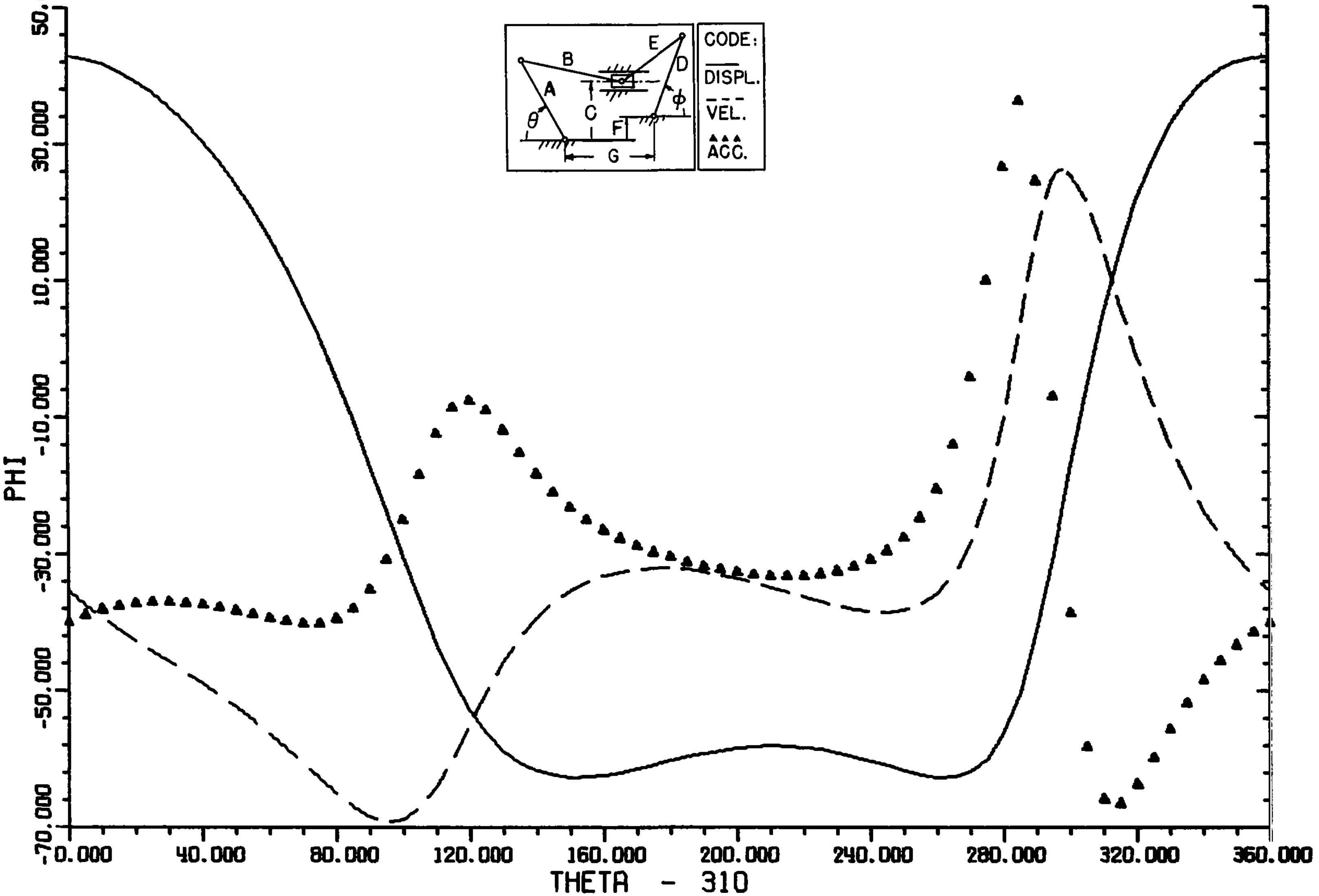


PLATE 6-18

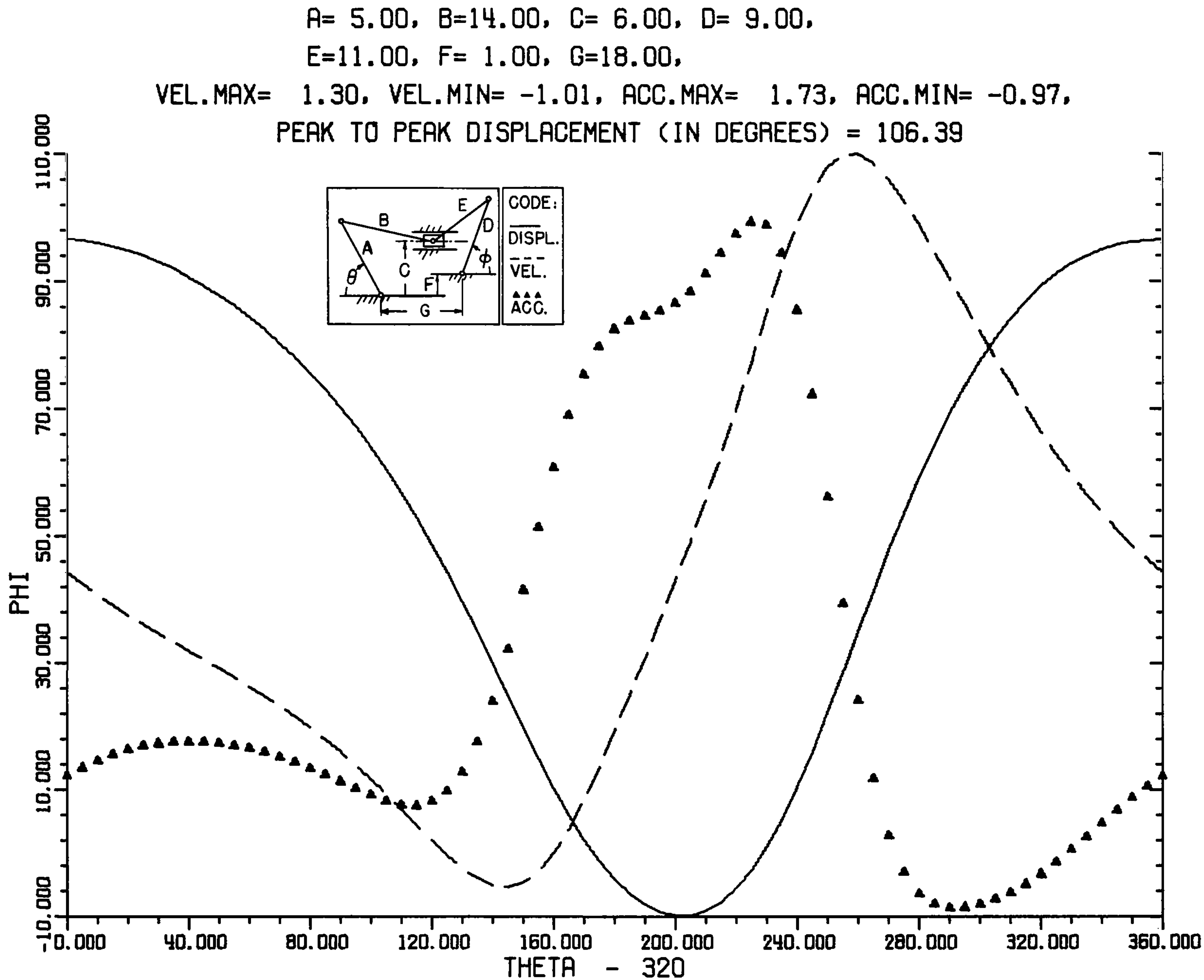

A= 5.00, B=14.00, C= 6.00, D= 9.00,
E=11.00, F= 1.00, G=18.00,
VEL.MAX= 1.30, VEL.MIN= -1.01, ACC.MAX= 1.73, ACC.MIN= -0.97,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 106.39
CODE:
DISPL.
VEL.
ACC.
PHI
THETA - 320

PLATE 6-19

A= 5.00, B=14.00, C= 6.00, D= 9.00,
E=11.00, F= 1.00, G=14.00,
VEL.MAX= 1.58, VEL.MIN= -1.10, ACC.MAX= 2.27, ACC.MIN= -1.64,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 107.66

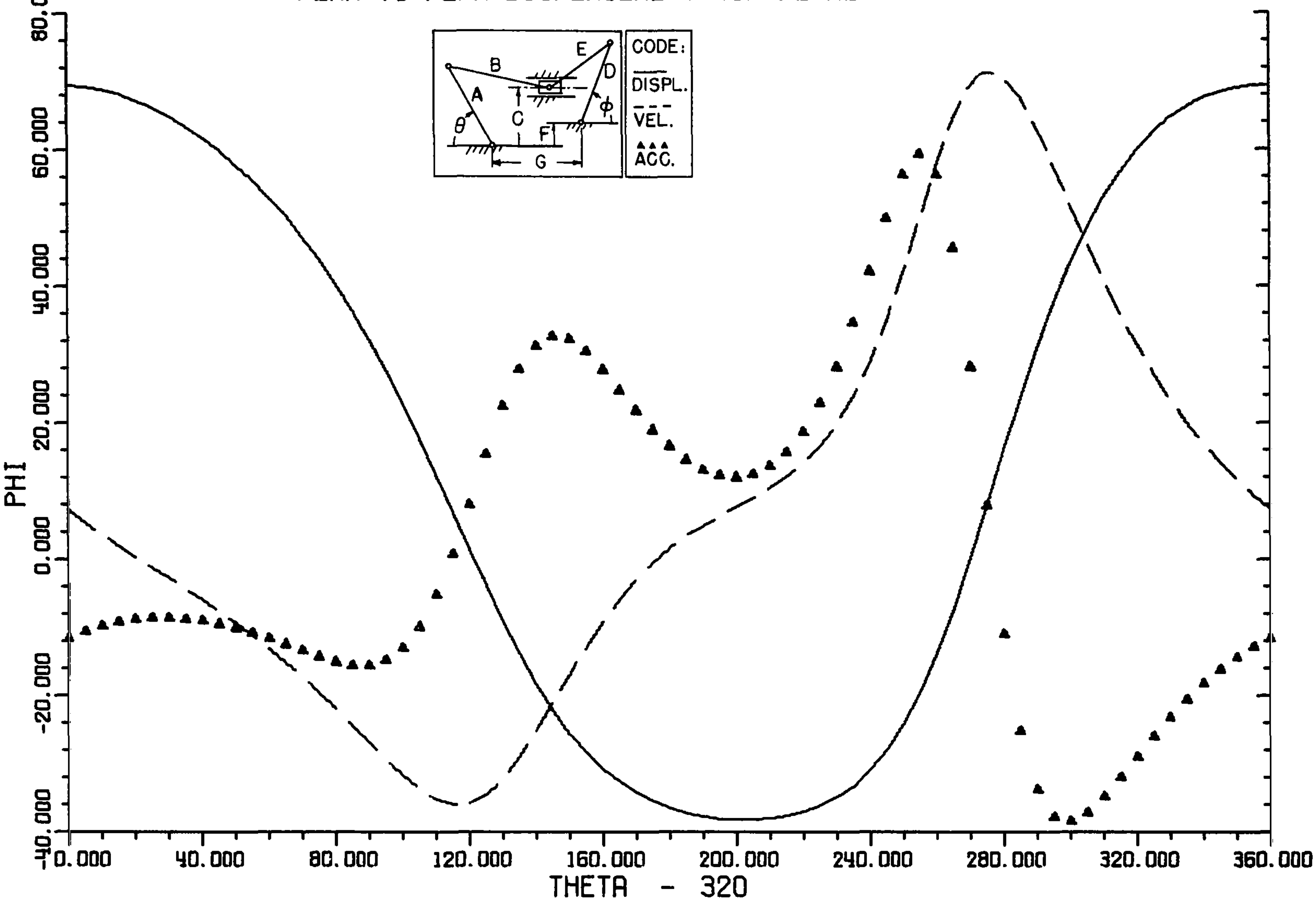


PLATE 6-20

A= 4.00, B=10.00, C= 3.00, D= 6.00,
E= 6.00, F= 2.00, G=14.00,
VEL.MAX= 1.55, VEL.MIN= -1.43, ACC.MAX= 5.34, ACC.MIN= -1.64,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 107.81

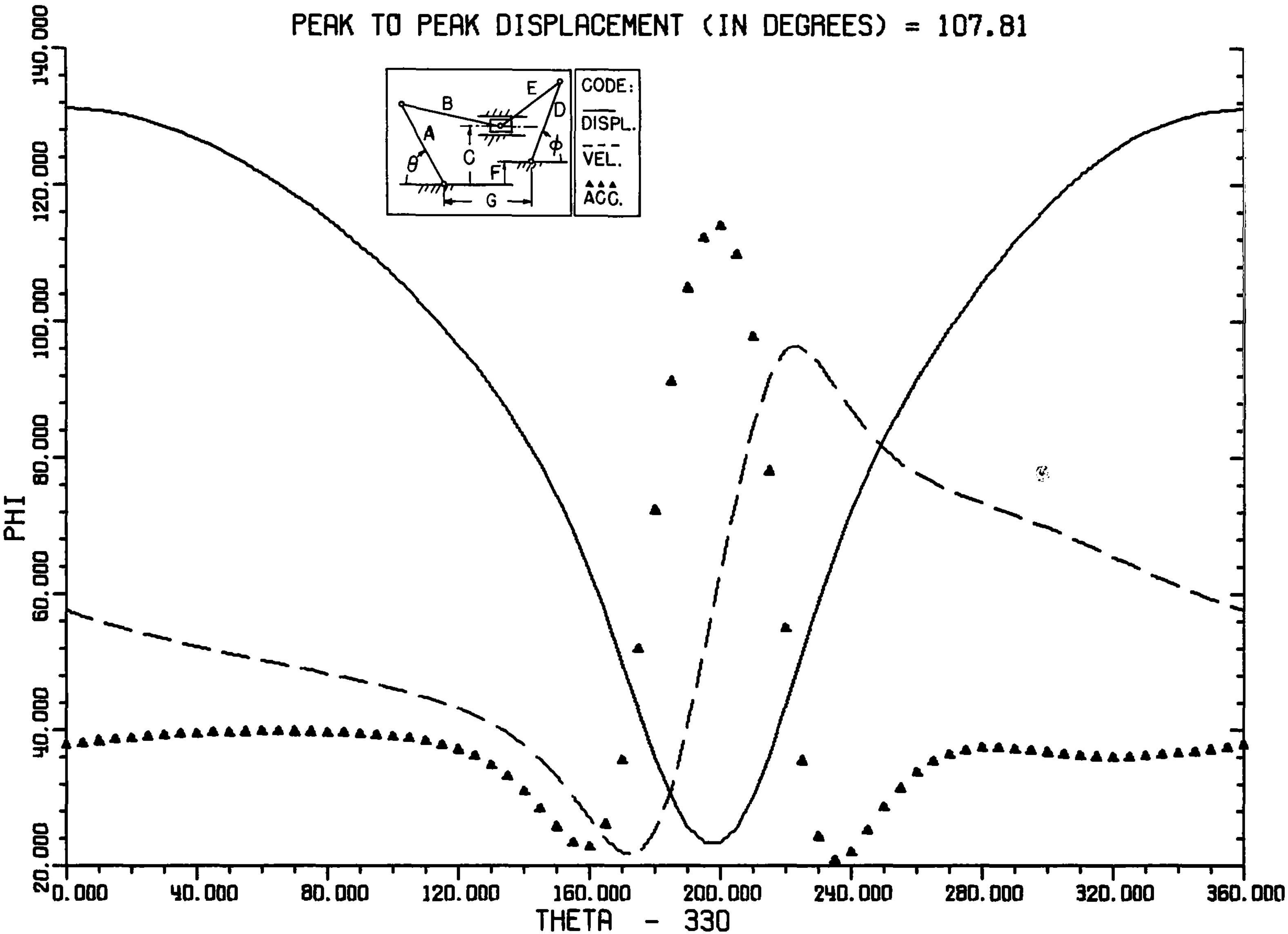


PLATE 6-21

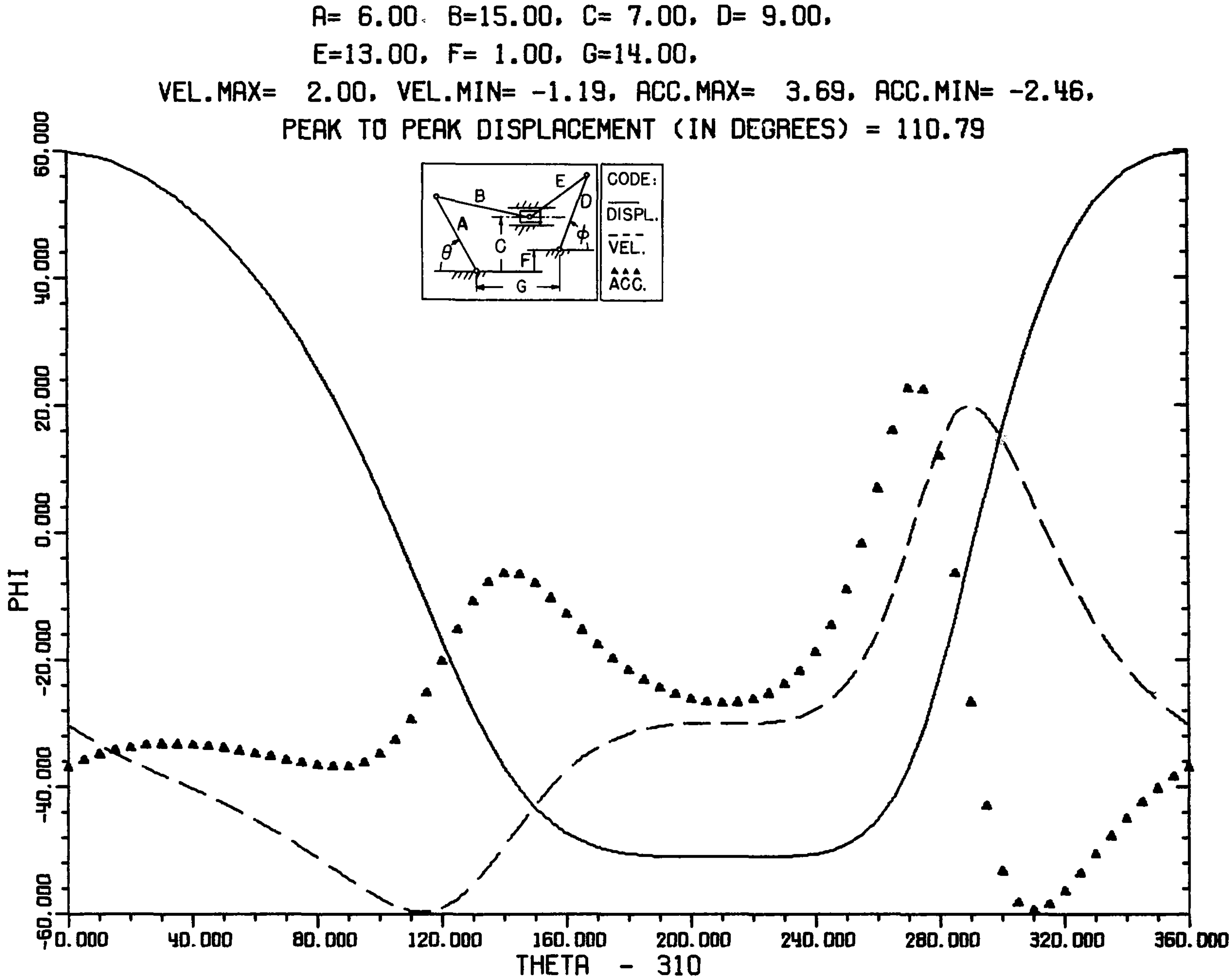

A= 6.00, B=15.00, C= 7.00, D= 9.00,
E=13.00, F= 1.00, G=14.00,
VEL.MAX= 2.00, VEL.MIN= -1.19, ACC.MAX= 3.69, ACC.MIN= -2.46,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 110.79
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
ϕ
PHI
60.000
40.000
20.000
0.000
-20.000
-40.000
-60.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 310

PLATE 6-22

A= 4.00, B=10.00, C= 5.00, D= 6.00,

E= 8.00, F= 1.00, G=14.00,

VEL.MAX= 1.38, VEL.MIN= -0.99, ACC.MAX= 1.93, ACC.MIN= -1.31,

PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 117.78

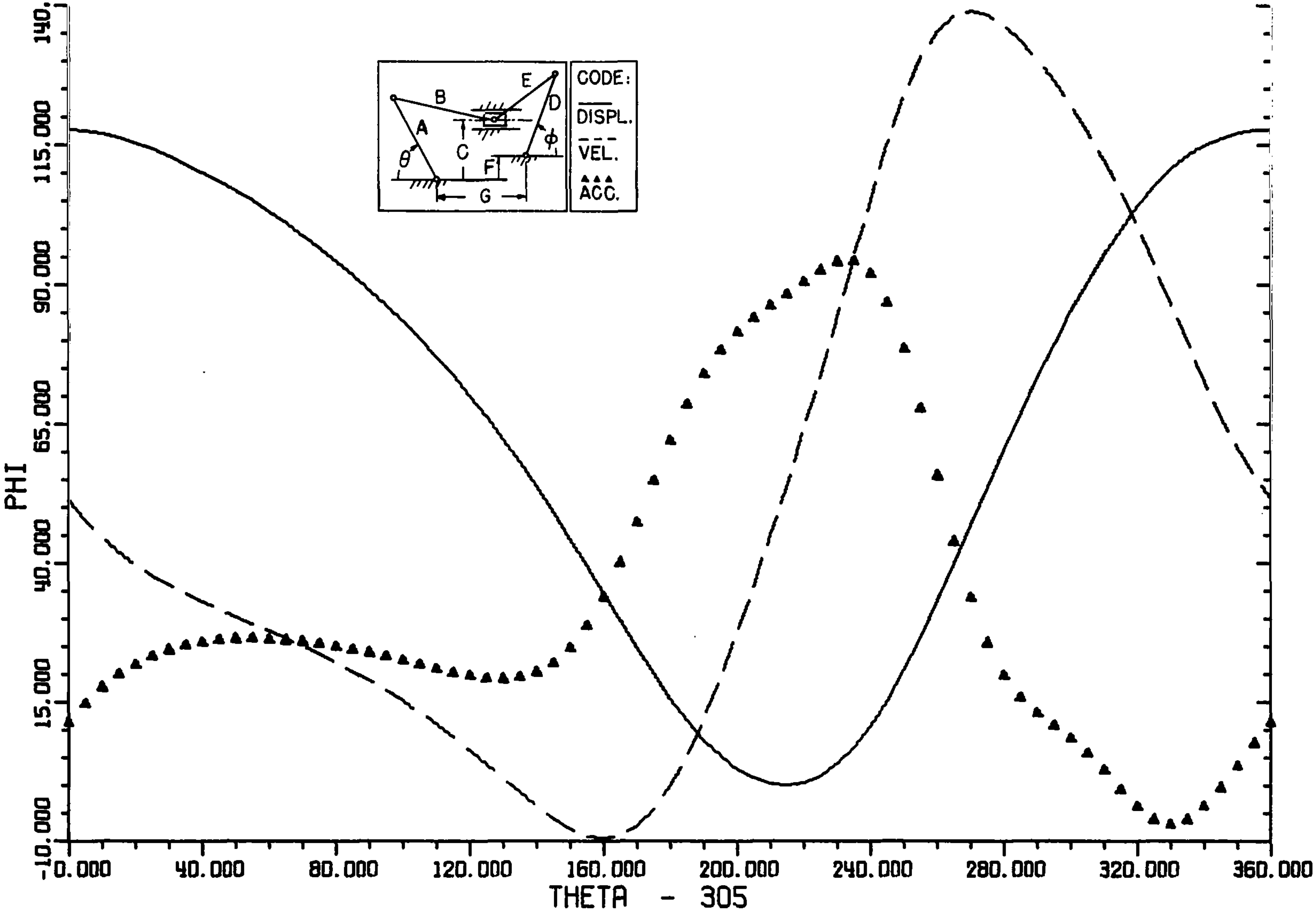
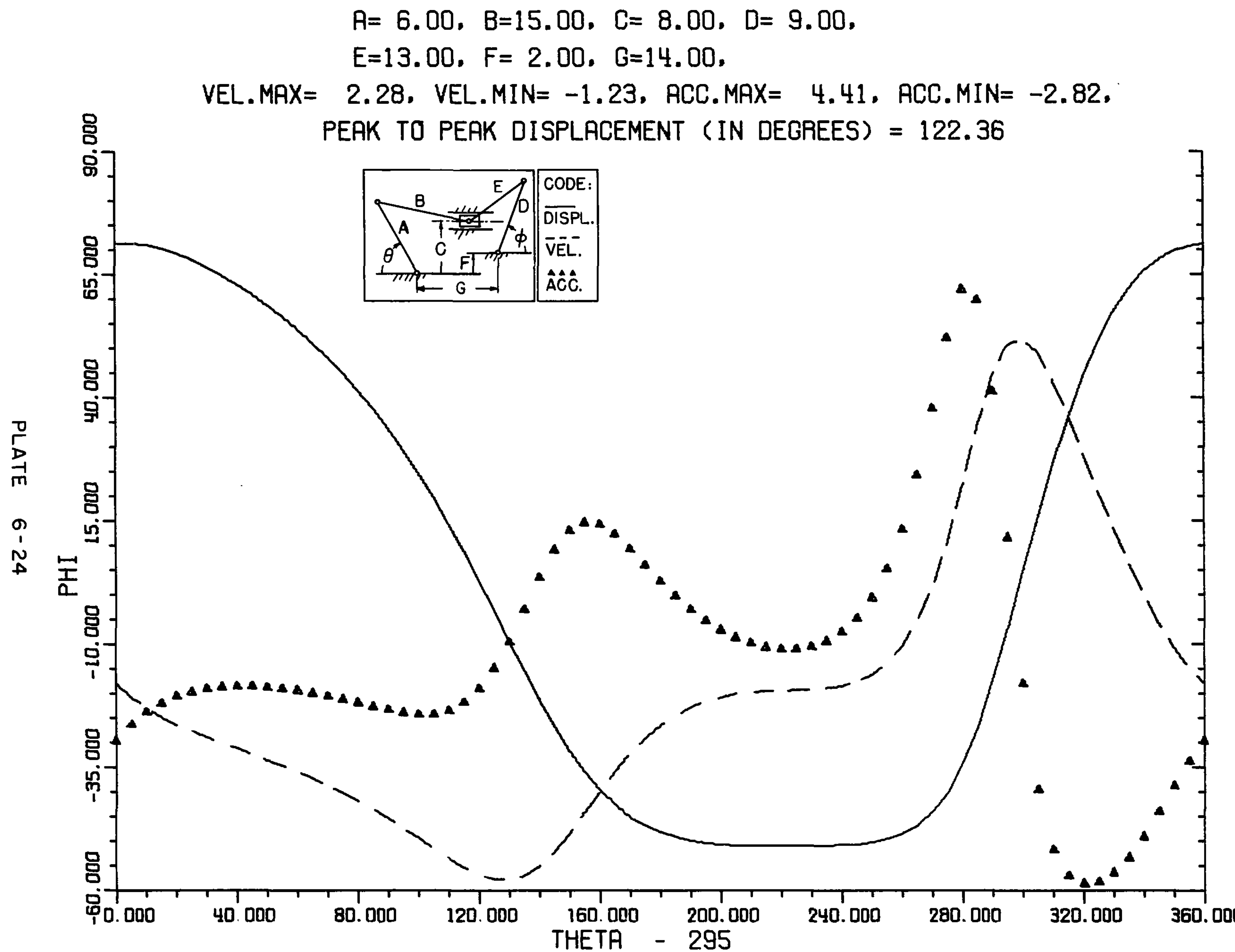


PLATE 6-23


A= 6.00, B=15.00, C= 8.00, D= 9.00,
E=13.00, F= 2.00, G=14.00,
VEL.MAX= 2.28, VEL.MIN= -1.23, ACC.MAX= 4.41, ACC.MIN= -2.82,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 122.36
CODE:
DISPL.
VEL.
ACC.
PHI
THETA - 295
PLATE 6-24

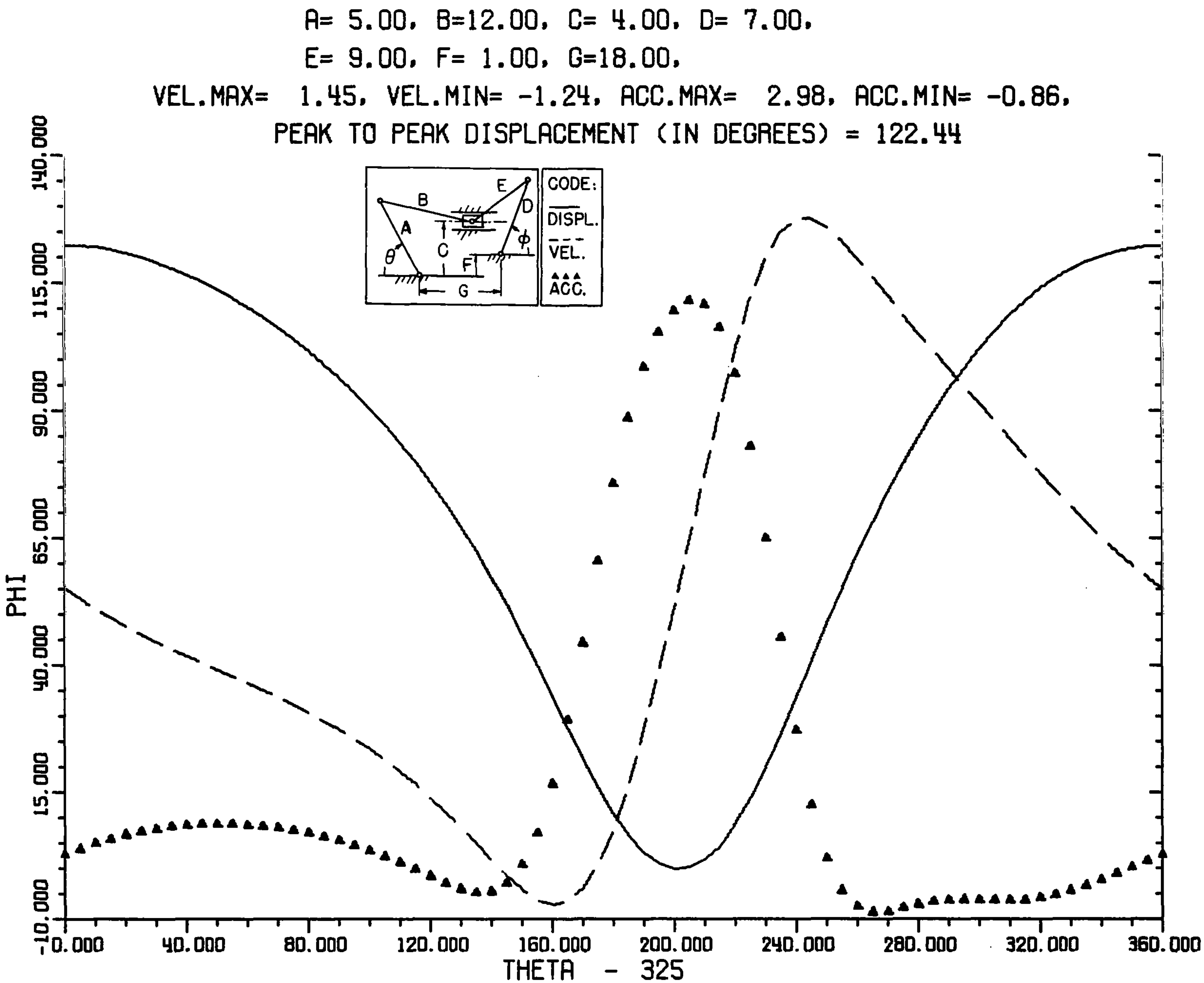

A= 5.00, B=12.00, C= 4.00, D= 7.00,
E= 9.00, F= 1.00, G=18.00,
VEL.MAX= 1.45, VEL.MIN= -1.24, ACC.MAX= 2.98, ACC.MIN= -0.86,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 122.44
A
B
C
D
E
F
G
θ
φ
CODE:
DISPL.
VEL.
ACC.
PHI
-10.000
15.000
40.000
65.000
90.000
115.000
140.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 325

PLATE 6-25

A= 5.00, B=14.00, C= 8.00, D= 9.00,
E=11.00, F= 5.00, G=10.00,
VEL.MAX= 3.60, VEL.MIN= -1.81, ACC.MAX= 11.58, ACC.MIN= -6.93,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 127.34

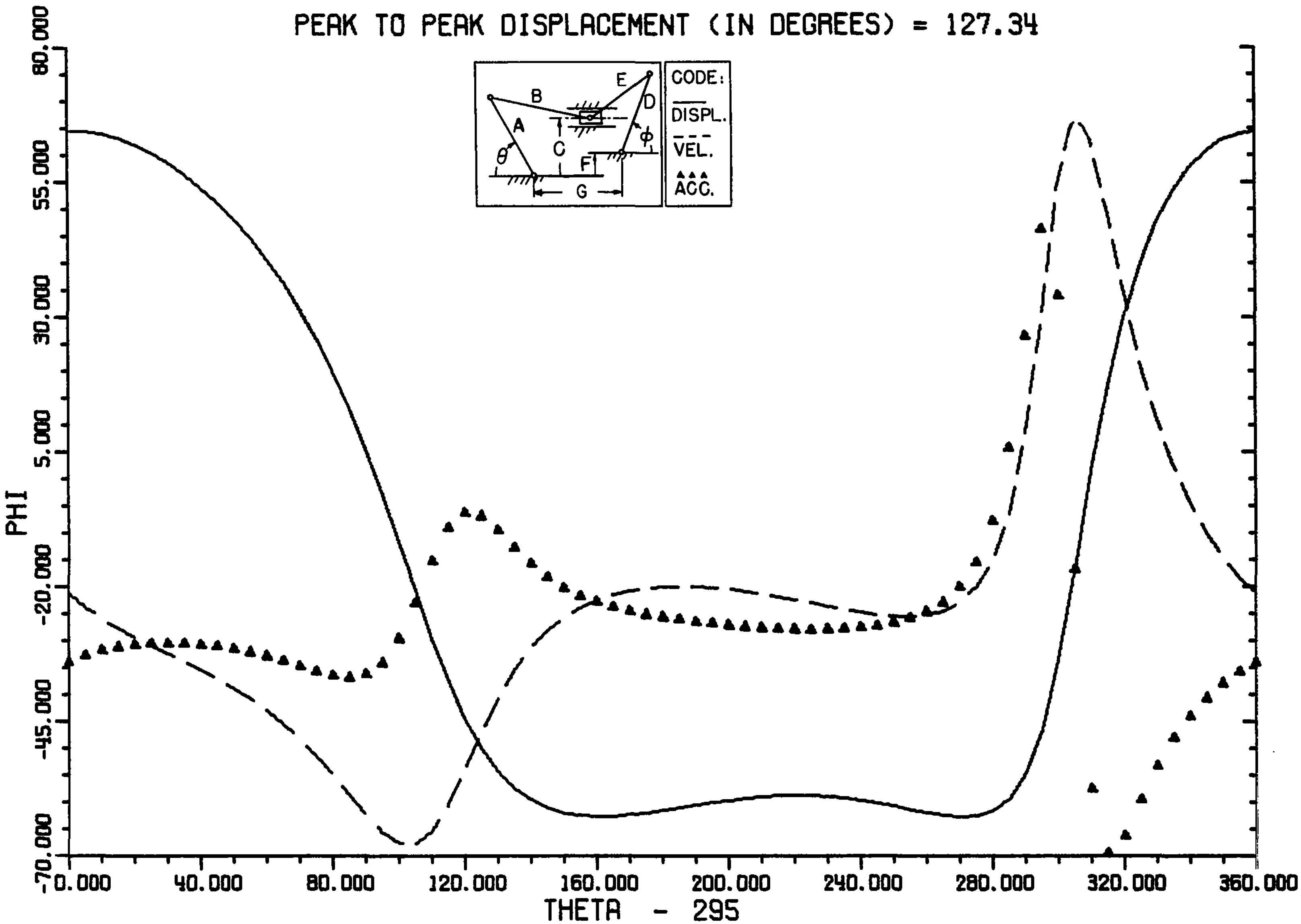


PLATE 6-26

A= 5.00, B=14.00, C= 4.00, D= 9.00,
E=11.00, F= 1.00, G=14.00,
VEL.MAX= 2.45, VEL.MIN= -1.96, ACC.MAX= 5.51, ACC.MIN= -3.32,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 127.95

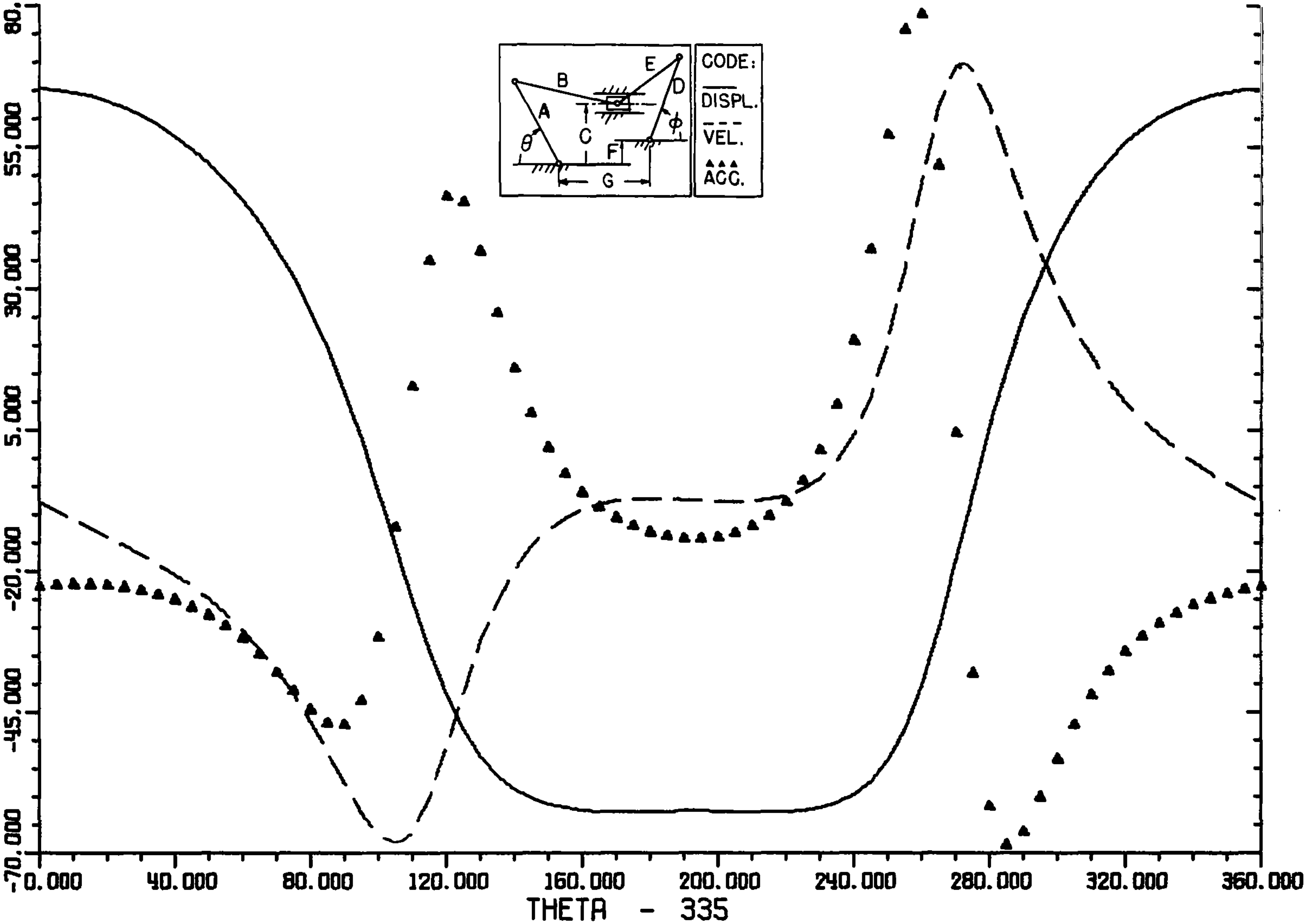


PLATE 6-27

A= 6.00, B=15.00, C= 6.00, D= 9.00,
E=13.00, F= 1.00, G=16.00,
VEL.MAX= 2.37, VEL.MIN= -1.64, ACC.MAX= 5.18, ACC.MIN= -2.73,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 131.49

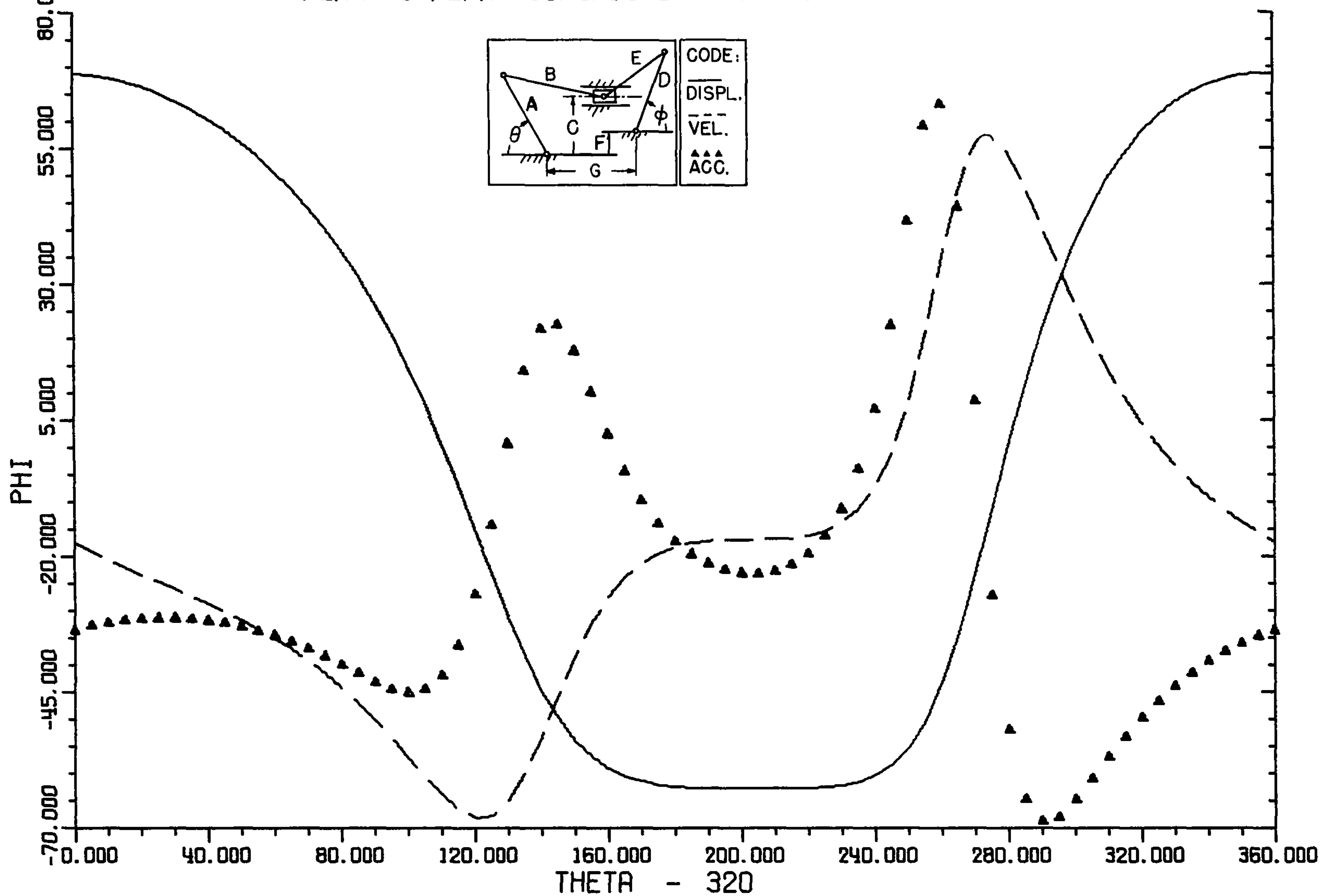


PLATE 6-28

A= 6.00, B=15.00, C= 8.00, D= 9.00,
E=13.00, F= 2.00, G=16.00,
VEL.MAX= 2.16, VEL.MIN= -1.29, ACC.MAX= 3.70, ACC.MIN= -2.22,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 133.56

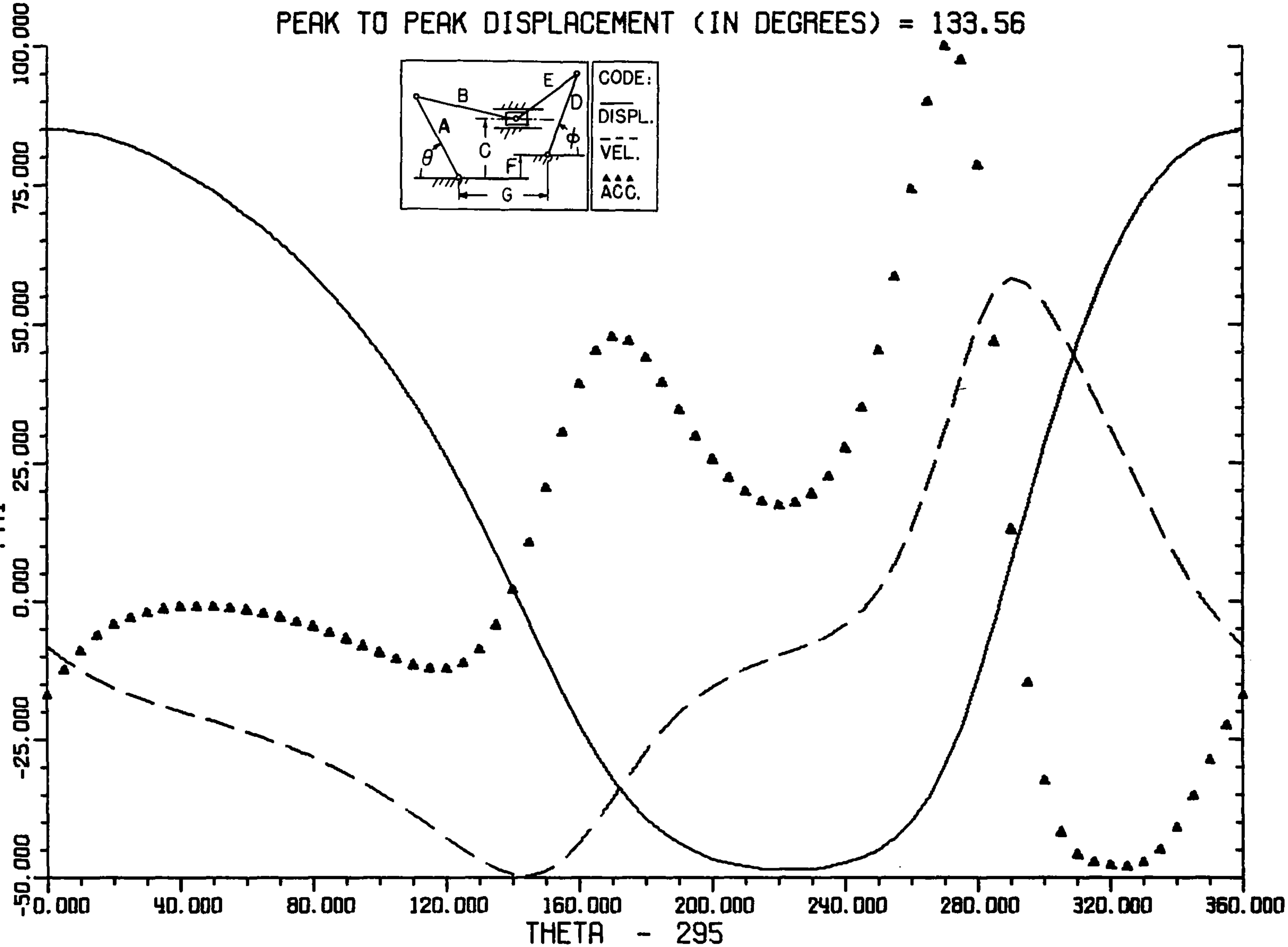


PLATE 6-29

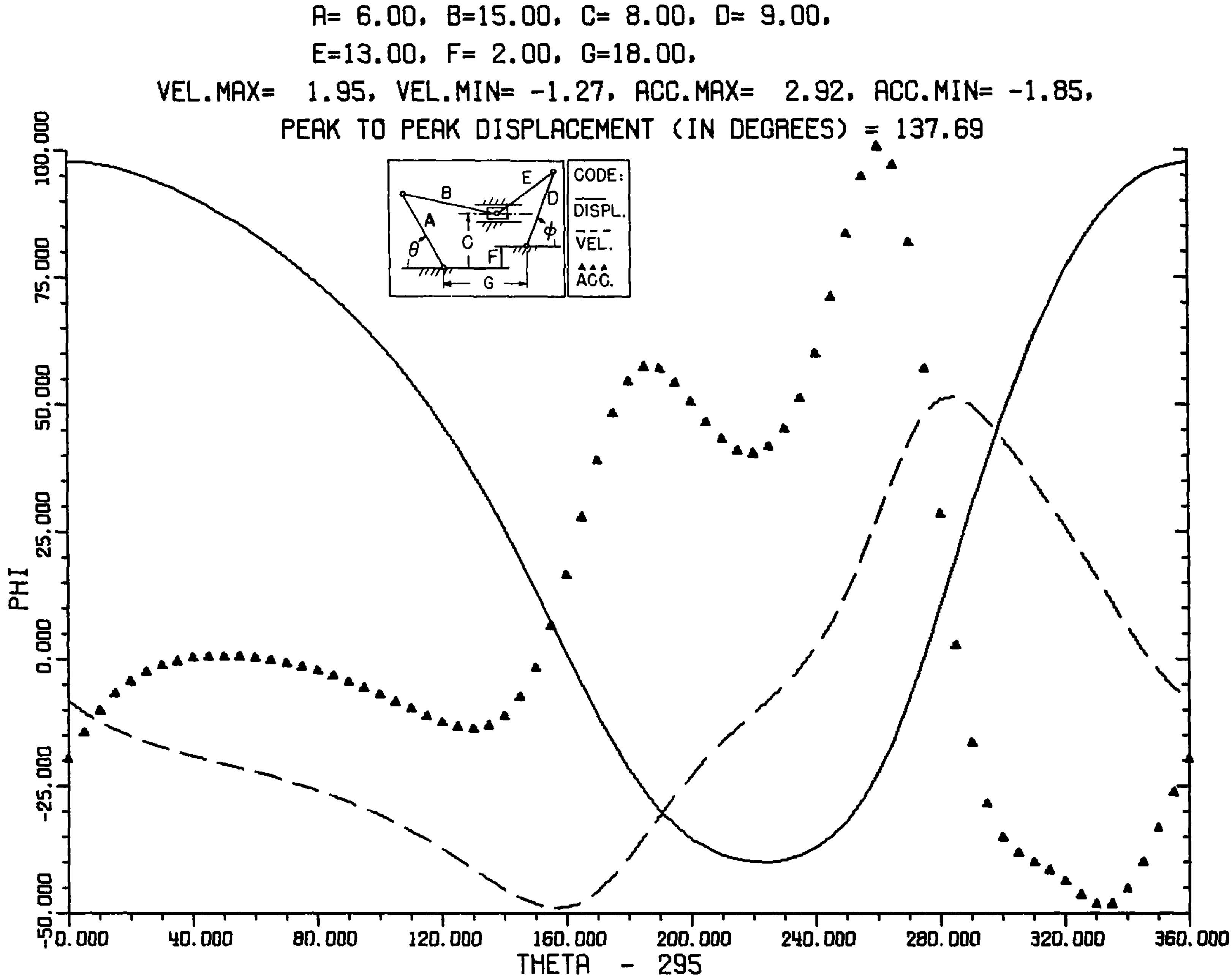

A= 6.00, B=15.00, C= 8.00, D= 9.00,
E=13.00, F= 2.00, G=18.00,
VEL.MAX= 1.95, VEL.MIN= -1.27, ACC.MAX= 2.92, ACC.MIN= -1.85,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 137.69
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
100.000
75.000
50.000
25.000
0.000
-25.000
-50.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 295

PLATE 6-30

A= 5.00, B=14.00, C= 6.00, D= 9.00,
E=11.00, F= 3.00, G=14.00,
VEL.MAX= 2.82, VEL.MIN= -2.01, ACC.MAX= 7.04, ACC.MIN= -4.09,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 139.34

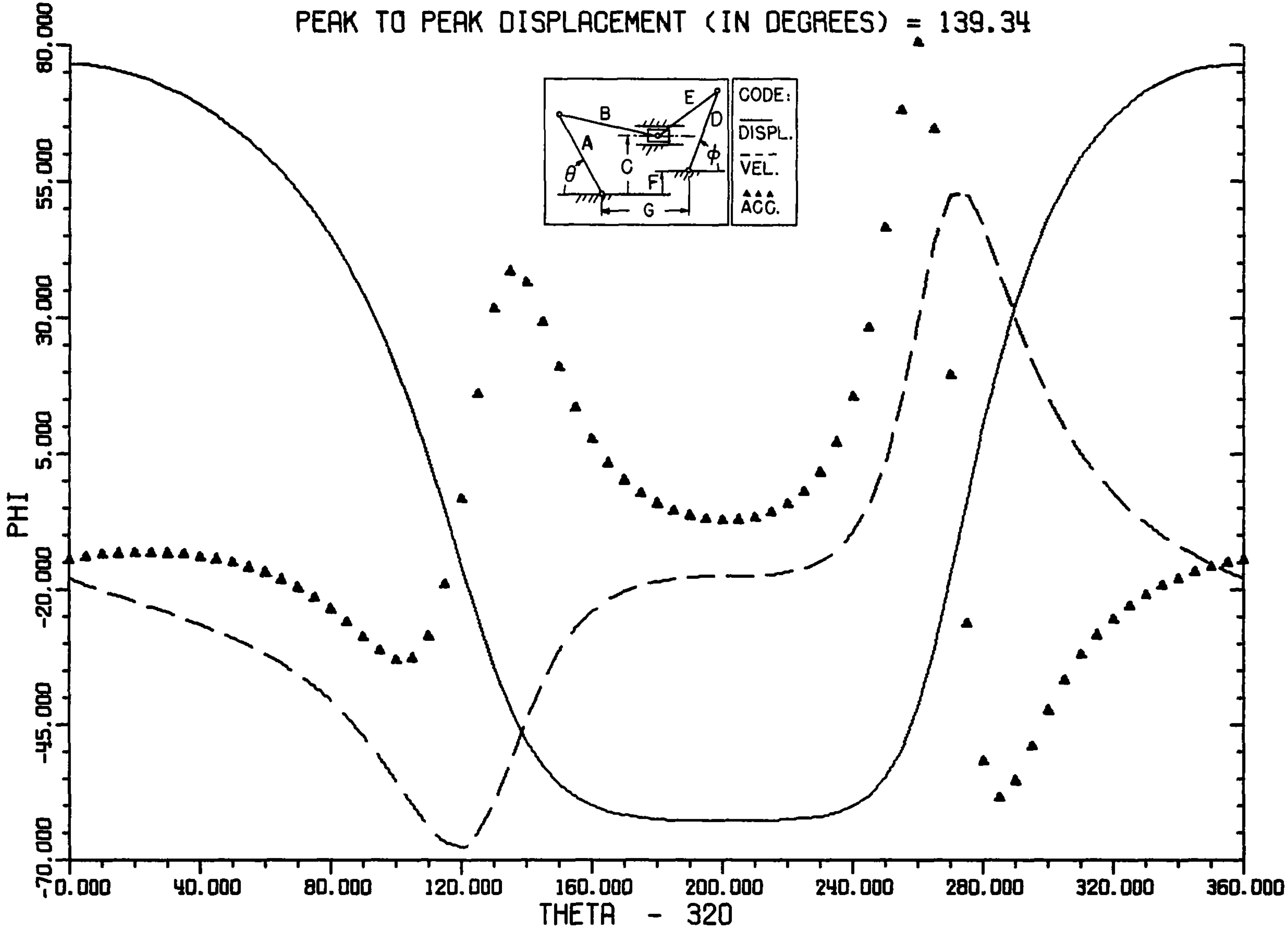


PLATE 6-31

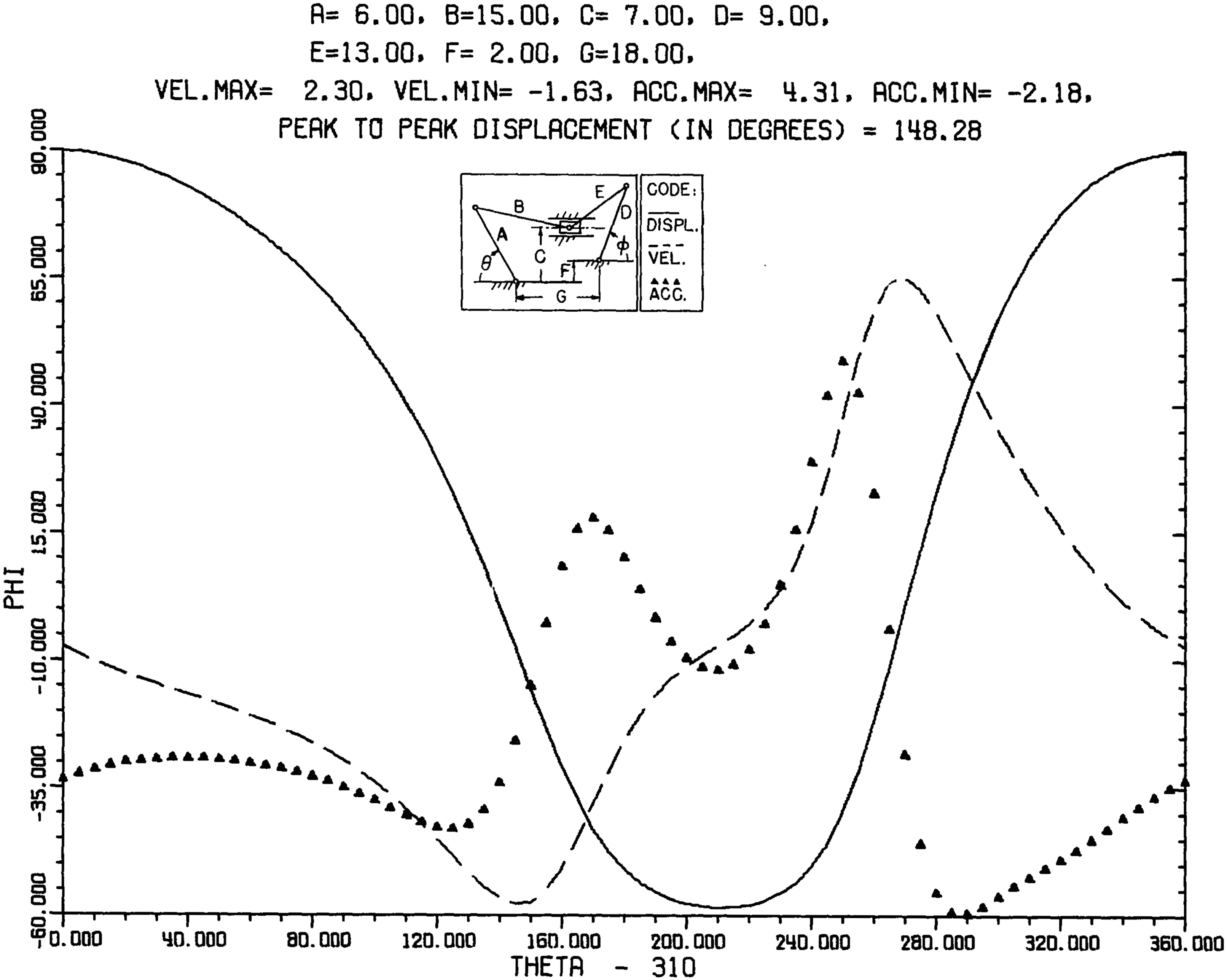
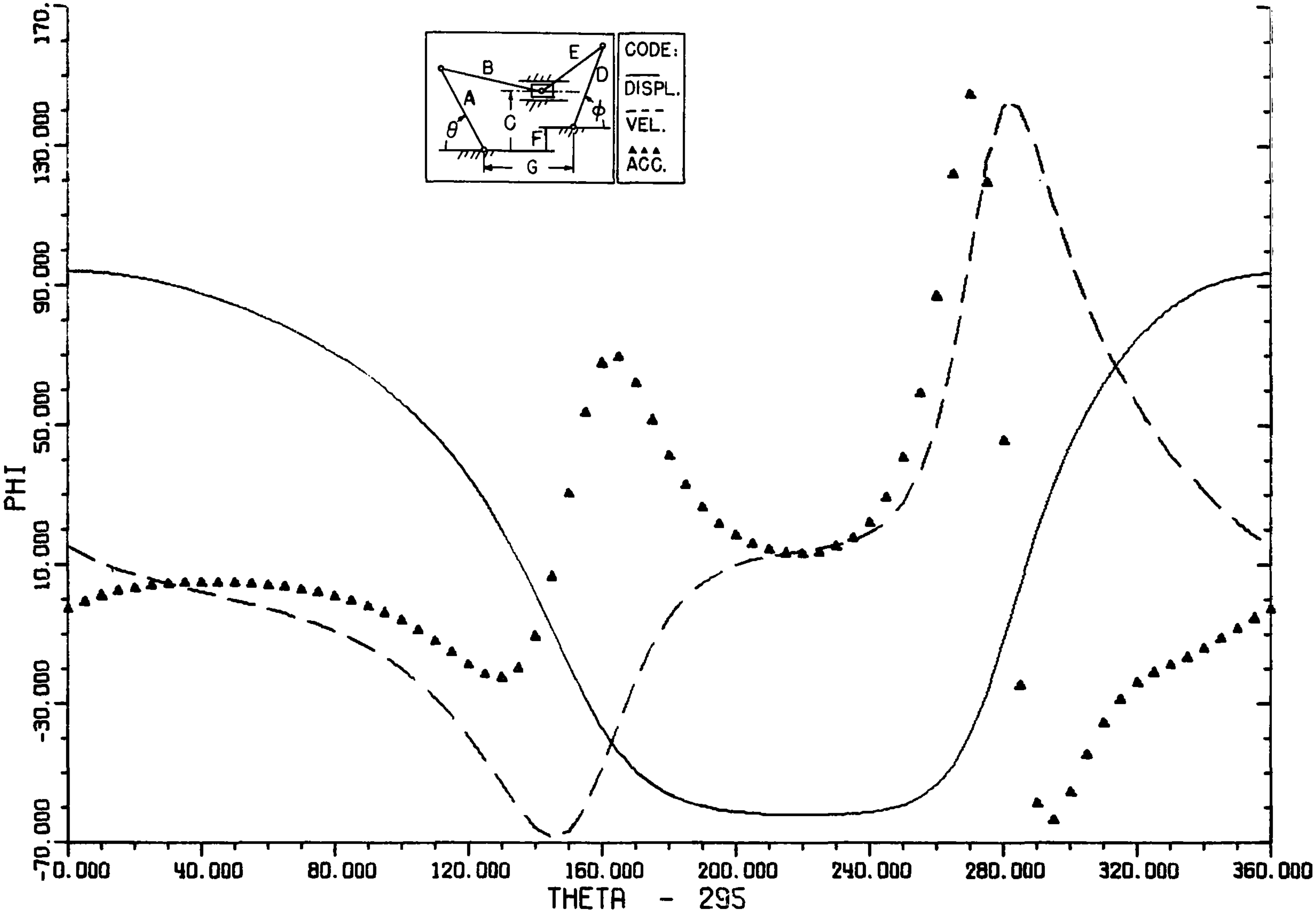

A= 6.00, B=15.00, C= 7.00, D= 9.00,
E=13.00, F= 2.00, G=18.00,
VEL.MAX= 2.30, VEL.MIN= -1.63, ACC.MAX= 4.31, ACC.MIN= -2.18,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 148.28
CODE:
DISPL.
VEL.
ACC.
PHI
THETA - 310
90.000
65.000
40.000
15.000
-10.000
-35.000
-60.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 6-32

PLATE 6-33

A= 5.00, B=14.00, C= 8.00, D= 9.00,
E=11.00, F= 5.00, G=14.00,
VEL.MAX= 3.19, VEL.MIN= -2.06, ACC.MAX= 8.42, ACC.MIN= -4.61,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 156.20



MECHANISM #7

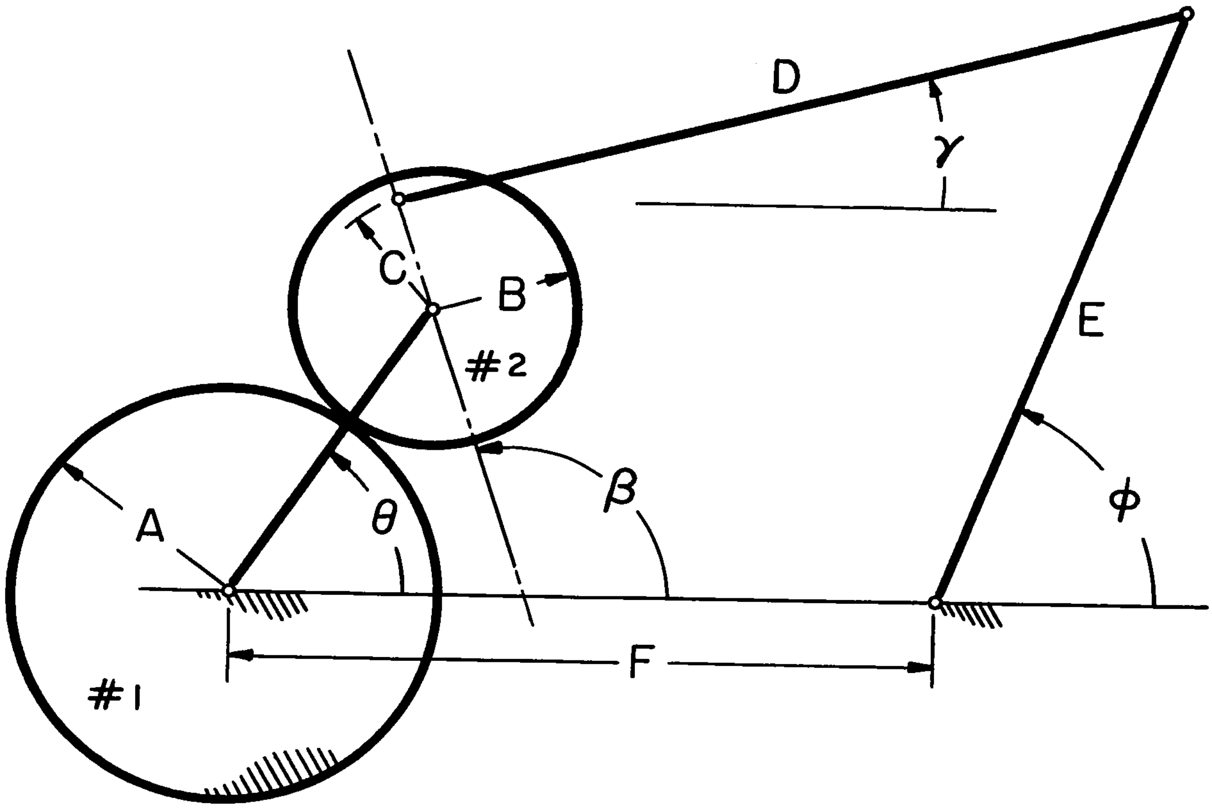


Figure 7-1

Figure 7-1 defines Mechanism #7 which consists of several links, a fixed gear labeled #1, and a moving gear labeled #2. For this mechanism θ, theta, is considered as the input and φ, phi, is considered as the output. Each link of the mechanism is identified by a capital letter which is also used for specifying the length of the link. The link lengths are noted as part of the title on each of the several graphs which follow.

Each graph for this mechanism shows φ versus θ as a solid line, the derivative of φ with respect to θ versus θ as a dashed line, and the second derivative of ϕ with respect to θ plotted versus θ as a series of small triangles. Each curve begins with the maximum displacement of ϕ. This has been accomplished by shifting the θ axis. The amount by which each curve has been shifted is indicated (in degrees) as part of the abscissa title. The zero value for velocity occurs at the beginning point of each velocity curve as this coincides with the maximum displacement. Both the variables, θ and ϕ, are presented in the units of degrees on each graph.

Scales for the derivatives have not been presented but each graph heading includes the maximum and minimum for both the velocity and acceleration, that is, for both the first and second derivative of ϕ with respect to θ. From these data scales for the derivative may be constructed. The units for the derivatives will be radians per radian for $d\phi/d\theta$ and radians per radian squared for $d^2\phi/d\theta^2$. A more conventional engineering unit for angular velocity may be derived as:

$$\frac{d\phi}{dt} = \frac{d\phi}{d\theta}\,\frac{d\theta}{dt}, \qquad \left[\frac{\text{radians}}{\text{second}} = \frac{\text{radians}}{\text{radian}} \times \frac{\text{degrees}}{\text{second}}\right] \tag{7-1}$$

in which

$$\frac{d\theta}{dt} = \text{rpm} \times 2\pi \times \frac{1}{60}$$

$$\left[\frac{\text{radians}}{\text{second}} = \frac{\text{rev}}{\text{minute}} \times \frac{\text{radians}}{\text{rev}} \times \frac{\text{minutes}}{\text{second}}\right]. \tag{7-2}$$

Substituting Eq. 7-2 into Eq. 7-1 will yield:

$$\frac{d\phi}{dt} = \frac{\pi}{30} \times \text{rpm} \times \frac{d\phi}{d\theta}, \qquad \frac{\text{radians}}{\text{second}} . \tag{7-3}$$

In words, the angular velocity of link E (radians/ second) is obtained as the product of $\pi/30$ times the angular speed of the input link (revolutions/ minute) and $d\phi/d\theta$ (radians/ radian). Values for this latter term may be obtained from a graph (the dashed line), or from equations which follow, or the extreme values may be obtained from the heading of a graph as VEL. MAX and VEL. MIN.

The angular acceleration of link E may be presented in more conventional engineering terms as:

$$\frac{d^2\phi}{dt^2} = \frac{d}{dt}\left[\frac{d\phi}{d\theta} \times \frac{d\theta}{dt}\right]$$

$$= \frac{d\phi}{d\theta} \times \frac{d^2\theta}{dt^2} + \frac{d^2\phi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2 . \qquad (7\text{-}4)$$

If the angular speed of the input link, the position of which is specified by θ, remains constant then $d^2\theta/dt^2$ is identically zero. The expression for the angular acceleration of the output link with the input link turning with constant speed simplifies to:

$$\frac{d^2\phi}{dt^2} = \frac{d^2\phi}{d\theta^2}\left[\frac{d\theta}{dt}\right]^2$$

$$= \frac{d^2\phi}{d\theta^2}\left[\frac{\pi}{30} \times \text{rpm}\right]^2, \quad \frac{\text{radians}}{\text{second}^2} . \qquad (7\text{-}5)$$

Values for the term $d^2\phi/d\theta^2$ may be obtained from a graph (the curve given by a series of small triangles) or from equations which are derived or the extreme values may be noted in the heading of a graph as ACC. MAX and ACC. MIN.

To completely define the mechanism requires specifying the orientation of the two gears. This is indicated in Figure 7-2 in which the mechanism is presented for $\theta = 0$. The angle G in addition to the dimensions of A, B, C, D, E, and F completely defines the mechanism. Note that G is given (in degrees) as part of the heading for each graph.

Referring to Figure 7-1 the relationship between θ and β may be established as:

$$\beta = \theta\left[1 + \frac{A}{G}\right] + G. \qquad (7\text{-}6)$$

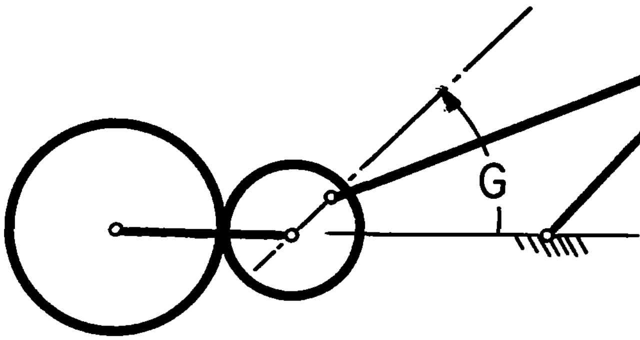


Figure 7-2

Let

$$H = A + B \cdot$$

Then the projections of the mechanism onto horizontal and vertical lines will give the relations:

$$E \cos \phi = H \cos \theta + C \cos \beta + D \cos \gamma - F$$

$$E \sin \phi = H \sin \theta + C \sin \beta + D \sin \gamma . \qquad (7\text{-}7)$$

Solving for D cos γ and D sin γ from these equations, squaring them, and then adding the resulting equations together will produce:

$$D^2 = E^2 + H^2 + C^2 + F^2 - 2CF \cos \beta - 2\,FH \cos \theta + 2EF \cos \phi$$

$$- 2EH \cos (\phi - \theta) - 2CE \cos (\phi - \beta) + 2\,CH \cos (\theta - \beta) . \qquad (7\text{-}8)$$

With β given in terms of θ by Eq. 7-6, the expression of Eq. 7-8 involves the two variables θ and ϕ. Upon specifying a value for θ the value for ϕ may be determined. The Newton-Raphson method has been very satisfactory for obtaining values for ϕ the results of which are indicated by the solid lined curves of the attached graphs.

Angular Velocity

An expression for $d\phi/d\theta$ may be obtained by differentiating Eq. 7-8 with respect to θ. Doing so results in:

$$\frac{d\phi}{d\theta} = \left\{ EH \sin(\phi - \theta) - FC \sin\beta \frac{d\beta}{d\theta} - FH \sin\theta + CE \sin(\phi - \beta) \frac{d\beta}{d\theta} \right.$$

$$\left. + \left[CH \sin(\theta - \beta)\right]\left[1 - \frac{d\beta}{d\theta}\right]\right\} \Big/ \left[EH \sin(\phi - \theta) - EF \sin\phi + CE \sin(\phi - \beta)\right] \tag{7-9}$$

in which

$$\frac{d\beta}{d\theta} = 1 + \frac{A}{B}.$$

Having determined ϕ in terms θ from Eq. 7-8, $d\phi/d\theta$ may be calculated using Eqs. 7-9 and 7-6. The angular velocity of the output, link E, may be evaluated with the application of Eq. 7-3.

Angular Acceleration

Eq. 7-9 may be differentiated with respect to θ to yield an expression for $d^2\phi/d\theta^2$ as:

$$\frac{d^2\phi}{d\theta^2} = \left\{ \left[EH \cos(\phi - \theta) - EF \cos\phi + CE \cos(\phi - \beta)\right]\left[\frac{d\phi}{d\theta}\right]^2 \right.$$

$$- \left[2\, EH \cos(\phi - \theta) + 2\, CE \cos(\phi - \beta)\frac{d\beta}{d\theta}\right]\left[\frac{d\phi}{d\theta}\right]$$

$$+ \left[CF \cos\beta + CE \cos(\phi - \beta)\right]\left[\frac{d\beta}{d\phi}\right]^2 + FH \cos\theta + EH \cos(\phi - \theta)$$

$$\left. - CH \cos(\theta - \beta)\left[1 - \frac{d\beta}{d\theta}\right]^2\right\} \Big/ \left[EF \sin\phi - EH \sin(\phi - \theta) - CE \sin(\phi - \beta)\right]. \tag{7-10}$$

This expression for $d^2\phi/d\theta^2$ has been used for constructing the curves, which are a series of small triangles, on the graphs which follow. As noted before the more conventional units for the angular acceleration of link E is obtained by using Eq. 7-5.

This mechanism offers a great variety of form for input-output motion transformations. A thorough understanding or "feeling" for using this mechanism is not easily attained. The fact that epicyclic gear trains are quite commonplace does add an incentive to learn more about Mechanism #7. Note that if the ratio of A to B is irrational, the resulting displacement curve (that is, ϕ versus θ) will not by cyclic. Even for an integer ratio of A to B less than 10, the motion transformations become extremely complex. Of course, this complexity might in itself be a challenge in that the possibilities of matching complicated displacements surely will require something other than an elementary mechanism.

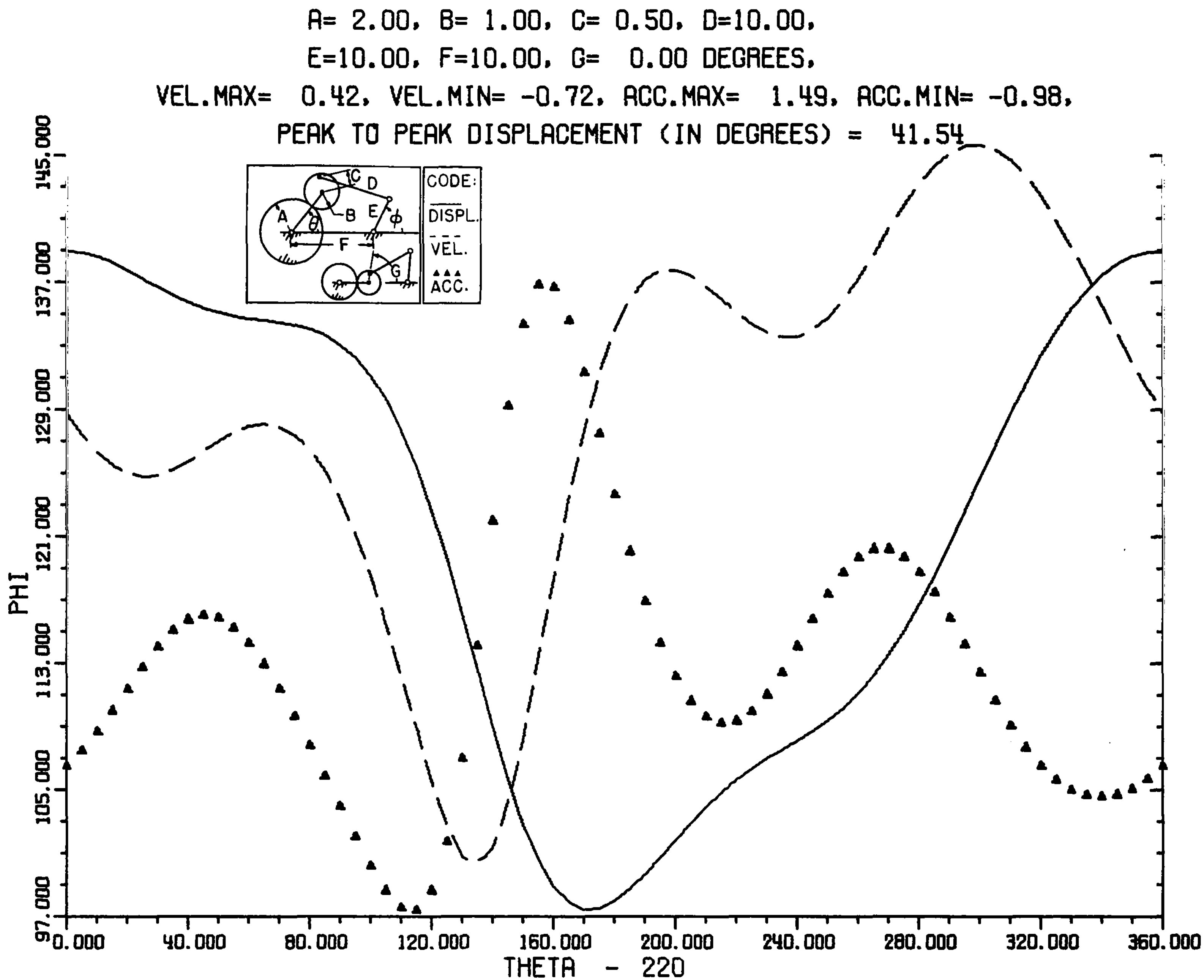

A= 2.00, B= 1.00, C= 0.50, D=10.00,
E=10.00, F=10.00, G= 0.00 DEGREES,
VEL.MAX= 0.42, VEL.MIN= -0.72, ACC.MAX= 1.49, ACC.MIN= -0.98,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 41.54
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
PHI
97.000
105.000
113.000
121.000
129.000
137.000
145.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 220

PLATE 7-1

A= 2.00, B= 1.00, C= 0.50, D=10.00,
E=10.00, F=10.00, G=270.00,
VEL.MAX= 0.37, VEL.MIN= -0.59, ACC.MAX= 1.42, ACC.MIN= -0.73,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 46.50

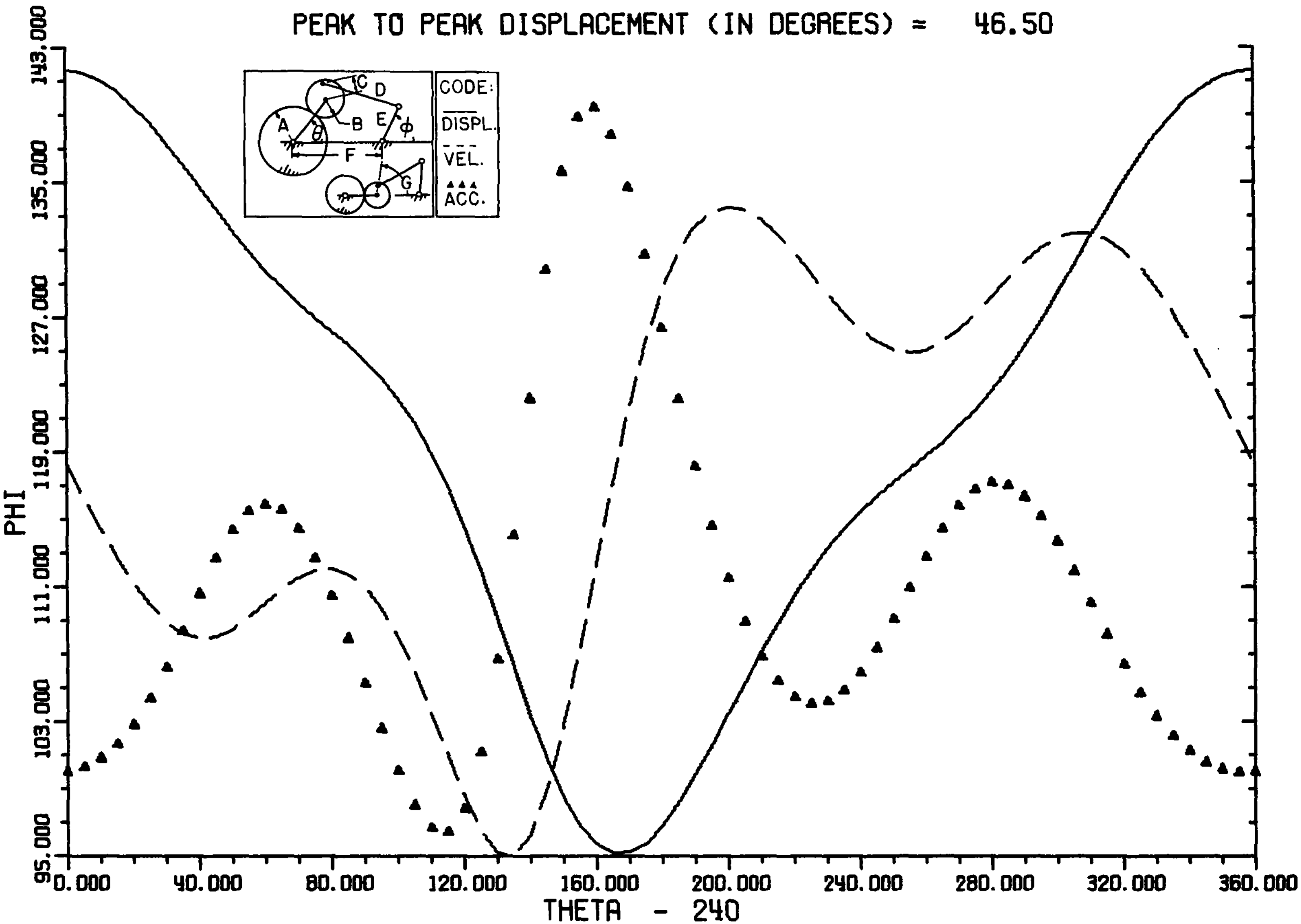


PLATE 7-2

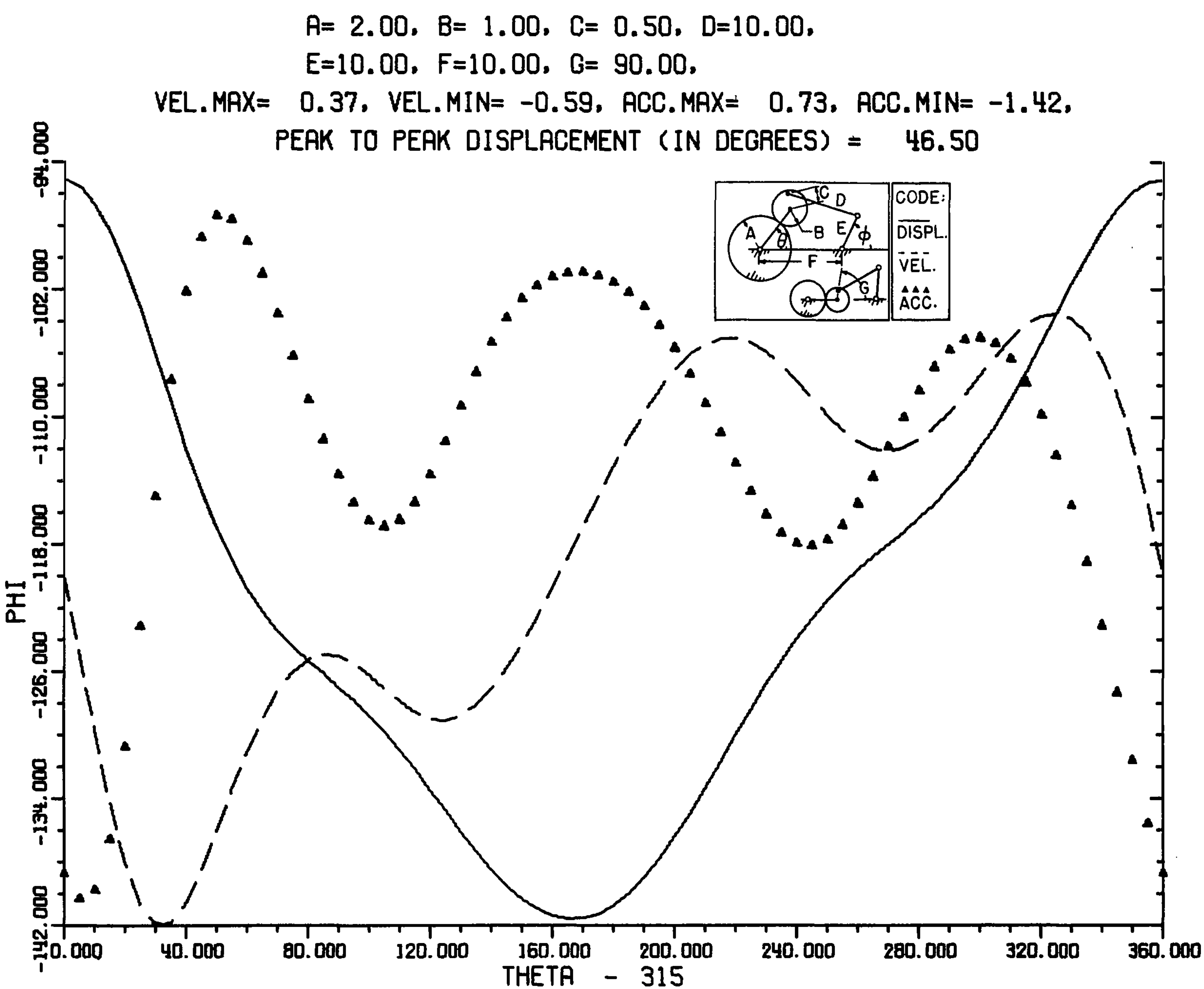

A= 2.00, B= 1.00, C= 0.50, D=10.00,
E=10.00, F=10.00, G= 90.00,
VEL.MAX= 0.37, VEL.MIN= -0.59, ACC.MAX= 0.73, ACC.MIN= -1.42,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 46.50
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
PHI
-94.000
-102.000
-110.000
-118.000
-126.000
-134.000
-142.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 315

PLATE 7-3

A= 2.00, B= 2.00, C= 1.00, D=10.00,
E=14.00, F=10.00, G=180.00 DEGREES,
VEL.MAX= 0.44, VEL.MIN= -0.40, ACC.MAX= 0.38, ACC.MIN= -0.76,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 47.96

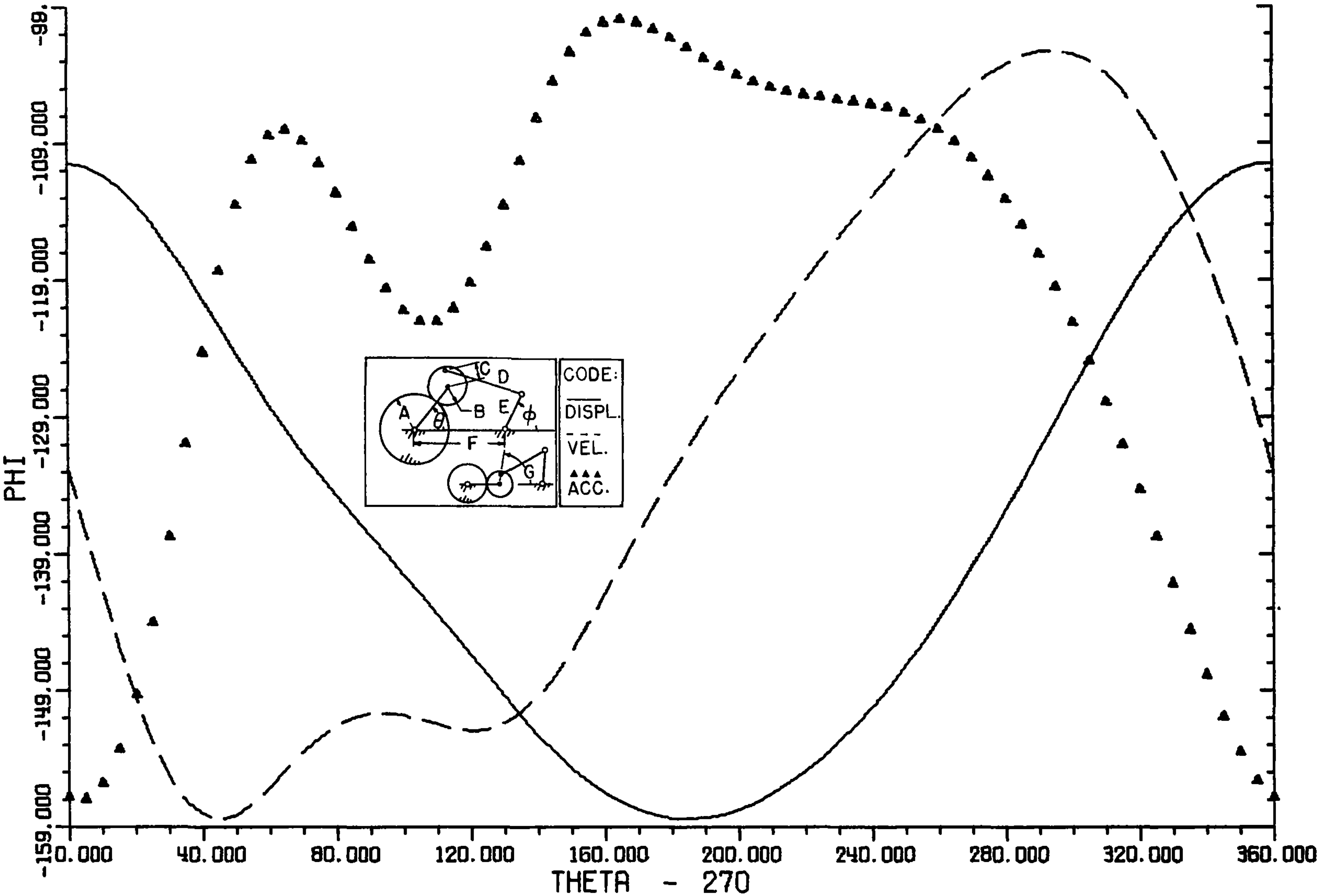
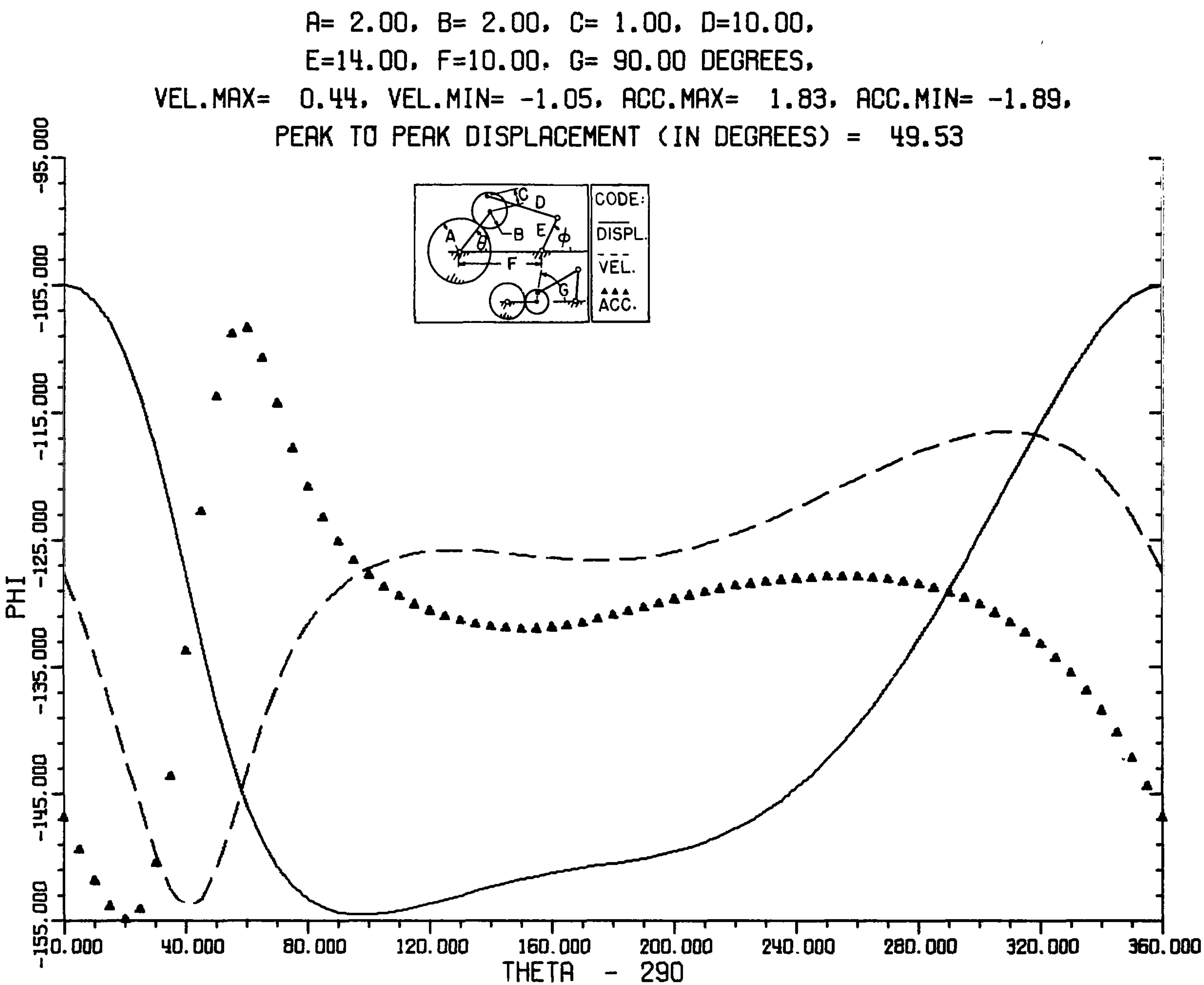


PLATE 7-4



A= 2.00, B= 2.00, C= 1.00, D=10.00,
E=14.00, F=10.00, G= 90.00 DEGREES,
VEL.MAX= 0.44, VEL.MIN= -1.05, ACC.MAX= 1.83, ACC.MIN= -1.89,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 49.53
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ
PHI
-95.000
-105.000
-115.000
-125.000
-135.000
-145.000
-155.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 290

PLATE 7-5

A= 2.00, B= 2.00, C= 1.50, D=10.00,
E=14.00, F=10.00, G=270.00 DEGREES,
VEL.MAX= 0.42, VEL.MIN= -1.25, ACC.MAX= 2.74, ACC.MIN= -2.22,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 52.03

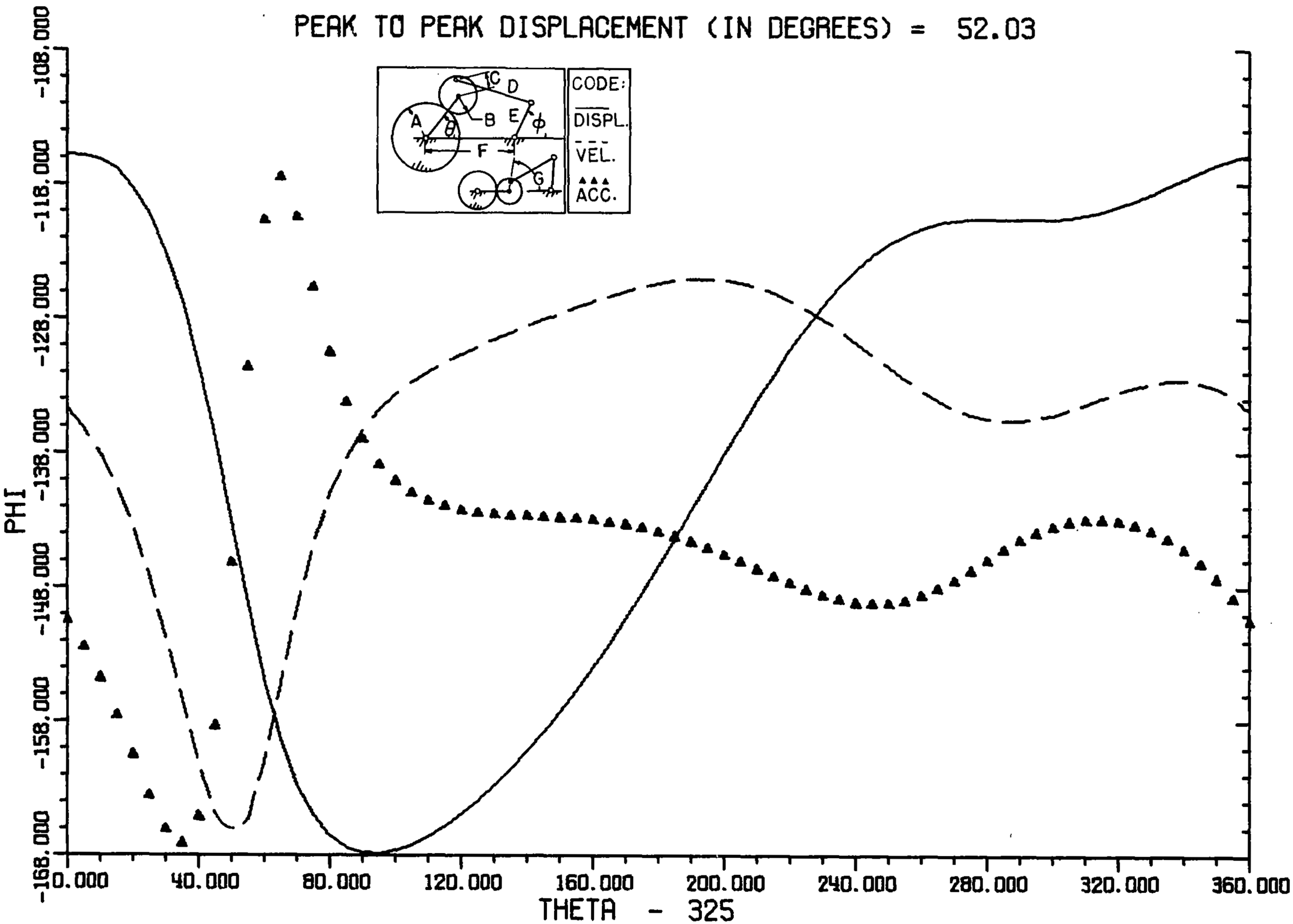


PLATE 7-6

A= 2.00, B= 2.00, C= 1.50, D=10.00,
E=14.00, F=10.00, G= 90.00 DEGREES,
VEL.MAX= 0.51, VEL.MIN= -1.35, ACC.MAX= 3.00, ACC.MIN= -2.74,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 53.81

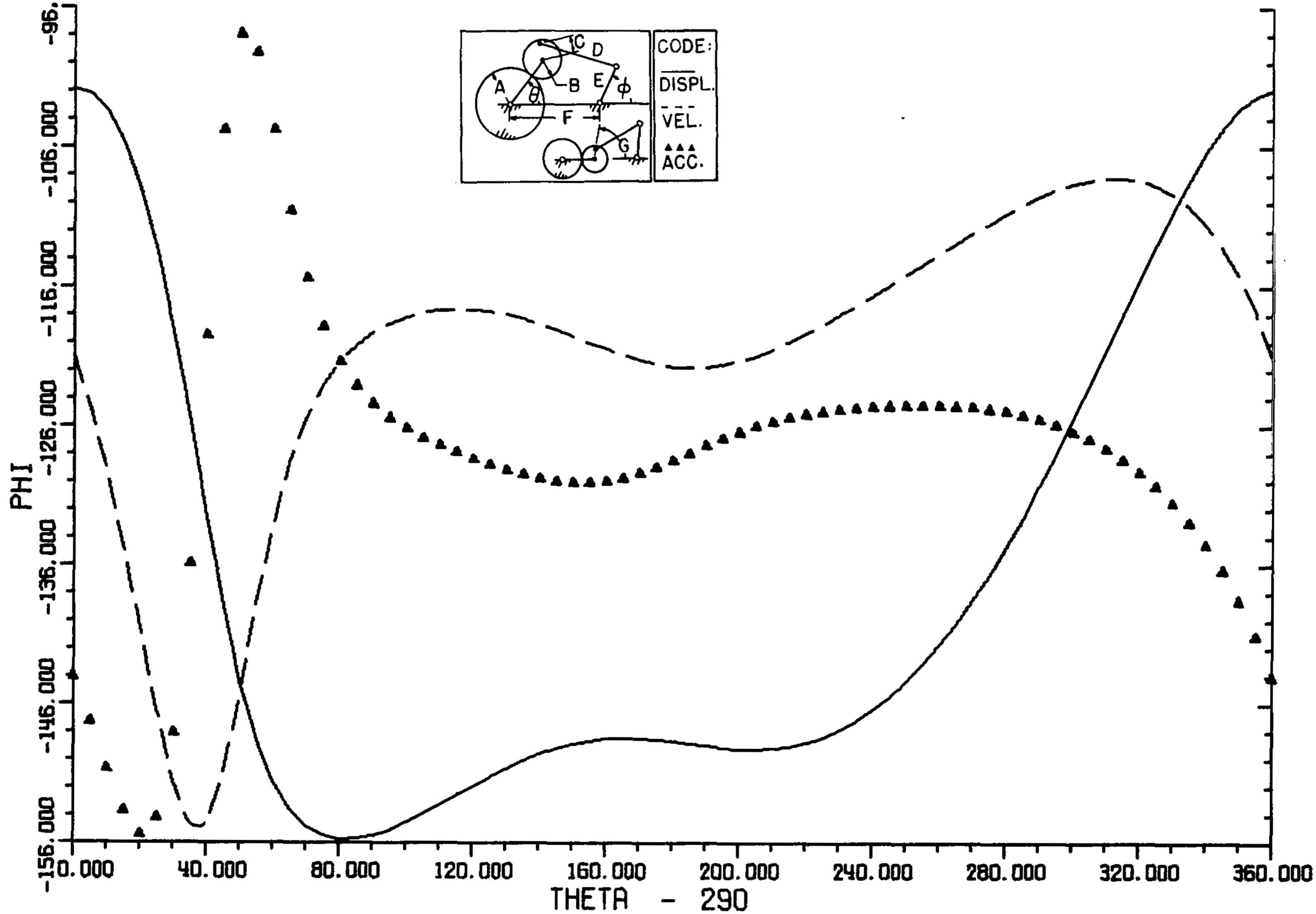


PLATE 7-7

A= 2.00, B= 2.00, C= 1.00, D=10.00,
E=14.00, F=10.00, G= 0.00 DEGREES,
VEL.MAX= 0.33, VEL.MIN= -1.46, ACC.MAX= 3.47, ACC.MIN= -2.72,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 57.32

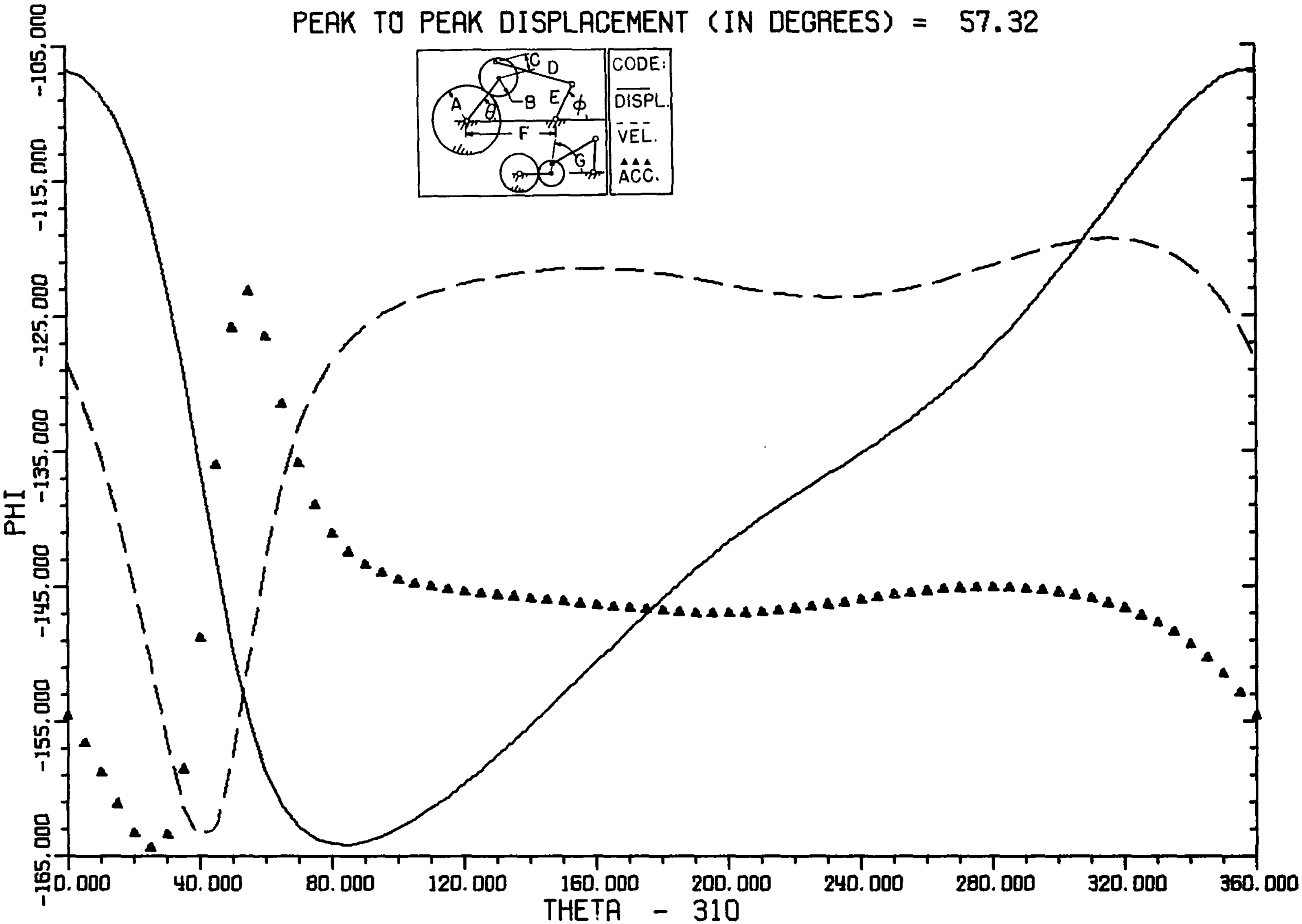


PLATE 7-8

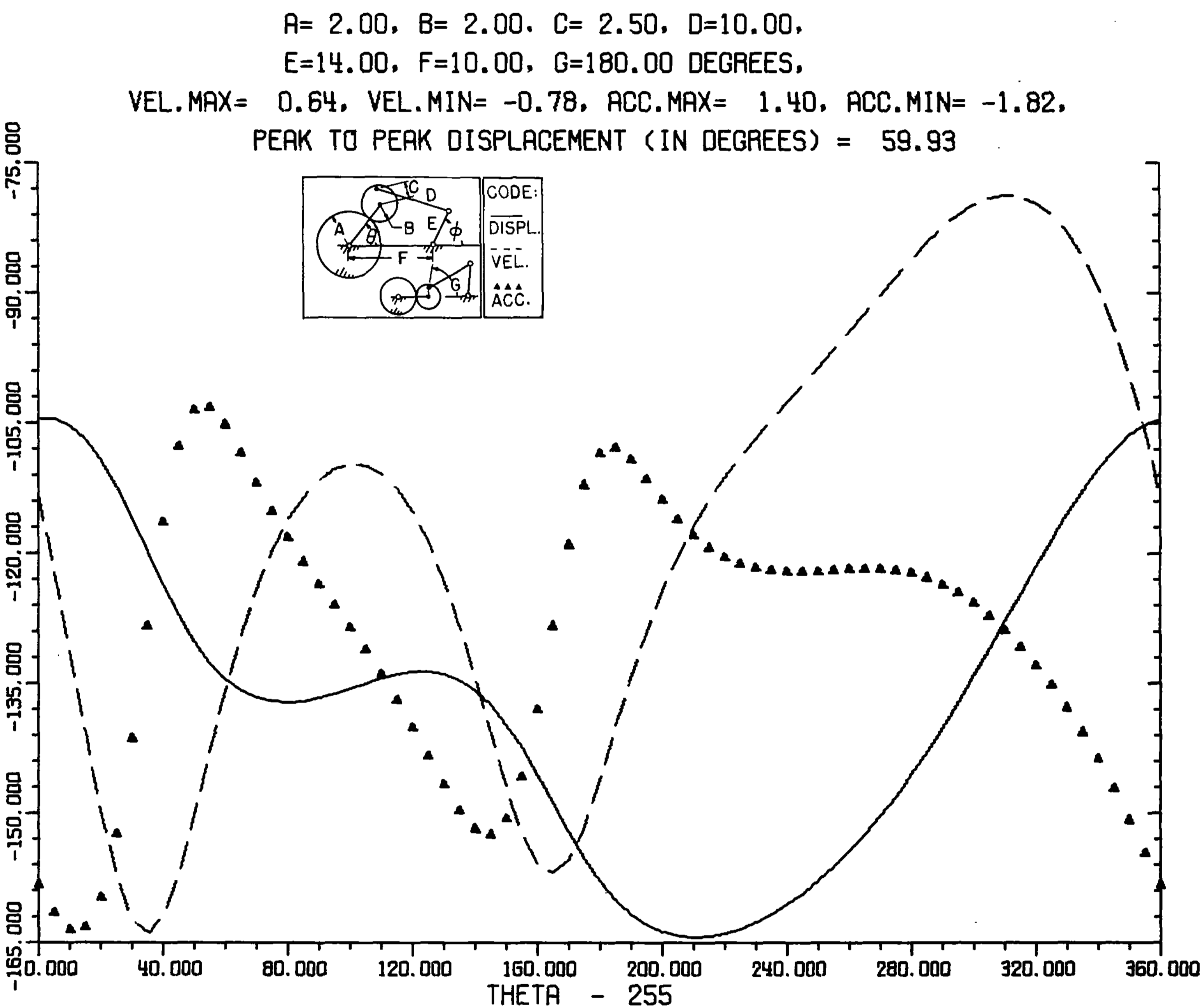

A= 2.00, B= 2.00, C= 2.50, D=10.00,
E=14.00, F=10.00, G=180.00 DEGREES,
VEL.MAX= 0.64, VEL.MIN= -0.78, ACC.MAX= 1.40, ACC.MIN= -1.82,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 59.93
CODE:
DISPL.
VEL.
ACC.
PHI
-75.000
-90.000
-105.000
-120.000
-135.000
-150.000
-165.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 255

PLATE 7-9

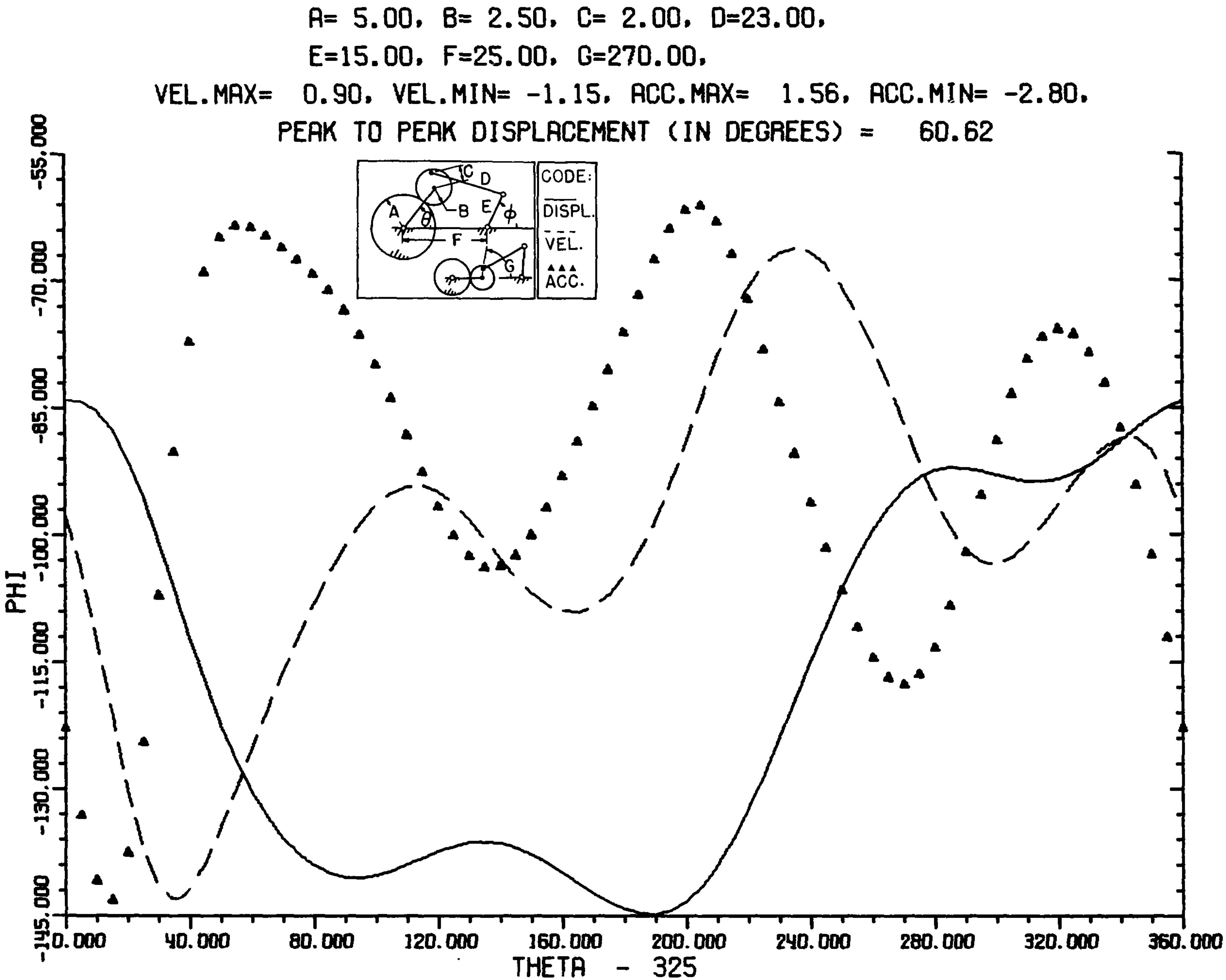

A= 5.00, B= 2.50, C= 2.00, D=23.00,
E=15.00, F=25.00, G=270.00,
VEL.MAX= 0.90, VEL.MIN= -1.15, ACC.MAX= 1.56, ACC.MIN= -2.80,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 60.62
CODE:
DISPL.
VEL.
ACC.
PHI
THETA - 325
-55.000
-70.000
-85.000
-100.000
-115.000
-130.000
-145.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000

PLATE 7-10

A= 5.00, B= 2.50, C= 1.50, D=27.00,
E=15.00, F=25.00, G= 90.00,
VEL.MAX= 0.77, VEL.MIN= -1.20, ACC.MAX= 2.93, ACC.MIN= -1.48,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 63.94

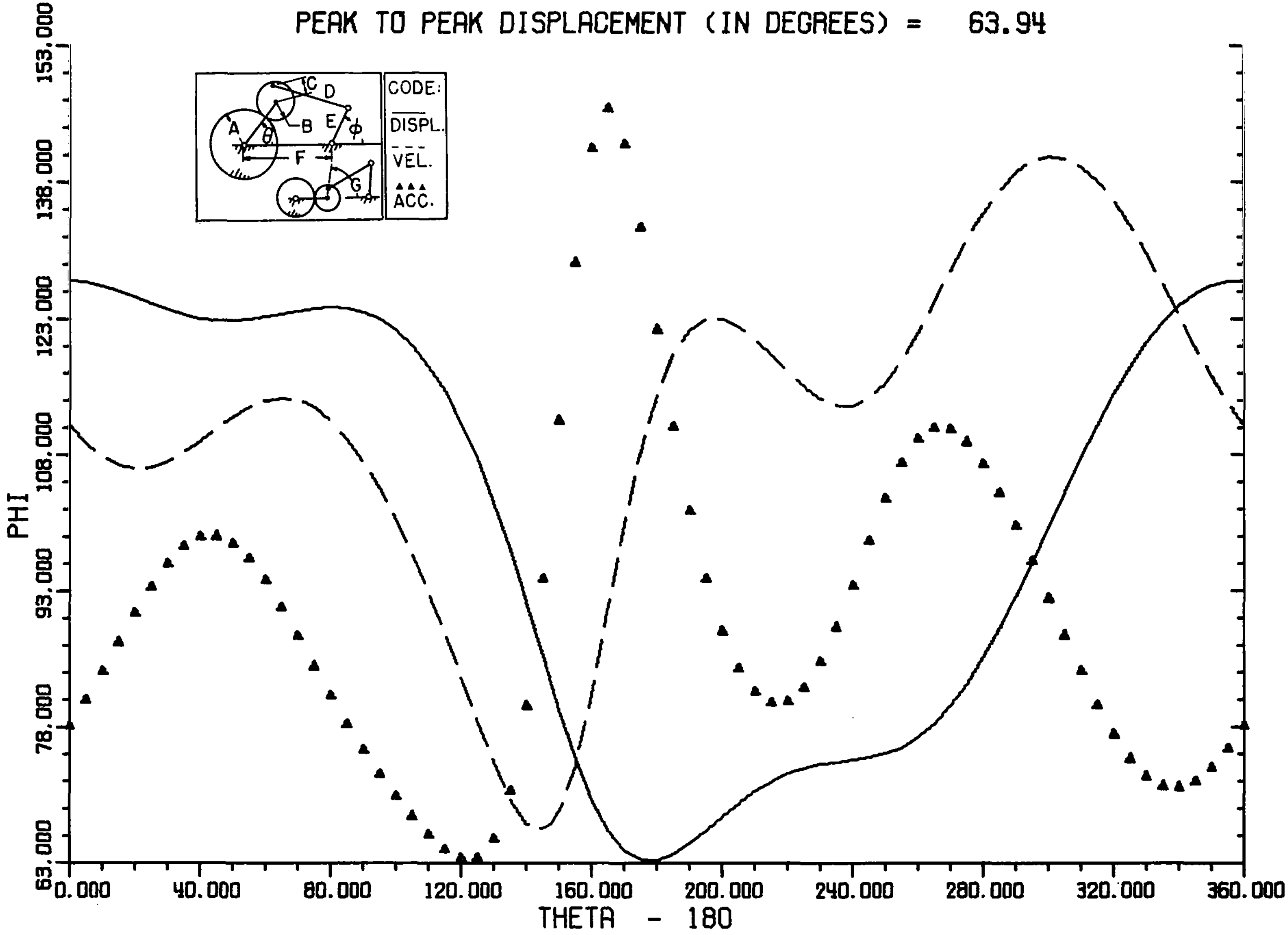
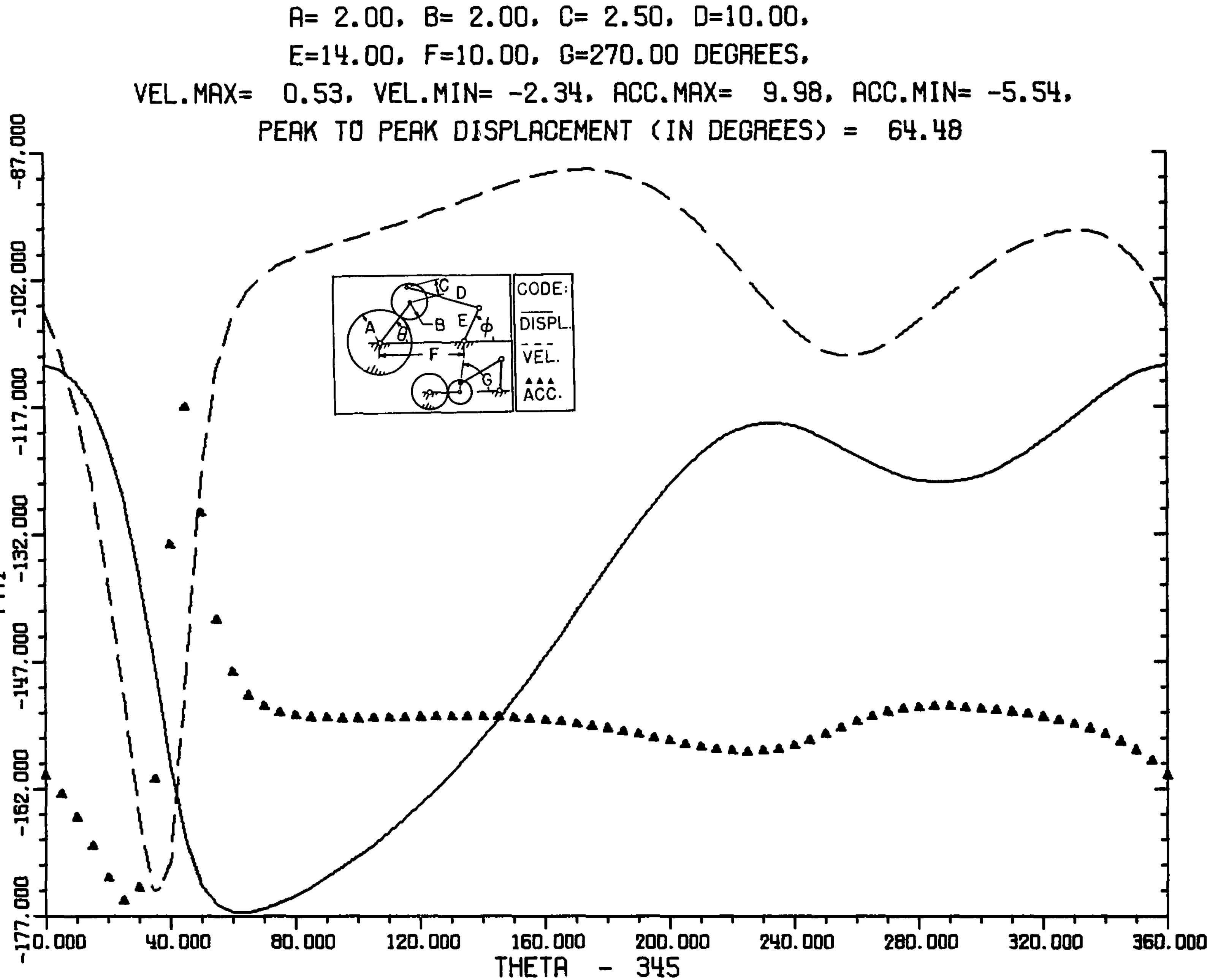


PLATE 7-11



A= 2.00, B= 2.00, C= 2.50, D=10.00,
E=14.00, F=10.00, G=270.00 DEGREES,
VEL.MAX= 0.53, VEL.MIN= -2.34, ACC.MAX= 9.98, ACC.MIN= -5.54,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 64.48
PHI
-87.000
-102.000
-117.000
-132.000
-147.000
-162.000
-177.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 345
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ

PLATE 7 - 12

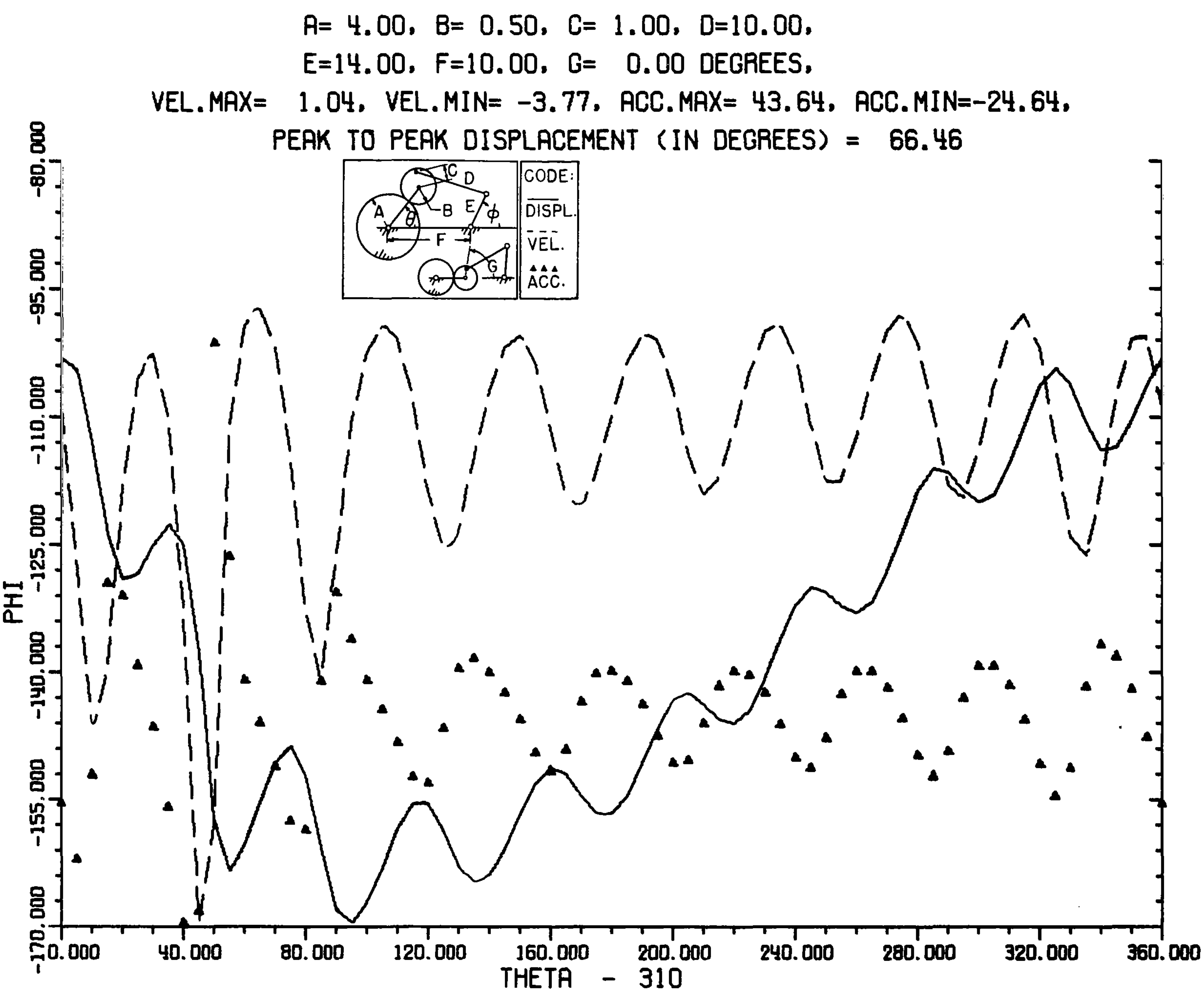

A= 4.00, B= 0.50, C= 1.00, D=10.00,
E=14.00, F=10.00, G= 0.00 DEGREES,
VEL.MAX= 1.04, VEL.MIN= -3.77, ACC.MAX= 43.64, ACC.MIN=-24.64,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 66.46
A
B
C
D
E
F
G
θ
φ
CODE:
DISPL.
VEL.
ACC.
PHI
-80.000
-95.000
-110.000
-125.000
-140.000
-155.000
-170.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 310

PLATE 7-13

A= 5.00, B= 2.50, C= 2.00, D=27.00,
E=15.00, F=25.00, G=270.00,
VEL.MAX= 0.87, VEL.MIN= -1.40, ACC.MAX= 1.86, ACC.MIN= -3.86,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 67.38

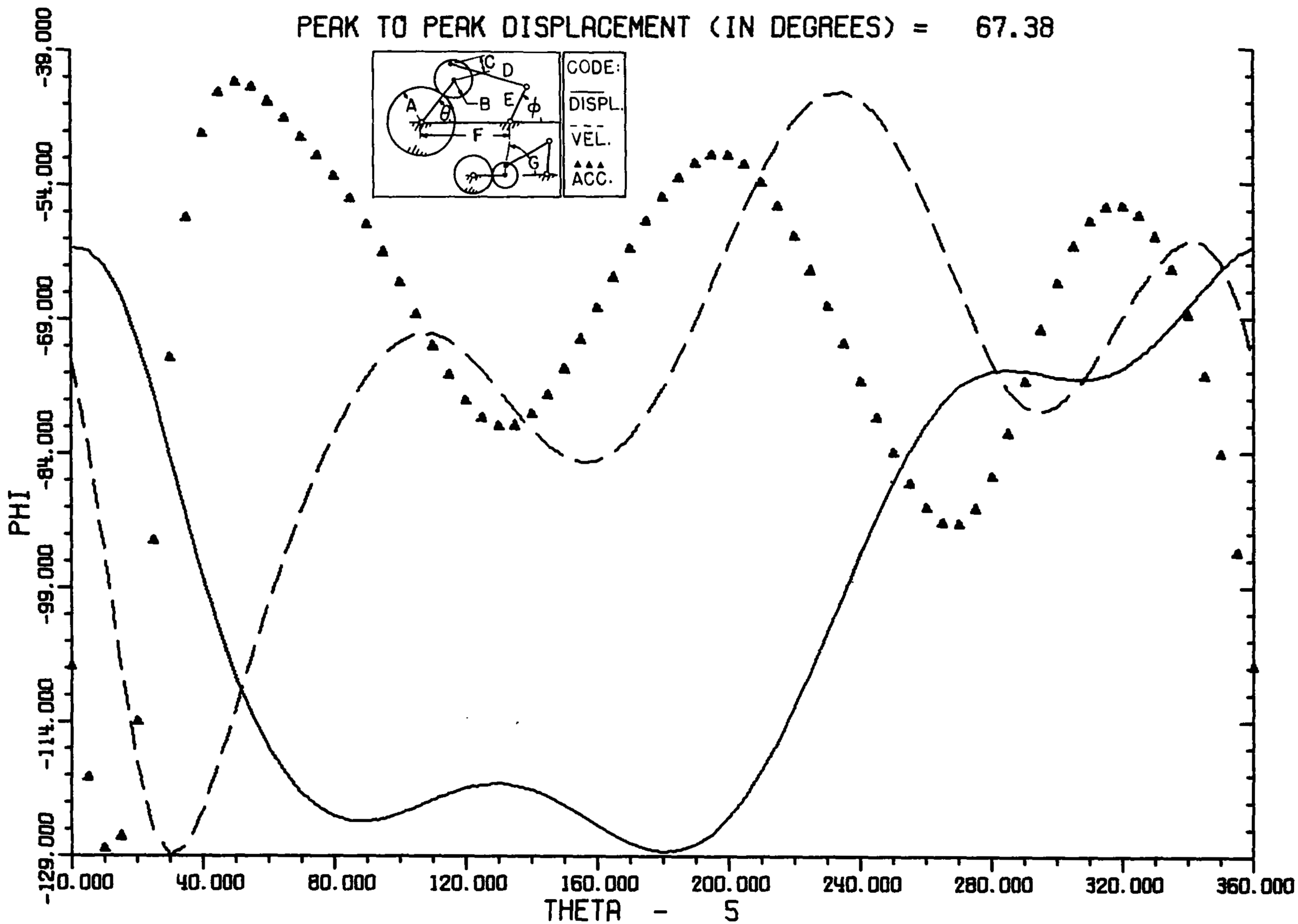


PLATE 7-14

A= 5.00, B= 2.50, C= 2.00, D=27.00,
E=15.00, F=25.00, G= 90.00,
VEL.MAX= 0.87, VEL.MIN= -1.40, ACC.MAX= 3.86, ACC.MIN= -1.86,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 67.38

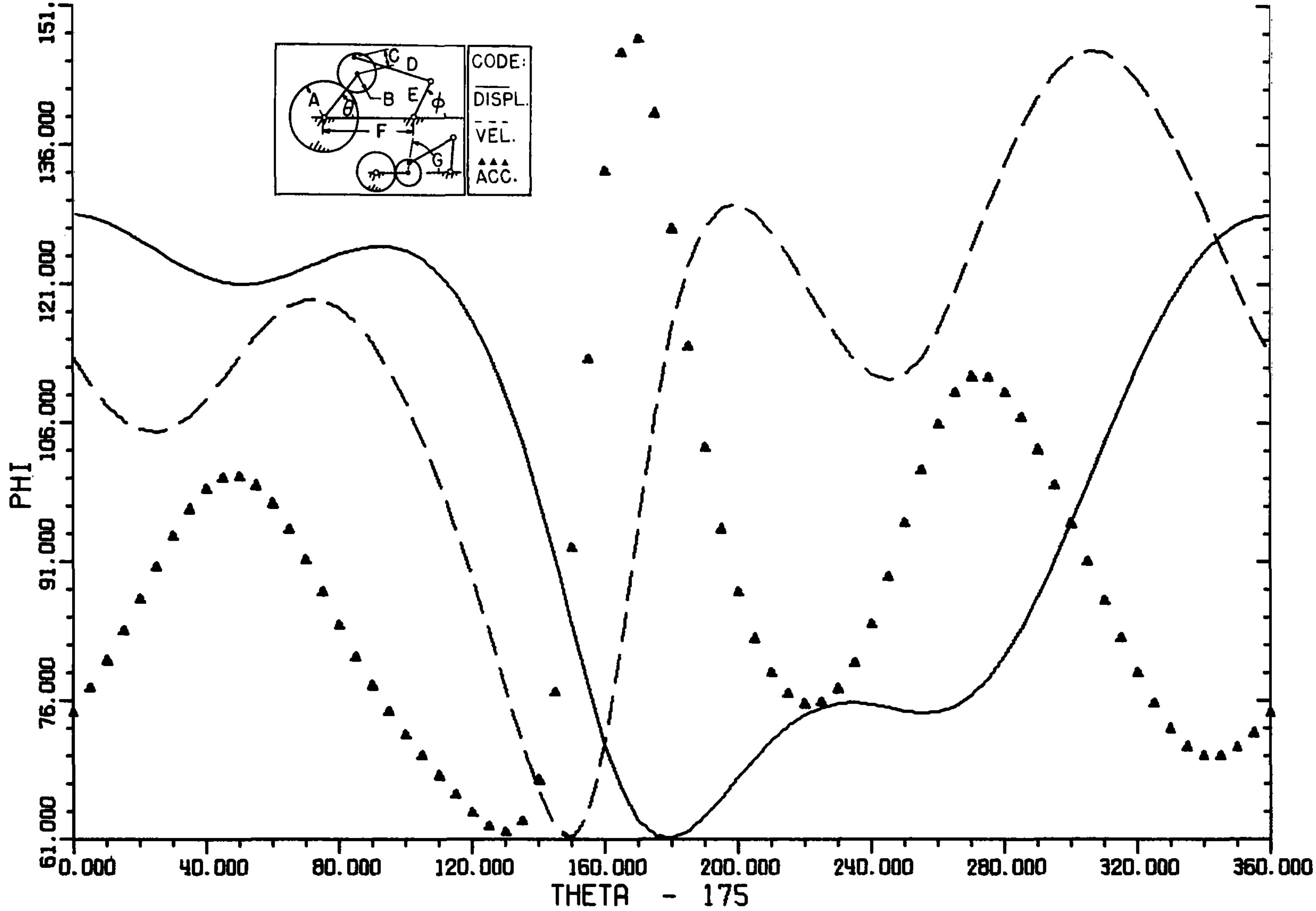


PLATE 7-15

A= 4.00, B= 0.50, C= 1.00, D=10.00,
E=14.00, F=10.00, G=180.00 DEGREES,
VEL.MAX= 1.02, VEL.MIN= -3.52, ACC.MAX= 29.60, ACC.MIN=-24.69,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 67.54

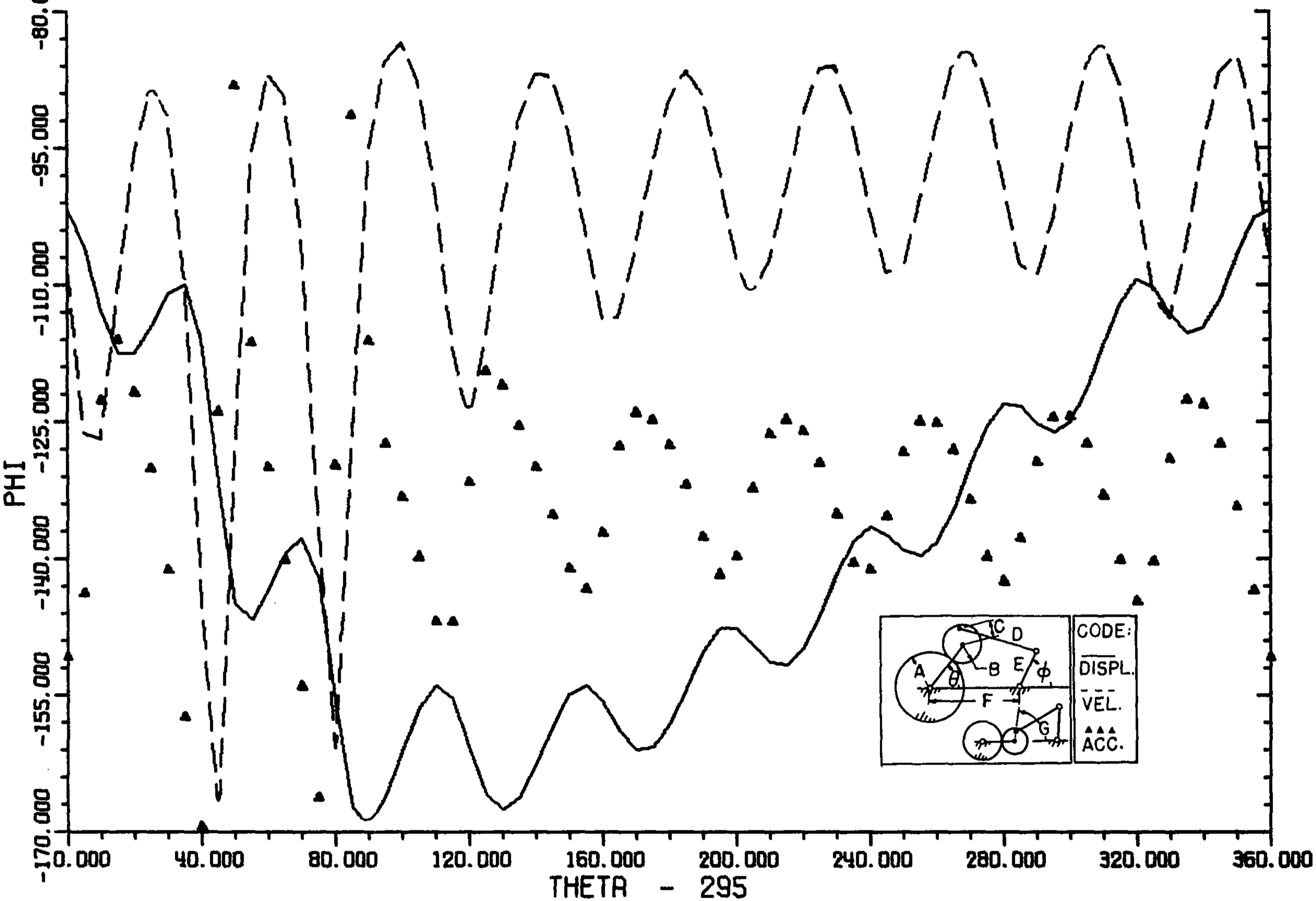
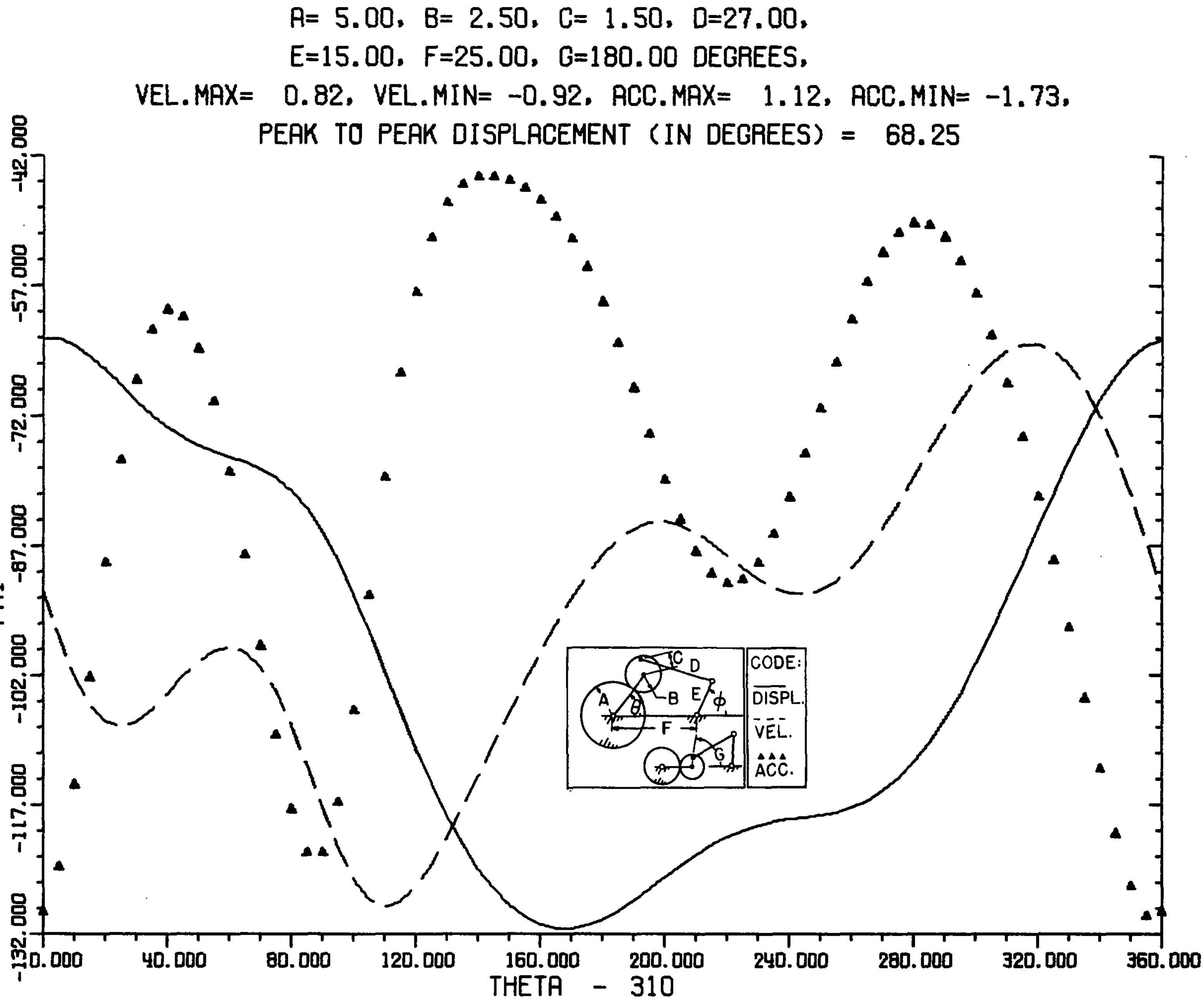


PLATE 7-16

PLATE 7-17


A= 5.00, B= 2.50, C= 1.50, D=27.00,
E=15.00, F=25.00, G=180.00 DEGREES,
VEL.MAX= 0.82, VEL.MIN= -0.92, ACC.MAX= 1.12, ACC.MIN= -1.73,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 68.25
PHI
-42.000
-57.000
-72.000
-87.000
-102.000
-117.000
-132.000
0.000
40.000
80.000
120.000
160.000
200.000
240.000
280.000
320.000
360.000
THETA - 310
CODE:
DISPL.
VEL.
ACC.
A
B
C
D
E
F
G
θ
φ

A= 5.00, B= 2.50, C= 2.00, D=27.00,
E=15.00, F=25.00, G=180.00 DEGREES,
VEL.MAX= 0.92, VEL.MIN= -1.05, ACC.MAX= 1.34, ACC.MIN= -2.28,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 71.75

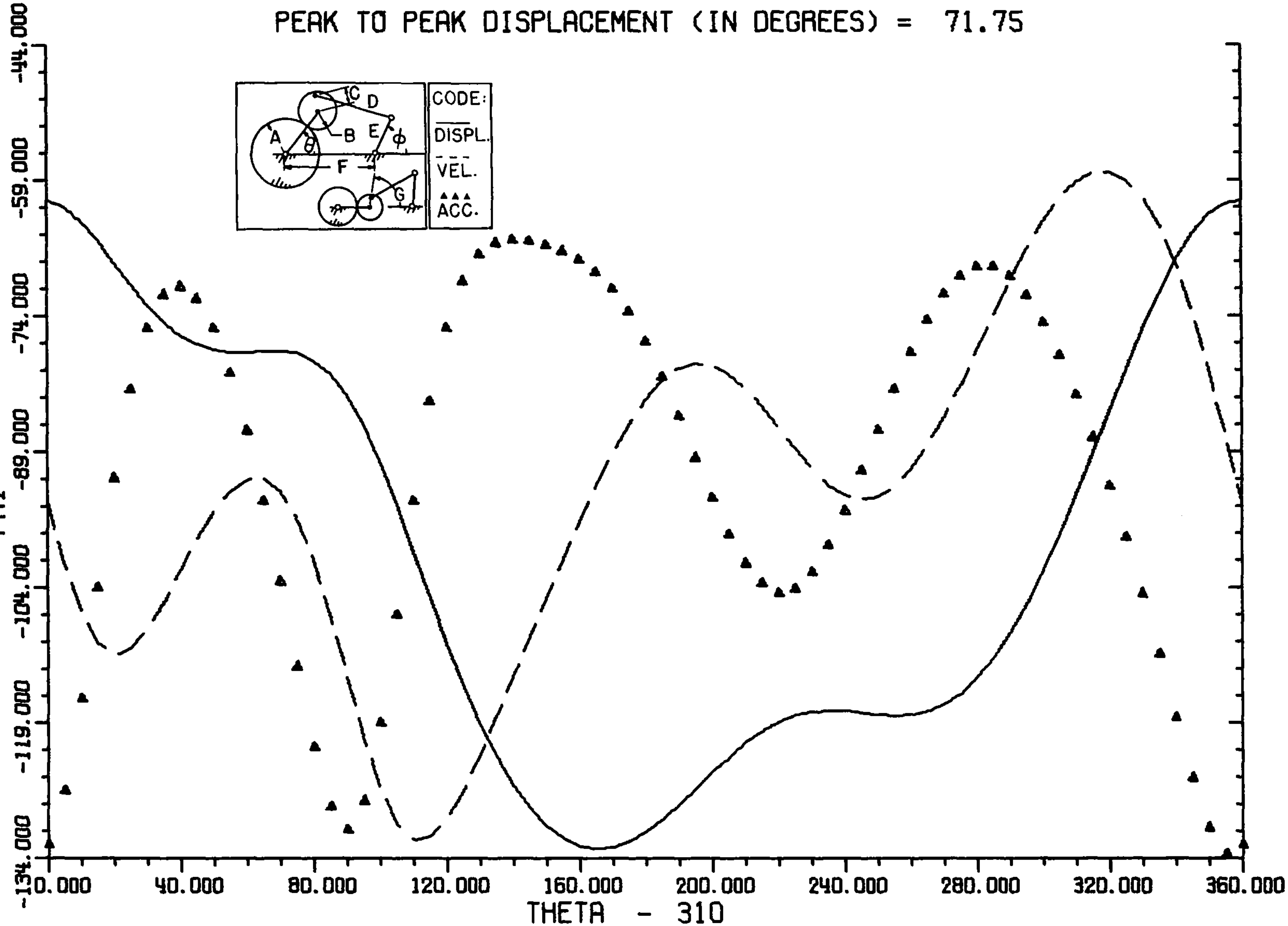


PLATE 7-18

A= 4.00, B= 0.50, C= 1.00, D=16.00,
E=15.00, F=10.00, G= 0.00 DEGREES,
VEL.MAX= 1.20, VEL.MIN= -3.00, ACC.MAX= 17.45, ACC.MIN=-24.33,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 72.47

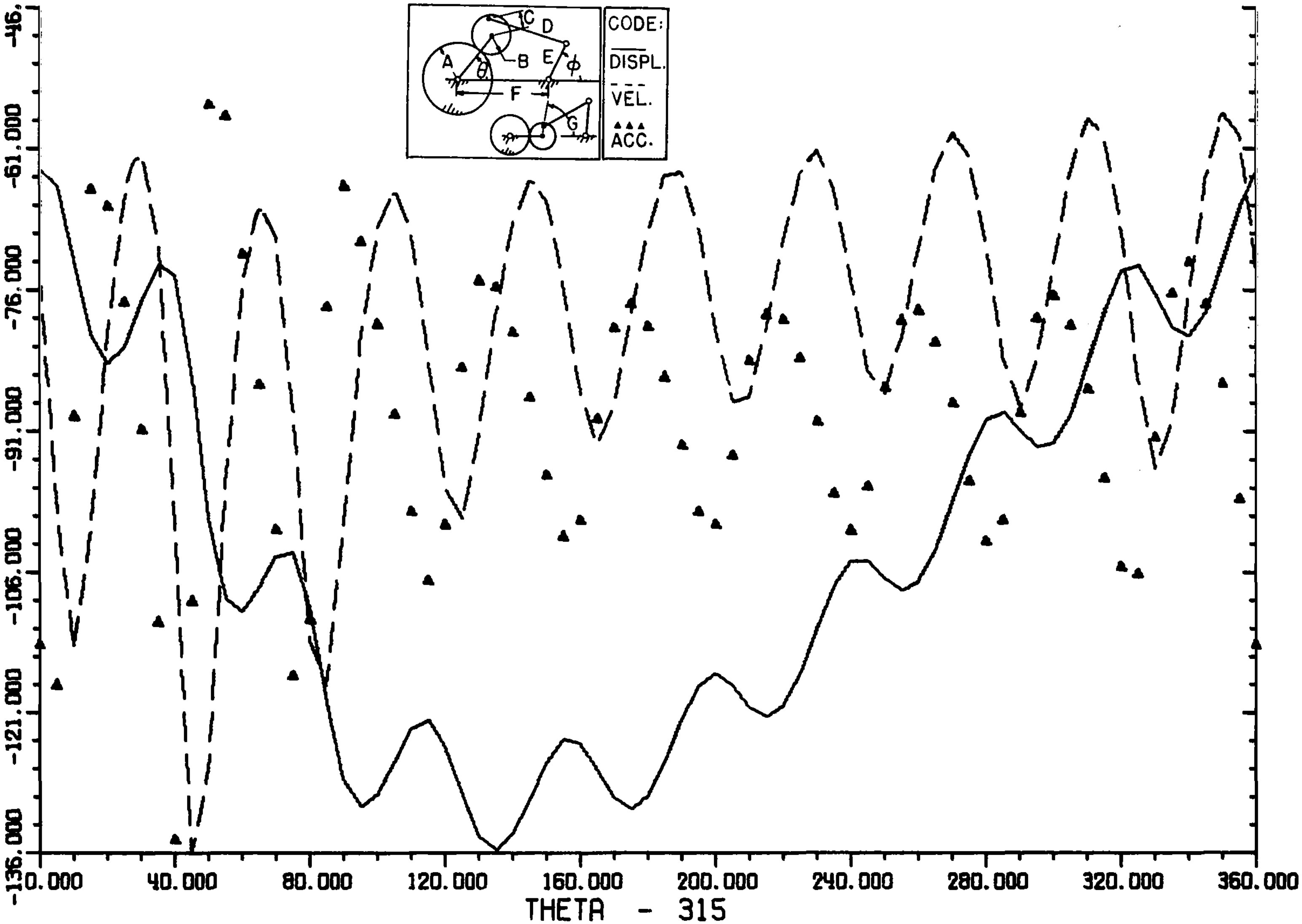


PLATE 7-19

A= 5.00, B= 2.50, C= 2.00, D=23.00,
E=15.00, F=25.00, G= 90.00,
VEL.MAX= 0.81, VEL.MIN= -0.91, ACC.MAX= 1.42, ACC.MIN= -2.98,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 77.88

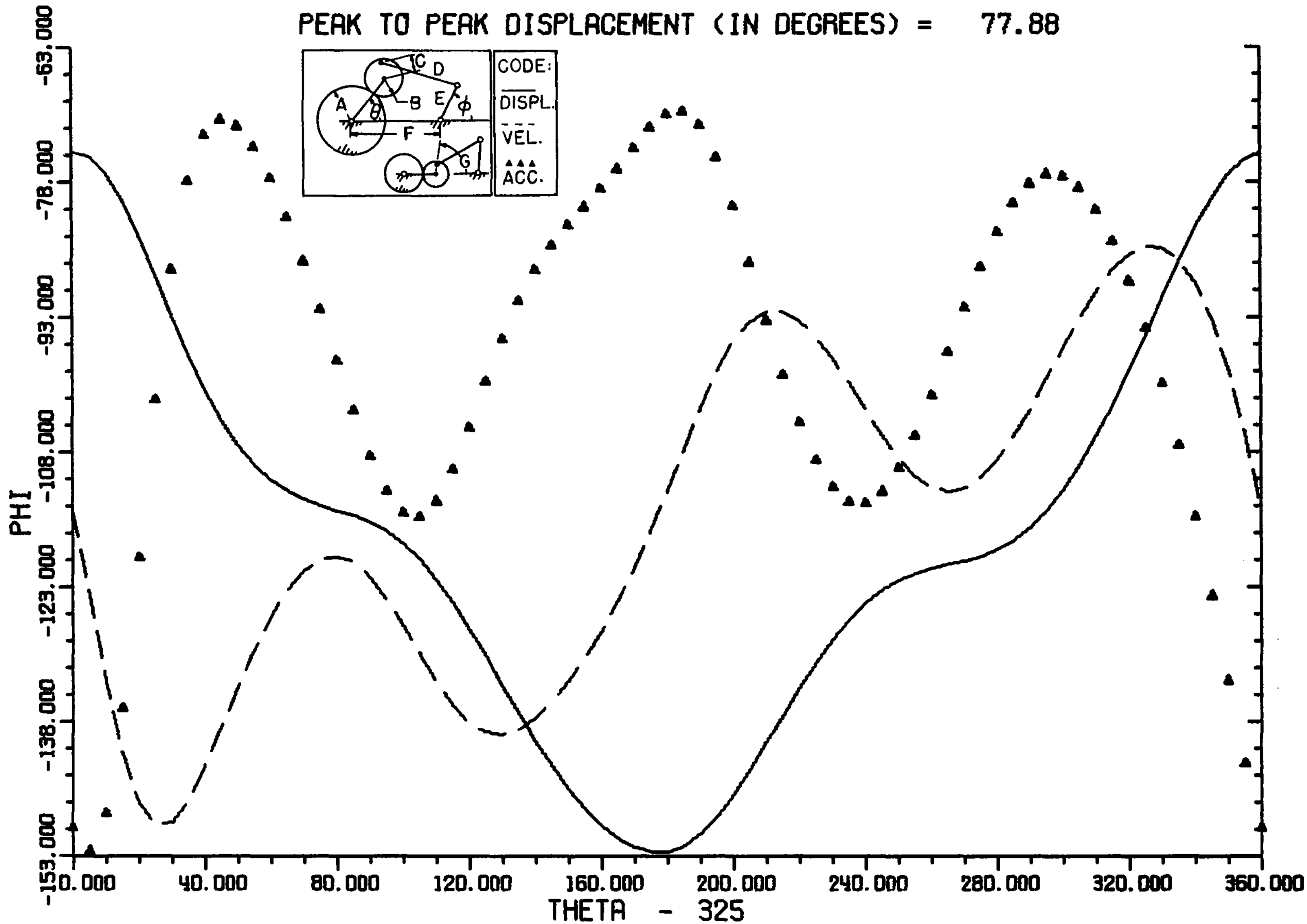


PLATE 7-20

A= 4.00, B= 2.00, C= 1.00, D=12.00,
E=15.00, F=10.00, G=270.00 DEGREES,
VEL.MAX= 0.55, VEL.MIN= -2.96, ACC.MAX= 8.16, ACC.MIN=-11.84,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 78.09

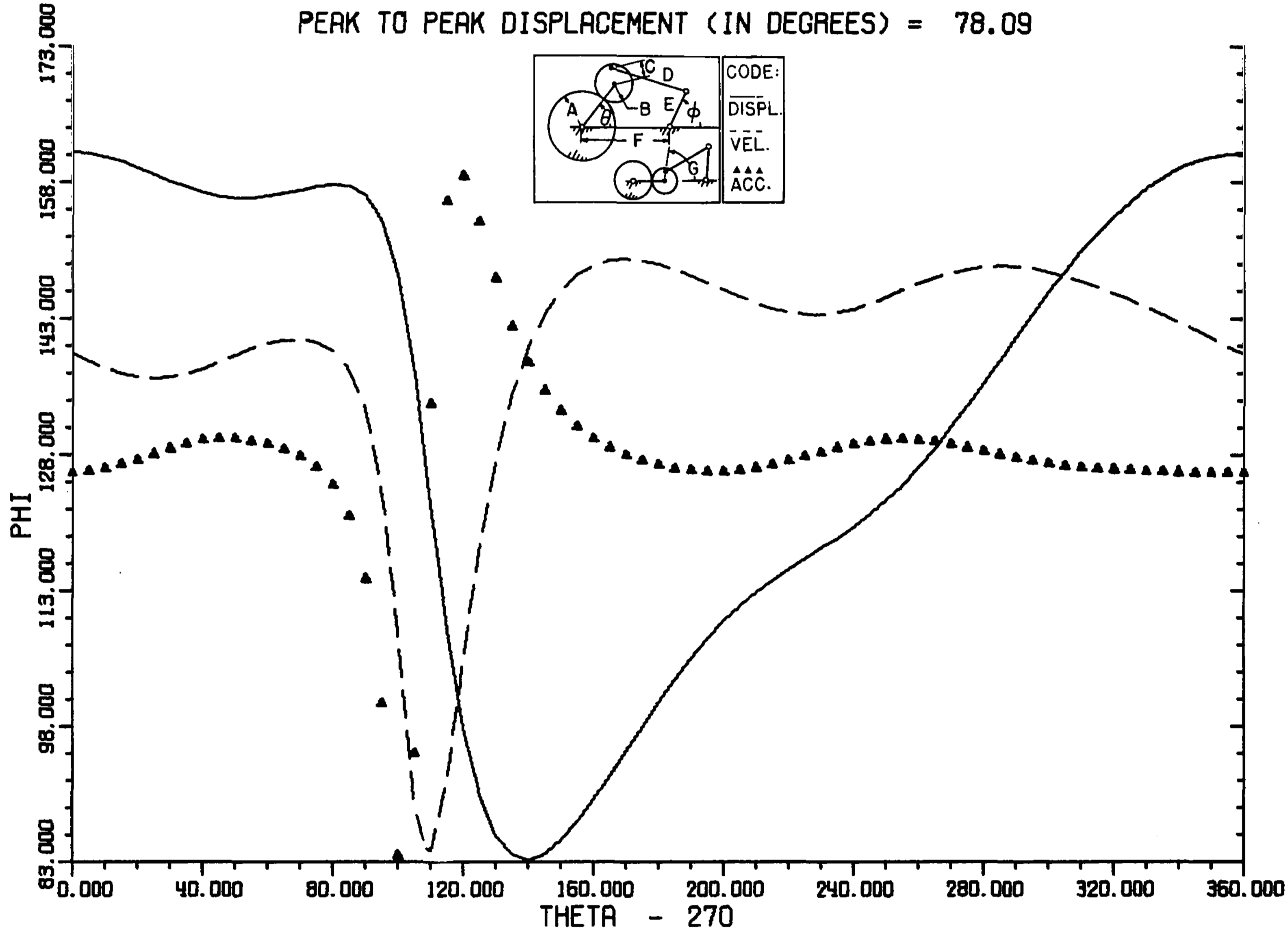


PLATE 7-21

A= 4.00, B= 2.00, C= 1.00, D=12.00,
E=15.00, F=10.00, G= 90.00 DEGREES,
VEL.MAX= 0.55, VEL.MIN= -2.96, ACC.MAX= 11.84, ACC.MIN= -8.16,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 78.09

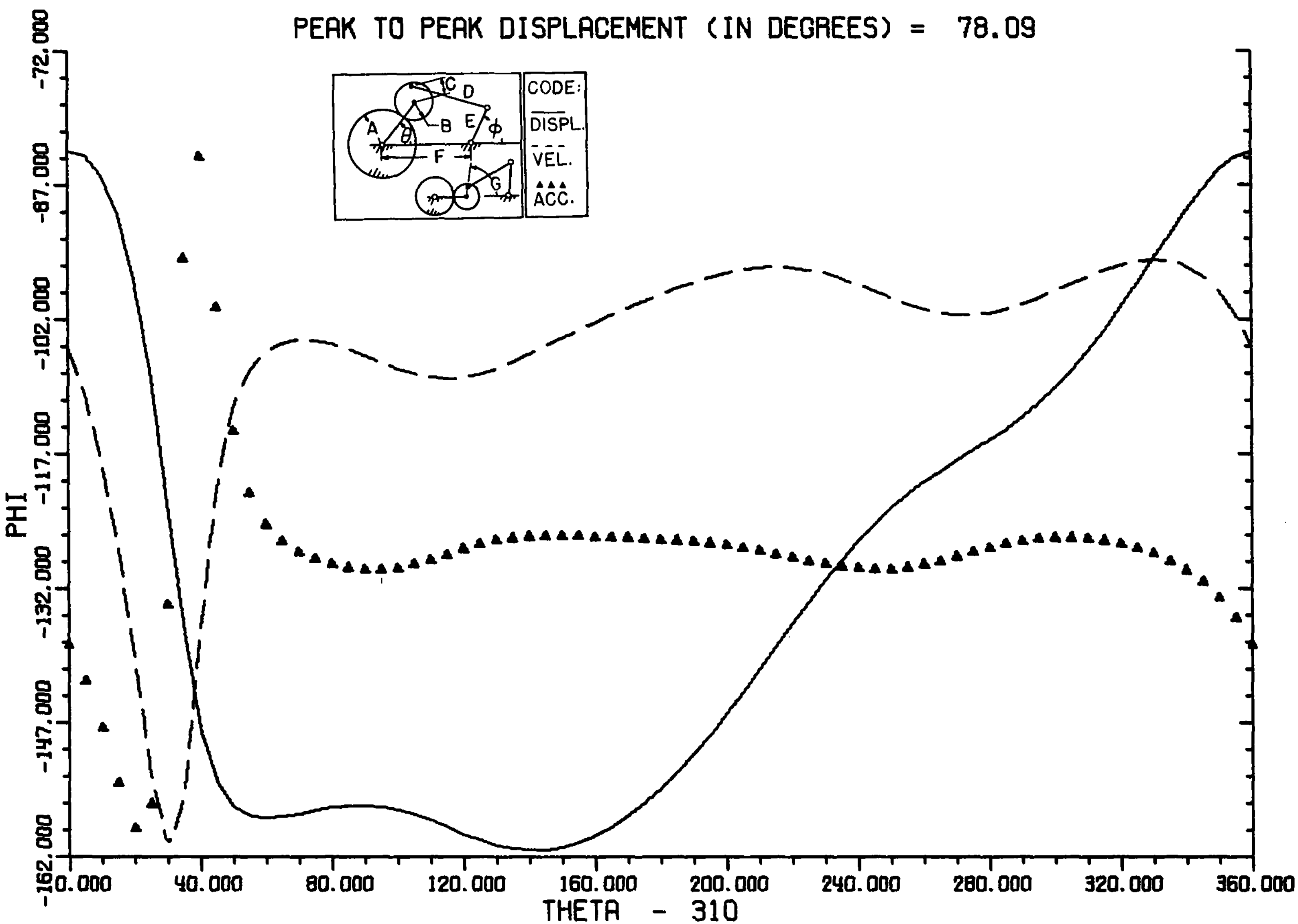


PLATE 7-22

A= 6.00, B= 3.00, C= 1.00, D=15.00,
E=15.00, F=12.00, G=270.00 DEGREES,
VEL.MAX= 0.80, VEL.MIN= -3.83, ACC.MAX= 11.48, ACC.MIN=-13.05,
PEAK TO PEAK DISPLACEMENT (IN DEGREES) = 100.61

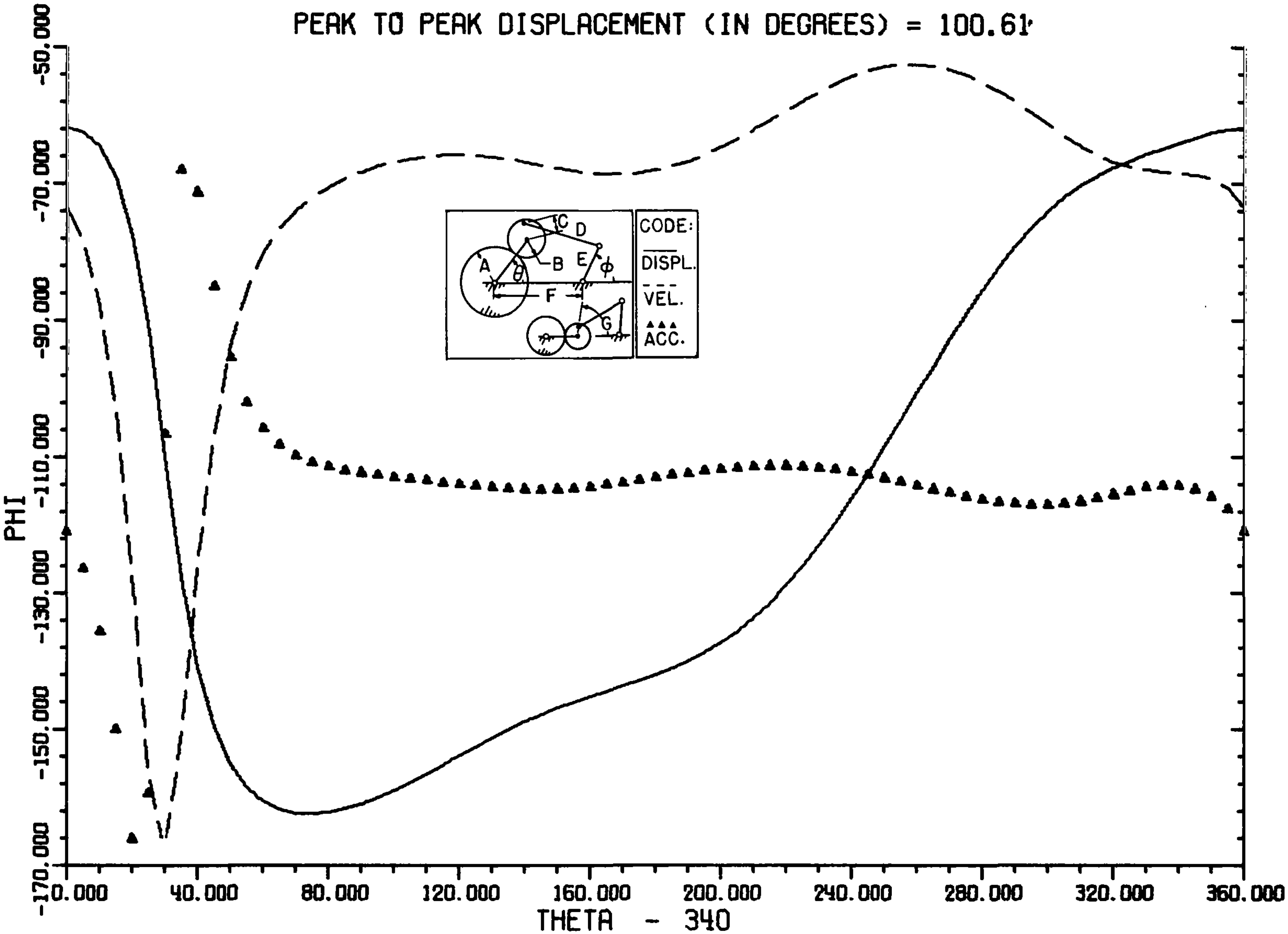


PLATE 7-23

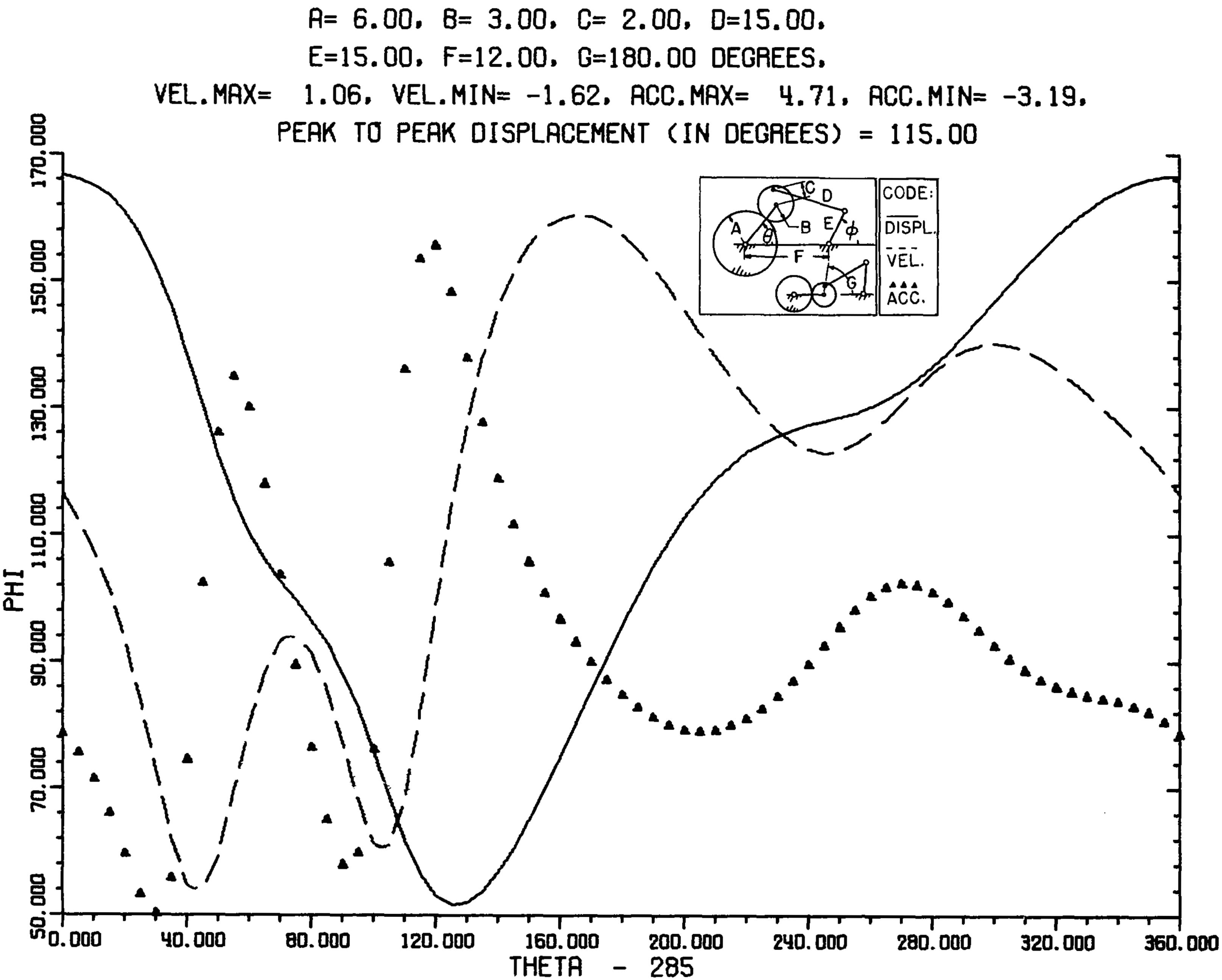


PLATE 7-24

NASA-Langley, 1966